Abteilung Algorithmik
Institut für Betriebssysteme
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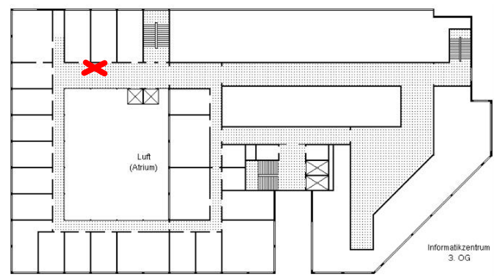
SS 12

Approximation Algorithms
Homework Set 3, 23. 05. 2012

Solutions are due Wednesday, June 13th, 2012, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (First-Fit-Decreasing for Bin Packing):
Show that the First-Fit-Decreasing Algorithm for Bin Packing presented in the tutorial on May 24, has an approximation factor of 3/2.

(15 points)

Exercise 2 (Greedy Set Cover Algorithm):
Apply the Greedy Set Cover Algorithm (Algorithm 2.3 from the lecture) to the following Set Cover instance:
$c(S_i) = |S_i| + 1$, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20\}$, and
$S_1 = \{1, 2, 3, 4\}$
$S_2 = \{5, 6, 7, 8\}$
$S_3 = \{9, 10, 11, 12\}$
$S_4 = \{13, 14, 15, 16\}$
$S_5 = \{17, 18, 19, 20\}$
$S_6 = \{1, 2, 3, 5, 6, 7, 9, 10, 11\}$
$S_7 = \{14, 15, 16, 17, 18, 19\}$
$S_8 = \{12, 13, 14, 15\}$
$S_9 = \{4, 5, 6\}$
$S_{10} = \{7, 8, 9\}$
$S_{11} = \{18, 19, 20\}$.

In case the maximum in step 2 is not uniquely defined, choose set S_i with minimum index.
What is the value of the computed set cover?
Can you give a better set cover?

(15 points)

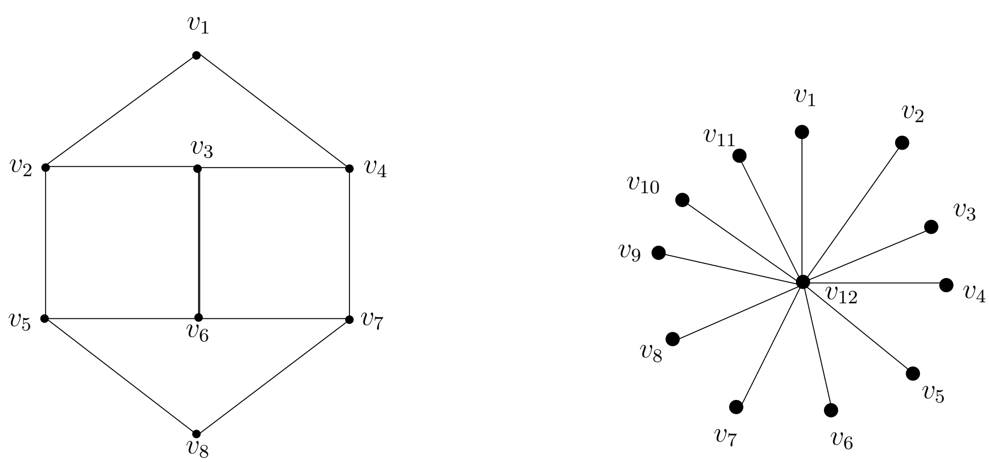


Abbildung 1: Right: G_1, Left: G_2.

Exercise 3 (2-Approximation for Vertex Cover):
Apply the 2-approximation algorithm for Vertex Cover (lecture, 1.12) to the two example graphs, G_1 and G_2, from Figure 1. What is the optimal size of a vertex cover?
(10 points)

Exercise 4 (Weighted Vertex Cover):
For the Weighted Vertex Cover Problem, we are given a graph $G = (V, E)$, each vertex is given a weight: $w : V \to \mathbb{R}^+$. We ask for a vertex cover C, such that the total weight of the cover is minimized: $\min \sum_{v \in C} w(v)$.
Consider the following algorithm for the weighted vertex cover problem: For each vertex v, $t(v)$ is initialized to its weight, and when $t(v)$ drops to 0, v is picked in the cover. $c(e)$ is the amount charged to edge e.

Algorithm

1. Initialization:
   - $C = \emptyset$
   - $\forall v \in V : t(v) = w(v)$
   - $\forall e \in E : c(e) = 0$
2. While C is no a vertex cover, do:
   - Pick an uncovered edge, say (u, v). Let $m = \min(t(u), t(v))$
   - $t(u) = t(u) - m$

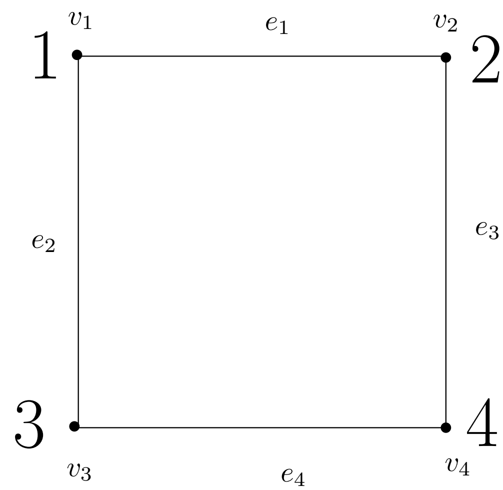


Abbildung 2: A graph for the Weighted Vertex Cover Problem. The large numbers define the weights, that is, $w(v_i) = i$.

- $t(v) = t(v) - m$
- $c(u, v) = m$
- Include in C all vertices having $t(v) = 0$.

3. Output C.

(a) Apply the algorithm to the example graph given in Figure 2; in case you can pick more than one uncovered edge, choose the edge with minimal index.

(b) Show that this is a 2-approximation algorithm. Hint: Show that the total amount charged to edges is a lower bound on OPT and that the weight of cover C is at most twice the total amount charged to edges.

(5+15 points)