Abteilung Algorithmik
Institut für Betriebssysteme
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SS 12

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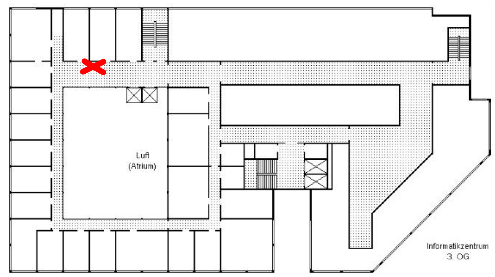
Approximation Algorithms
Homework Set 2, 09. 05. 2012

Solutions are due Wednesday, May 23th, 2012, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Minimum-Degree Spanning Trees):
Consider the local search algorithm for finding a minimum-degree spanning tree as presented in the tutorial on May 10. Complete the proof of Theorem 3: The algorithm finds a locally optimal tree in polynomial time.
For this consider the potential function $\Phi(T)$: $\Phi(T) = \sum_{v \in V} 3^{d_T(v)}$ for a tree T.
We concluded that $\Phi(T) \leq n3^{\Delta(T)}$, hence, the initial potential is at most $n3^n$. In addition, a Hamiltonian path has the lowest possible potential: $2 \cdot 3 + (n-2)3^2 > n$.

(a) Show that for each move, the potential function of the resulting tree is at most $1 - \frac{2}{27n^3}$ times the potential function previously.

(b) Consider the situation after $\frac{27}{2}n^4 \ln 3$ local moves and conclude that we obtained a locally optimal tree.

(10+10 Punkte)

Exercise 2 (Vertex Cover in Trees):

(a) Give a polynomial-time algorithm for the Vertex Cover Problem when only trees are used as an input. (Hint: What statement is possible on the leafs of the trees and their participation in an optimal Vertex Cover?)

(b) Let T_n be the complete binary tree of depth n (that is, on each path from the root r to an arbitrary leaf we have n nodes). Show: for n odd the root is never included in an optimal Vertex Cover.

(10+10 Punkte)

Exercise 3 (Independent Set Problem):
A set of vertices $U \subseteq V$ is called *independent*, if for all $u, v \in U$: $\{u, v\} \notin E$. For the Independent Set Problem we ask for an independent set of maximum cardinality.
Show: C is a Vertex Cover of $G = (V, E)$ iff $U = V \setminus C$ is an independent set.
Moreover, show: C is an optimal solution for the Vertex Cover Problem iff $U = V \setminus C$ is an optimal solution for the Independent Set Problem.

(10 Punkte)

Exercise 4 (NP-Completeness of the Dominating Set Problem):
Dominating Set Problem:
Instance: Graph $G = (V, E)$, positive integer $K \leq |V|$.
Question: Is there a subset $V' \subseteq V$ such that $|V'| \leq K$ and such that every vertex $v \in V \setminus V'$ is joined to at least one member of V' by an edge in E?

Vertex Cover Problem:
Instance: Graph $G = (V, E)$, positive integer $C \leq |V|$.
Question: Does G contain a vertex cover of size at most C?

Show the Dominating Set Problem to be NP-complete by reducing Vertex Cover to it.

(10 Punkte)