Abteilung Algorithmik
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SoSe 15

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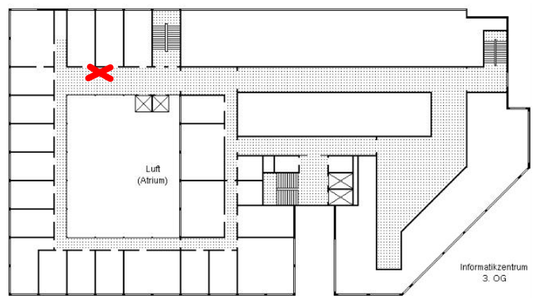
Approximation Algorithms
Homework Set 5, 29. 06. 2015

Solutions are due on Monday, July 13th, 2015, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Randomized grid shifting): Provide an analysis of the randomized version of the grid shifting technique (for packing) of Hochbaum and Maass in two dimensions. That is, instead of testing all possible grids that can be placed on top of the input, consider one chosen at random — akin to Arora's TSP. Assume you are packing 2×2-squares.
Hint: Consider a random grid of cell size m. What is the expected amount of squares that can be removed from the optimal solution due to intersections with the grid. How is this amount related to the solution output by the approximation algorithm. In the end we want to prove that the expected size of the approximation is at least $(1 - \varepsilon)OPT$, for $0 < \varepsilon \leq 1$. **(15 pts.)**

Exercise 2 (Technical lemma for Arora's PTAS): The following lemma was not proven in class. Provide its proof.
Lemma For nice Euclidean TSP instances we have that $T \leq 2 \cdot OPT$, where:

$$T = \sum_{l:vertical} t(l) + \sum_{l:horizontal} t(l)$$

and $t(l)$ is the number of times the optimal tour crosses a given line l of the unit grid imposed over the nice instance.
Hint: Observe that T *roughly* measures the length of the optimal tour under the l_1 metric. You may also use that $\sqrt{2(a^2 + b^2)} \geq a + b$. **(15 pts.)**

Exercise 3 (PTAS for Euclidean Steiner tree:) In the Euclidean Steiner Tree Problem (ESTP), we are given as input a set P of n points in the plane (called terminals). We are allowed to choose other set S of points in the plane (called non-terminals) such that the weight of the MST on $P \cup S$ is minimized. Here the interpoint distance is given by the Euclidean distance between the points. Show that Arora's PTAS for TSP can be adapted to obtain a PTAS for the ESTP. In particular, (1) show how to produce a nice instance of ESTP, just as for TSP and with the same definition. For this you have to argue about the degree of the vertices of S, and $|S|$. (2) What is different now in the dynamic program?
In this exercise we care mostly about understanding of the structural intricacies present in the ESTP, and not the running time of the algorithm for example.
Hint: Remember that for the Steiner Forest Problem there is an exact exponential algorithm. **(10 + 20 pts.)**