1) We modify the proof from the lecture.
We keep the subdivision into $m \times m$ – supertiles, building a grid anchored on a point (i,j), $(i,j) \in \{0,\dots,m-1\}^2$. We consider all possible choices for (i,j).

We compute an optimal solution for every supertile S in every possible choice for (i,j).
We then combine the solutions for each supertile into a global solution $Tile^m_{i,j}$.
We consider all possible choices of (i,j):

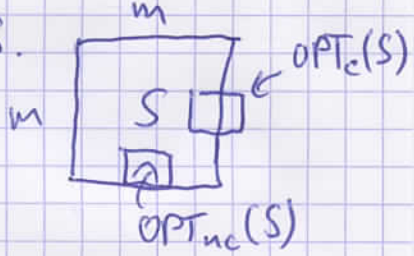
$$Tile^m = \min_{i,j} \{Tile^m_{i,j}\}.$$

$$\rightsquigarrow m^2 Tile^m \le \sum_{i,j=0}^{m-1} Tile^m_{i,j}.$$

Subdivide OPT into non-crossing and crossing objects: $OPT = OPT_{nc} + OPT_c$.

For each supertile S, $OPT_{nc}(S)$ are the covering objects contained in S in OPT, and $OPT_c(S)$ are intersected by the boundary of S.
For each S, we have

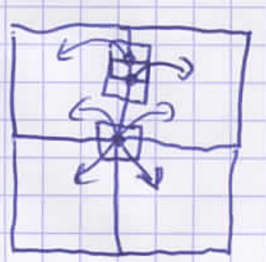
$$Tile^m_{i,j}(S) \le OPT_{nc}(S) + OPT_c(S).$$



$$Tile^m_{i,j} = \sum_S Tile^m_{i,j}(S) \le OPT + 4 \cdot Cut^m_{i,j}$$

$$\Leftrightarrow Tile^m_{i,j} - 4\, Cut^m_{i,j} \le OPT$$

lecture: $Cut^m_{i,j} = (2m-1) \cdot OPT$.



$$m^2 \cdot Tile^m - 4(2m-1)OPT \le \sum_{i,j=0}^{m-1} Tile^m_{i,j} - 4 \sum_{i,j=0}^{m} Cut^m_{i,j} \le m^2 \cdot OPT$$

$$Tile^m - \frac{8m-4}{m^2}\cdot OPT \le OPT$$

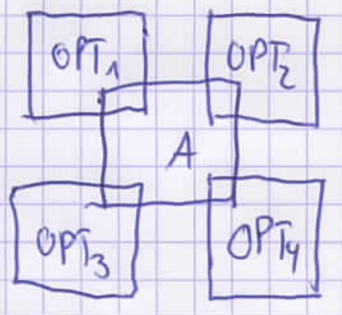
$$\iff Tile^m \le OPT\cdot\left(1+\frac{8}{m}-\frac{4}{m^2}\right) \le \cancel{OPT\left(1+\frac{8}{m}\right)}\ OPT\left(1+\frac{8}{m}\right)$$

For $1+\varepsilon$-Approximation: $m = \frac{8}{\varepsilon}$

$\rightsquigarrow$ PTAS for covering a polyomino with 2×2-squares.

2) For contradiction, assume our algorithm A computes a solution $|A| < \frac{OPT}{4}$.

Our algorithm produces a solution that is inclusion-wise maximal; every square in OPT must be ~~contained~~ intersected by a square in our solution A.



Every square in A can intersect at most 4 squares of OPT!

$\rightsquigarrow$ Contradiction to our assumption $|A| < \frac{OPT}{4}$.