Algorithms Group **Summer '23**
Departement of Computer Science - IBR
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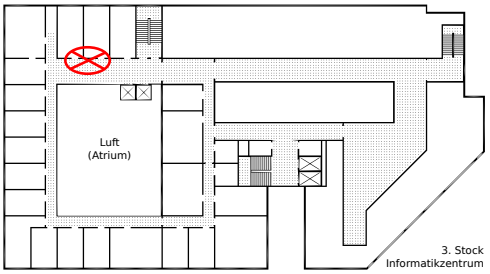
Approximation Algorithms: Homework 4 (21. June)

Solutions are due on July, 5th until 11:00 AM in the cupboard for handing in practice sheets. Please put your name on all pages.



Exercise 1 (Set Cover):
Consider the minimum set cover problem and its integer programming formulation discussed in the lecture last week. Use complementary slackness condition and prove that the following rounding based approximation algorithm has factor f, the maximum frequency among all elements.

Algorithm 1: Set Cover Rounding

Data: $\mathcal{E}, \mathcal{S}$
$x^* \leftarrow$ optimal LP solution , $C \leftarrow \emptyset$;
forall $S \in \mathcal{E}$ **do**
  if $x^*_S > 0$ **then**
    $C \leftarrow C \cup \{S\}$;
return C;

Note that, here we put **all** sets with positive LP value in the cover. **(20 P.)**

Exercise 2 (Tight example):
Consider the primal dual algorithm we discussed for the minimum weighted set cover problem. Find a example showing that the approximation factor we found is tight. **(20 P.)**

Exercise 3 (Integrality Gap):
Consider again the minimum set cover problem over the instance $(\mathcal{E}, \mathcal{S})$ and the integer linear programming formulation we used in the lecture last week. Prove that the integrality Gap of set cover could be of size $\Omega(log\ n)$, where n is the number of elements in the universe $\mathcal{E}$. **(20 P.)**