Abteilung Algorithmik
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Online-Algorithms
2nd Homework Assignment, 30. April 2013

Due on 15. May 2013 until 13:00 in the box in front of IZ 338
Don't forget to label each sheet with your name!

Exercise 1 (The Bahncard Problem II):
The algorithm SUM tends to be pessimistic about the future: it buys at the latest possible time, namely after it has seen enough regular requests to know for sure that OPT would already have bought a Bahncard.
Let us try an optimistic variant: OSUM.
OSUM buys a Bahncard at a regular request (t, p) iff

$$p \geq \frac{C - s(1-\beta)}{2(1-\beta)}, \quad (1)$$

where s is the regular T-cost at t^-, so $s = rr^{\sigma}_{\text{SUM}}(t^-)$. With t^- we denote the fact that we do not want the current request at time t to be included in the summation when computing $rr^{\sigma}_{\text{SUM}}(t)$

OSUM will never buy its ith Bahncard later than SUM, but often it will buy earlier.

(a) For the BC(240 Euro, 1/2, 1 year) problem, consider the request sequence (June 22, 250 Euro), (June 26, 100 Euro), (July 17, 50 Euro), (July 31, 200 Euro). How would SUM and OSUM buy tickets and Bahncards?

(b) Show OSUM is $(2-\beta)$-competitive for BP(C, β, T).

Hint: In general, we like to use the idea from the proof for SUM. However, OSUM might buy more Bahncards than OPT, so we can no longer charge the cost of OSUM's Bahncards to the expensive phase. Therefore, we introduce *critical phases*. If OSUM buys a Bahncard at time t, let t' be the maximum of $t - T$ and the expiration time of OSUM's previous Bahncard.

The proof idea is the following:

- If the interval $I = (t', t]$ has a non-empty intersection with an expensive phase, then charge OSUM's cost for this Bahncard to this expensive phase.
- Otherwise: I is called a *critical phase* and we charge OSUM's cost for this Bahncard to this critical phase.

(5+10 points)

Exercise 2 (Randomized Algorithms):

a) For some problem P, two online algorithms A_1 and A_2 are given, with a competitve ratio of 2 and 3, respectively. Design a randomized online algorithm with competitive ratio 9/4.

b) During the last lab we presented the RSUM algorithm for the Bahncard Problem. What is the competitive factor of this algorithm for the real-world Bahncard Problem?

c) For which β has RSUM a better competitive factor than SUM?

d) Consider a randomized variant of OSUM, R-OSUM, which, with probability $q = \frac{1}{1+\beta}$, buys a Bahncard at time t iff OSUM would buy one at time t. Give a competitive factor for R-OSUM and prove this factor.

(5 + 5 + 5 + 10 points)

Exercise 3 (k-Server Problem):
Consider the k-server problem: The algorithm can move k servers in space; in the beginning they are placed on fixed points of a set M in space.
Given is a sequence of requests $\sigma = r_1, r_2, \ldots, r_n$, where each request corresponds to a point in the plane. A request r_i is considered *served*, when a server has reached the point r_i.
The algorithm has to serve the requests in the given order by moving the servers. The cost of the algorithm is the sum of all distances that the servers have to move (according to some specified metric).

Show that a greedy strategy for the k-server problem is not necessarily competitive. (A greedy strategy chooses the cheapest possibility, i.e., it moves the server that is closest to the request.)

Hint: Consider an example with $k = 2$ servers and an infinite sequence on 3 well-chosen request points.

(20 points)