Algorithms Department **Summer 2021**
Department of Computer Science - IBR
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Approximation Algorithms
Exercise 2
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Please hand in your solutions until May 19, 11:30 am by e-mail to `keldenich@ibr.cs.tu-bs.de`.

Exercise 1 (Minimum-Cardinality Maximal Matching):
In the lecture, we used a maximal matching to approximate the Vertex Cover problem. Maximality of the matching is necessary to guarantee feasibility of the resulting vertex cover. This motivates the search for the smallest possible maximal matching of a given undirected graph G.

(a) Show that, for any two maximal matchings A, B with $|A| \leq |B|$, $|B| \leq 2|A|$.
Hint: Assume $|A| < {}^{|B|}/_{2}$ and consider the number b of edges in B that are incident to a vertex matched in A.

(b) Give a 2-approximation algorithm for the problem of finding the smallest possible maximal matching.

(12+3 P.)

Exercise 2 (Set Packing):
In the lecture, you saw the Set Cover problem, where we want to cover a given universe U of elements by sets from a weighted family of subsets with minimum total weight. There is an analogous maximization problem called Set Packing, for which we are given a universe $U = \{e_1, \ldots, e_m\}$ and a family of subsets $S_i \subseteq U$ with associated weights c_i. The goal is to find a subset $\mathcal{S} \subseteq \{S_1, \ldots, S_n\}$ of the sets with maximum total weight that is pairwise disjoint, i.e., $S_i \cap S_j = \emptyset$ for all $S_i \neq S_j$ in $\mathcal{S}$.
In the lecture, we described a greedy algorithm for Set Cover. In this exercise, we consider two similar approaches to Set Packing. In each step, we add a set that maximizes

(a) c_i,

(b) $c_i/|S_i|$.

Show that neither approach works well, i.e., both approaches are at most ${}^{1}/_{\Omega(m)}$-approximation algorithms.

(c) Show that Set Packing is NP-hard by reduction from Independent Set.

(4+6+5 P.)