



Online Algorithms Tutorial 1

Phillip Keldenich

Organization

Part I — Organization

Small tutorial & exercise sheets

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Small tutorial

- Held by Jannik Heroldt (j.heroldt@tu-bs.de)
- Mondays, every other week, starting from next week (23.4.2018)
- Same time & place as the big tutorial

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- Not graded
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Exercise sheets

- Will appear after the big tutorial to be handed in two weeks later
- Hand them in via the box in front of Room IZ337
- Or via E-Mail to both keldenich@ibr.cs.tu-bs.de and j.heroldt@tu-bs.de
- Please register for the mailing list!

Material & Videos

Material & Videos

Material site

- Course website: <https://www.ibr.cs.tu-bs.de/courses/ss18/oa/index.html>
- Material site: <https://www.ibr.cs.tu-bs.de/courses/ss18/oa/mat>
- Username: „online“, Password: „OA18202SkiRental“
- No script, but there are books
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Video of lectures / tutorials

- We are videotaping lectures & tutorials
- Accessible shortly after the event on the material site
- If you do not wish to be in the video: sit behind the camera

Mailing list

<https://mail.ibr.cs.tu-bs.de/mailman/listinfo/oa>

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Exercise sheets

- You hand in your solutions to the exercises
- They will be graded individually
- To pass the module, you need 50% of the points

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Exam

- Oral or written exam at the end
- You do not need 50% of the exercise points to take the exam
- Grade only depends on the exam

Organization

Part II — Introduction

Recapitulation

Recapitulation

Online algorithm

- Informally: Algorithm that works with incomplete knowledge
- What about a formal definition?

➡ See board.

Motivation & Discussion

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Often criticized: focus on the worst case

- We only look at the worst case — is that sensible?
- May restrict our view to *unlikely* inputs
- So, average case analysis — but what is a *likely* input?
- Do we play against an adversary in the real world?
- In security contexts, we might!

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Alternative concepts

- Randomized online algorithms (average case analysis)
- Other, non-standard scenarios (more later)

Ski Rental

Part III — Ski Rental

The BahnCard Problem

The BahnCard Problem

BahnCard Problem

- Generalization of Ski Rental
- Buying only reduces cost by a factor (e.g., 0.5 for BahnCard 50)
- Only lasts for a finite time (e.g., a year)
- Different costs for different travels

The BahnCard Problem

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Lemma 3.2: The optimal solution never buys a BahnCard if it still owns one.

The BahnCard Problem — Offline

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Question: Is the following offline algorithm optimal?

- For every request (t_i, c_i) , check whether we have a valid BahnCard
- If yes, simply buy the reduced ticket
- Otherwise, buy a BahnCard if $[t_i, t_i+T]$ is expensive

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Answer: No! Proof: See board.

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Answer: No! Proof: See board.

Correct algorithm: Exercise sheet 1.

The BahnCard Problem — Lower bound

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Algorithm SUM

- For a request (t_i, p_i) , buy BahnCard iff
 - we do not own one,
 - the cost of all *regular* requests in $(t_i - T, t_i]$ is at least the critical cost c^*

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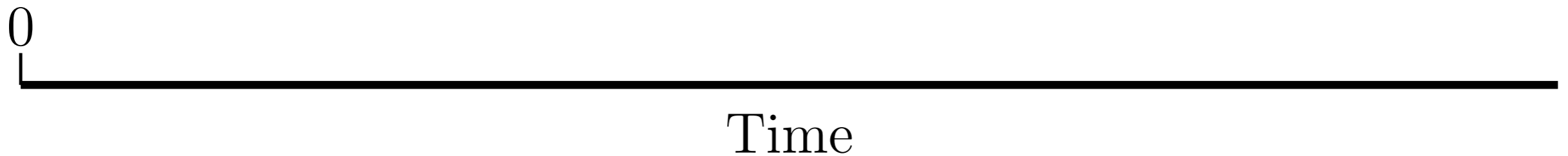
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Question: Competitive ratio?

Theorem 3.4: SUM is $(2 - \beta)$ -competitive.

