Abteilung Algorithmik
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SoSe 15

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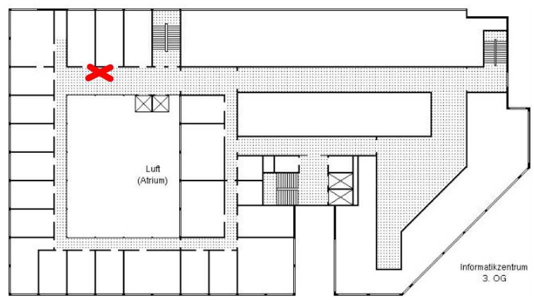
Approximation Algorithms
Homework Set 1, 28. 04. 2015

Solutions are due on Monday, May 11th, 2015, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Independent Set): Let $G = (V, E)$ be a graph. A set of vertices $I \subseteq V$ is called *independent* if for all $u, v \in I$: $\{u, v\} \notin E$. The Independent Set Problem (IS) asks for an independent set of maximum cardinality. (1) Show that C is a Vertex Cover of G iff $I = V \setminus C$ is an independent set. (2) Prove that IS is NP-Complete. **5+5 pts.**

Exercise 2 (Vertex Cover): We have seen in the lecture that the (minimum) Vertex Cover Problem (VC) is in general NP-complete. Show however that, when the input graph is a tree, VC can be solved in polynomial time. **10 pts.**

Exercise 3 (Vertex Cover): We consider two greedy algorithms for the Vertex Cover problem in a graph $G = (V, E)$:

Greedy 1:
$C := \emptyset$
while $E \neq \emptyset$ **do**
  Choose an edge $e \in E$ and choose a vertex v of e.
  $C := C \cup \{v\}$
  $E := E \setminus \{e \in E : v \in e\}$
end
return C

Show that for both algorithm a constant approximation factor cannot be guaranteed, not even in bipartite graphs. **15+15 pts.**

Greedy 2:
$C := \emptyset$
while $E \neq \emptyset$ **do**
  Choose a vertex with maximal degree in the *current* graph.
  $C := C \cup \{v\}$
  $E := E \setminus \{e \in E : v \in e\}$
end
return C

Exercise 4 (Diameter of Sets of Points): Let P be a set of n points in $\mathbb{R}^d$ (assume d is constant). The *diameter* (Λ) of P is a pair of points $p, q \in P$ that realizes the maximum distance between any two points of P (two points that are furthest apart). The diameter of P can trivially be computed in $O(n^2)$ time (assuming that the distance between points can be computed in $O(1)$). However, show that in $O(n)$ time a 2-approximation of the diameter can be computed. That is, a number Λ' such that: $\Lambda' \leq \Lambda \leq 2 \cdot \Lambda'$. **10pts.**