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SoSe 15

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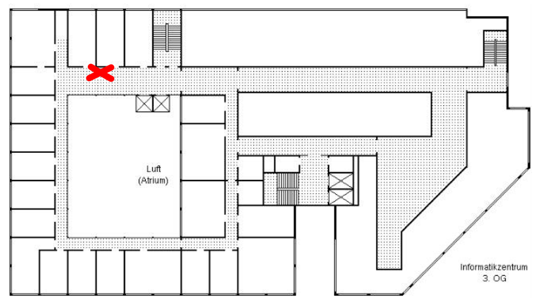
Approximation Algorithms
Homework Set 4, 15. 06. 2015

Solutions are due on Monday, June 29th, 2015, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1: The objective of this exercise is to study simultaneous approximation of problems with more than one objective function.
Consider a polyomino P, i.e., a polygon with axis-parallel edges and integer vertices. We want to solve a bicriteria optimization problem called *Covering with Travel Cost*, which can be described as follows. We are given a square-shaped scanner of size 2×2, which can be centered at any point s_i in the plane; a scan at location s_i covers all points that are covered by the scanner, i.e., that are within L_∞ distance 1 of s_i. We want to find a set $S = \{s_1, \ldots, s_n\}$ of scan points, such that:

a) S is feasible, i.e., the union of all scans at positions from S covers P.
b) The cardinality n of S is small.
c) The distance L necessary to travel S is small.

Let $n_{\min}$ be the smallest cardinality of a feasible set of scan points, and let $L_{\min}$ be the shortest possible length of a tour that visits a feasible set of scan points.

(a) Argue (based on a variant of the grid-shifting technique Hochbaum and Maas) that there is a Polynomial-Time Approximation Scheme (PTAS) for $n_{\min}$, i.e., for any fixed $\varepsilon > 0$, there is a polynomial-time algorithm that can find a feasible set S' of cardinality at most $(1+\varepsilon)n_{\min}$ for any P.
(b) Argue (based on a result of Arora or Mitchell) that there is a PTAS for finding a shortest tour of S'.
(c) Show that this will in general *not* yield a PTAS for $L_{\min}$, i.e., for *both* objectives at once.
(d) Develop an algorithm that computes a feasible set S'' and a tour of S'' of length L'', such that $|S''| \leq 2.5 n_{\min}$ and $L'' \leq 2.5 L_{\min}$.

Hint: For (a), briefly sketch how to modify the grid shifting technique, such that it works for covering instead of packing. For (c) and (d), consider the following paper. S.P. Fekete, J.S.B. Mitchell, C. Schmidt: Minimum covering with travel cost. Journal of Combinatorial Optimization, 24(1): 32-51 (2012), for which a free PDF can be found at http://arxiv.org/pdf/1101.2360v1.pdf. **(2+1+2+15 pts.)**

Exercise 2 (k-edge-coloring of bipartite graphs): The k-edge-coloring problem is in general an NP-complete problem. For certain particular classes of graphs, however, this problem can be solved efficiently. Design an algorithm that, in polynomial time, produces a $\Delta(G)$-edge coloring of a given bipartite graph G.
Hint: Remember that for an r-regular bipartite graph, a *perfect* matching *always* exists and can be found in polynomial time. **(20 pts.)**

Exercise 3: Consider a complete undirected graph G in which all edges have length either 1 or 2 (Observe that G satisfies the triangle inequality). Give a 4/3-approximation for TSP in this particular kind of graphs.
Hint: Start with a minimum 2-matching in G. A 2-matching is a subset M_2 of edges so that every vertex in G is incident to exactly two edges in M_2. A 2-matching can be computed in polynomial time. **(20 pts.)**