The BahnCard Problem — Online



The BahnCard Problem — Online

Proof:

- Decomposition into phases $[0, \tau_1)$, $[\tau_1, \tau_2)$, \dots , $[\tau_k, \infty)$
- τ_i is when OPT buys its i^{th} BahnCard

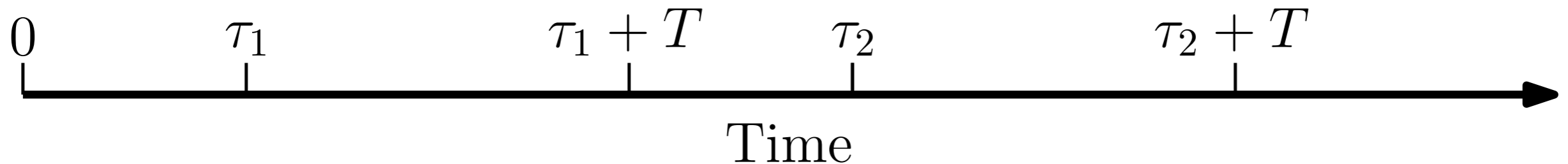
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Time

The BahnCard Problem — Online

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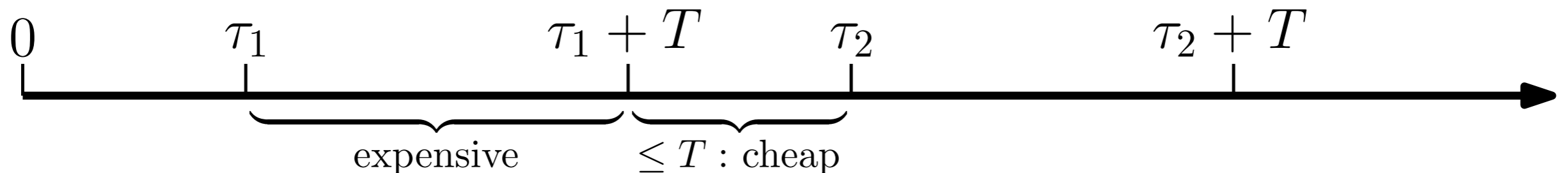
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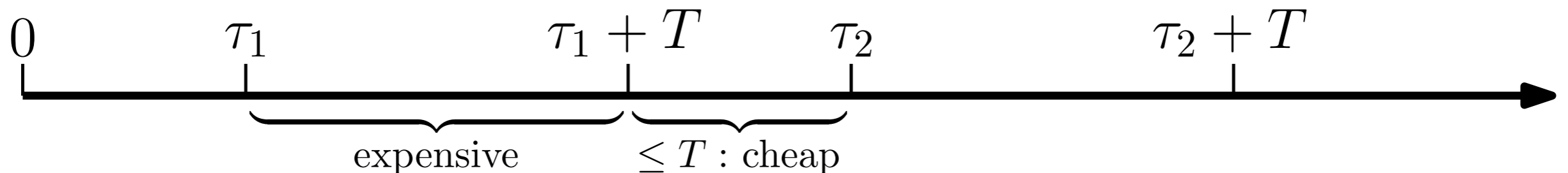
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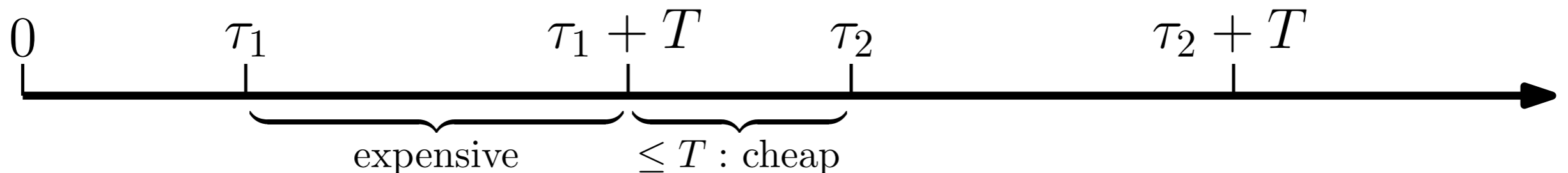
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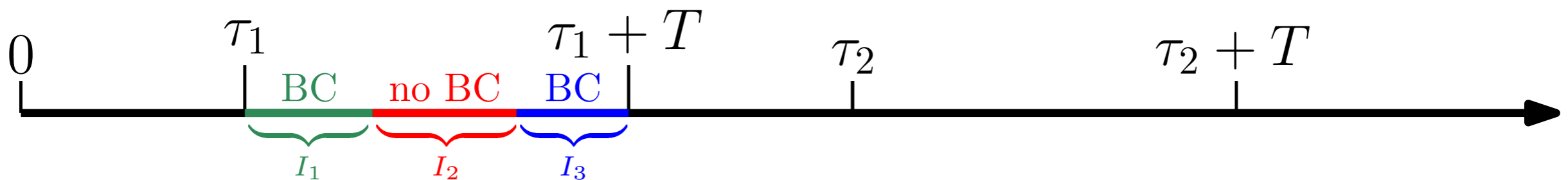
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➔ Details: Exercise sheet 1.

