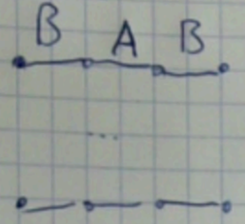
# Approximation algorithms

## Solution to HW 2



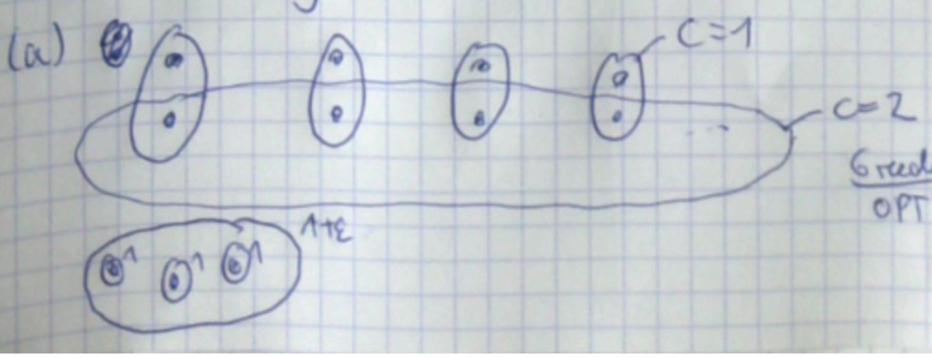

Factor between smallest and largest maximal matching can be 2

1) (a) Towards a contradiction, assume $A$ and $B$ are maximal matchings of a graph $G$, and $|A| < |B|/2$.
Every edge in $B$ has a matched endpoint in $A$, otherwise $A$ cannot be maximal.
$B$ has exactly $2|B|$ different endpoints.
Any edge in $A$ can cover at most two endpoints in $B$; we need to cover at least $|B|$ endpoints in $A$.
$\rightsquigarrow$ Contradiction to $|A| < |B|/2$, because any edge in $A$ can cover at most 2 endpoints in $B$.

(b) Compute a maximal matching (e.g., using a Greedy $O(n+m)$-algorithm).
Since by (a), any maximal matching is at most twice as large as the smallest maximal matching, this is a 2-approximation algorithm.

2) Set Packing

(a)

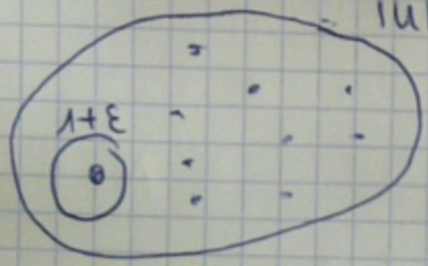



Greedy: 2

OPT: $\frac{m}{2}$

$$\frac{\text{Greedy}}{\text{OPT}} = \frac{2}{m/2} = \frac{4}{m} = \Omega\left(\frac{1}{m}\right)$$

(b)




$|U| = m = c$

Greedy: $1+\varepsilon$ $\rightsquigarrow$ $\Omega\left(\frac{1}{m}\right)$
OPT: $m$

(c) Reduction: IS $\to$ Set Packing

$(G=(V,E), k) \longmapsto$

Universe $U := E$

Set family $\mathcal{S} := \{ S_i = \{ e \mid e \text{ incident to } v_i \} \mid v_i \in V \}$, $c(S_i) = 1$

$\ell := k$ (the total weight we want to reach).

Observation: $S_i \cap S_j = \emptyset$ iff $v_i, v_j$ are not adjacent

$\Rightarrow$ $S \subseteq \mathcal{S}$ is a feasible Set Packing iff $\{ v_i \mid S_i \in S \}$ is an independent set in $G$.

$\Rightarrow$ $G$ has independent set of size $k$ iff $(U, \mathcal{S})$ has a set packing of weight $k$.