Algorithms Department **Summer 2021**
Department of Computer Science - IBR
TU Braunschweig

Prof. Dr. Sándor P. Fekete
Dr. Phillip Keldenich
Dominik Krupke

Approximation Algorithms
Exercise 1
April 22, 2021

Please hand in your solutions until May 5, 11:30 am by e-mail to `keldenich@ibr.cs.tu-bs.de`.

Exercise 1 (Complexity and Reductions: Independent Set):
Let $G = (V, E)$ be a graph. A set of vertices $I \subseteq V$ is called *independent* if for all $u, v \in I : \{u, v\} \notin E$. The INDEPENDENT SET PROBLEM (IS) asks for an independent set of maximum cardinality.

a) Show that C is a vertex cover of G if and only if $I = V \setminus C$ is an independent set.

b) In Lecture 1, we have shown that it is NP-complete to decide whether a given graph has a vertex cover of a given size ℓ. Prove that it is NP-complete to decide whether a given graph G has an independent set of a given size $k \in \mathbb{N}$.

(5+5 P.)

Exercise 2 (Removing Directed Cycles):
Consider the following problem P. We are given a directed graph $G = (V, A)$ without loops, i.e., without arcs $(v, v) \in A$. The goal is to find a maximum-cardinality subset $A' \subseteq A$ of directed arcs such that the (directed) subgraph $G' = (V, A')$ has no directed cycles.
Give a polynomial-time approximation algorithm for P that produces a set A' with at least $\frac{|A^*|}{2}$ arcs, where $A^* \subseteq A$ denotes an optimal solution.
Hint: Assign a unique label from $\{1, \ldots, n\}$ to each vertex and consider what kinds of arcs must be present in any directed cycle. **(10 P.)**

Exercise 3 (Diameter of Point Sets):
Let P be a set of n points in the d-dimensional Euclidean space $\mathbb{R}^d$ (assume d is constant). The *diameter* Λ of P is a pair of points $p, q \in P$ that realizes the maximum distance between any two points of P. Assuming that the distance between points can be computed in $O(1)$, the diameter of P can trivially be computed in $O(|P|^2)$ time.
Show that in $O(|P|)$ time, we can compute a 2-approximation Λ' of the diameter with $\Lambda' \leq \Lambda \leq 2 \cdot \Lambda'$.
For the special case of $d = 2$, show that we can compute a c-approximation for a factor $c < 2$ in $O(|P|)$ time. **(10 P.)**