Abteilung Algorithmik **Summer term 2016**
Institut für Betriebssysteme und Rechnerverbund
TU Braunschweig

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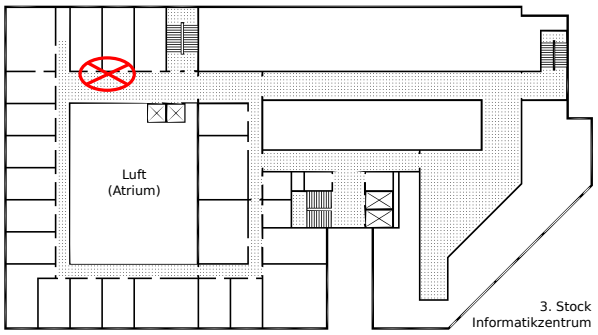
Online Algorithms
5th Homework Assignment, 22nd of June, 2016

Solutions are due on Wednesday, the 6th of July, 2016, until 11:30 in the homework cupboard. You can also hand in your solution in person before the small tutorial begins. External students can hand in the homework via e-mail to `c.rieck@tu-bs.de`.

Please make sure your homework submissions are clearly labeled using your name and/or matriculation number.



Exercise 1 (Visibility in Polygons): In the big tutorial, we made use of the following lemma: If a tour $G \subseteq P$ sees every point on the boundary δP of a simple polygon P, i.e. $\delta P \subseteq \bigcup_{g \in G} \text{Vis}(g)$, then G sees the entire polygon P, i.e. $P = \bigcup_{g \in G} \text{Vis}(g)$.

a) Prove that the lemma holds in every simple polygon P.

b) Prove that simplicity is indeed necessary, i.e. find a non-simple polygon Q and a corresponding tour $G \subseteq Q$ that sees the boundary of Q, including the boundary of all holes, but does not see some point w in the interior of Q.

(10+10 points)

Exercise 2 (Online Bin Covering): Consider the problem Online Bin Covering. You are given a sequence of items $w_i \in (0, 1)$ and an infinite number of bins with unlimited capacity. Your goal is to pack the items into the bins, maximizing the number of bins that are *covered*, i.e. that receive items of total weight at least 1. Formally, you have to construct a mapping $p : \{1, \dots, n\} \to \mathbb{N}, i \mapsto p(i)$, maximizing $|\{j \in \mathbb{N} \mid \sum_{p(i)=j} w_i \geq 1\}|$, with $p(i)$ solely depending on $w_1, \dots, w_i$.

a) Find a 2-competitive online algorithm and prove its competitiveness.

b) Prove that there cannot be an online algorithm with a competitive ratio $r < 2$.

(10+10 points)

Exercise 3 (Greedy online rectilinear watchman): In the big tutorial, we presented the online algorithm GREEDY ONLINE for computing a rectilinear watchman route in a simple rectilinear polygon P, starting from a given point $s \in P$ called the *entry* of the polygon. The algorithm keeps the following internal state:

- z, the current position ($z_0 = s$),
- M, the area that is visible by the tour so far ($M_0 = \text{Vis}(s)$),
- f, the *frontier* point, which is the right endpoint of
- C, the maximal *continuous* part of visible boundary in clockwise direction starting from s,
- b, the *blocking corner* which is undefined iff $M = P$ or f is an endpoint of a polygon edge, and
- E, the necessary extension that we travel to before the next step of the algorithm.

In each step, the algorithm determines the part of the boundary in clockwise order that is not already covered and an associated necessary extension. It distinguishes two cases: In the first case, the frontier f_i in the ith step is a polygon vertex. In that case, the algorithm travels to the extension of $F(f_i)$, which is the part of the boundary after f_i in the clockwise order. In the second case, the frontier f_i is is the interior of an edge. In that case, there is a blocking corner b_i that blocks the view onto that edge, and the algorithm travels to the extension of $B(b_i)$, which is the part of the boundary before b_i in the clockwise order.

Apply the algorithm to the polygon P given in figure 1. You may either create a sufficiently accurate copy of P by hand, or hand in your solution on a printed-out version of the exercise sheet, or hand in your solution electronically. For each step, include at least f_i, E_i, z_i and b_i if it exists.

```
z_0 ← s
M_0 ← Vis(z_0)
Initialize C_0, f_0
i ← 0
while M ≠ P do
    if f_i is a vertex of P then
        E_{i+1} := Ext(F(f_i))
    else
        Find blocking corner b_i
        E_{i+1} := Ext(B(b_i))
    end if
    Travel to closest point z_{i+1} on E_{i+1}
    i ← i + 1
    Compute M_i, C_i, f_i
end while
Travel back to s
```

Algorithm 1: GREEDY ONLINE(P, s)

(20 points)

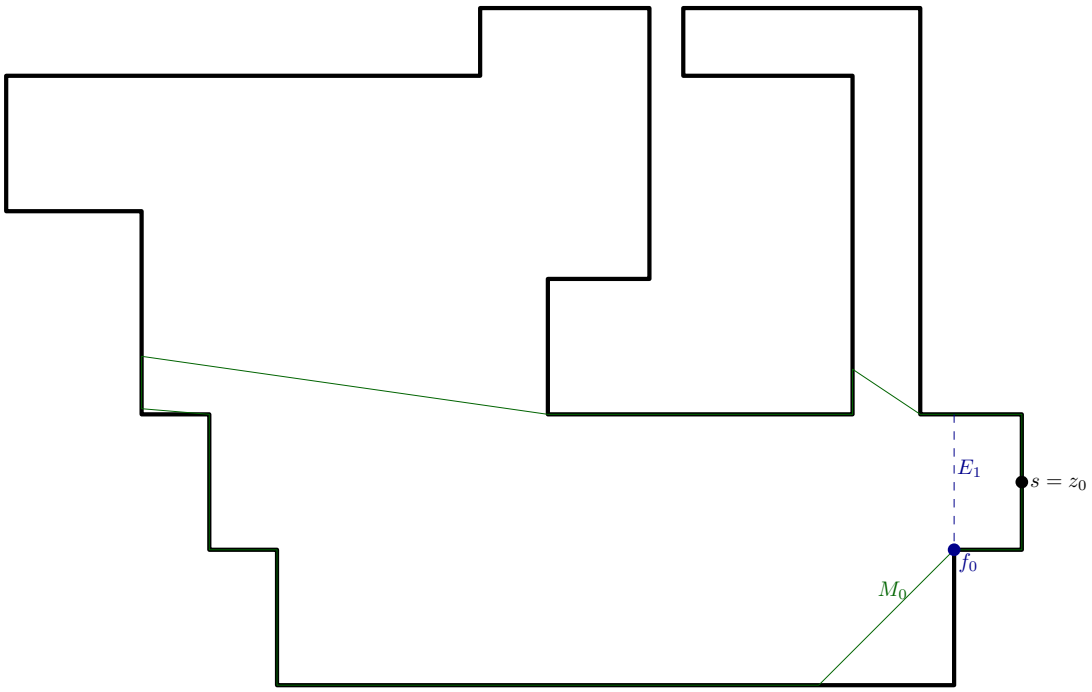


Figure 1: Polygon P