Abteilung Algorithmik
Institut für Betriebssysteme
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SS 12

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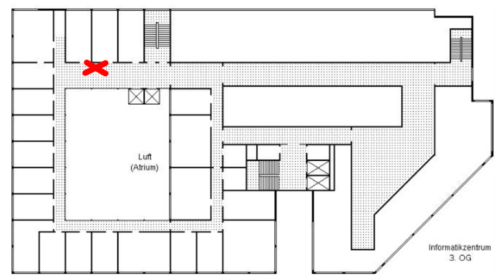
Approximation Algorithms
Homework Set 1, 25. 04. 2012

Solutions are due Wednesday, May 9th, 2012, until 13:00 in the cupboard for handing in practice sheets. **Please put your name on all pages!**



Exercise 1 (Vertex Cover):
We consider two greedy algorithms for the Vertex Cover problem in a graph $G = (V, E)$:

Greedy 1:
$C := \emptyset$
while $E \neq \emptyset$ **do**
  Choose an edge $e \in E$ and choose a vertex v of e.
  $C := C \cup \{v\}$
  $E := E \setminus \{e \in E : v \in e\}$
end
return C

Greedy 2:
$C := \emptyset$
while $E \neq \emptyset$ **do**
  Choose a vertex with maximal degree in the *current* graph.
  $C := C \cup \{v\}$
  $E := E \setminus \{e \in E : v \in e\}$
end
return C

Show that for both algorithm a constant approximation factor cannot be guaranteed, not even in bipartite graphs.

(15+15 Punkte)

Exercise 2 (The Geometric Traveling Salesman Problem II):
In the first tutorial and in homework set 0 we considered a TSP-approximation that doubles the edges of an MST of the given graph $G = (V, E)$.

Consider the output from the MST computation. This graph is not Eulerian, because any tree must have nodes of degree one. Let O be the set of odd-degree nodes in the MST. For any graph, the sum of its node degrees must be even, because each edge in the graph contributes 2 to this total. The total degree of the even-degree nodes must also be even, but then the total of degree of the odd-degree nodes must also be even. In other words, we must have an even number of odd degree nodes; $|O| = 2k$ for some positive integer k.
Suppose that we pair up the nodes in O: $(i_1, i_2), (i_3, i_4), \ldots, (i_{2k-1}, i_{2k})$. Such a collection of edges that contain each node in O exactly once is called a *perfect matching* (as known from Network Algorithms!!) of O. One of the classic results of combinatorial optimization is that given a complete graph (on an even number of nodes) with edge costs, it is possible to compute the perfect matching of minimum total cost in polynomial time.
Given the MST, we identify the set O of odd-degree nodes with even cardinality, and then compute a minimum-cost perfect matching on O. If we add this set of edges to our MST, we have constructed an Eulerian graph on our original set of cities: it is connected (because the spanning tree is connected) and has even degree (because we added a new edge incident to each node of odd degree in the spanning tree). We can now shortcut this graph to produce a tour of no greater cost.
Prove that the given algorithm for the metric Traveling Salesman Problem is a $\frac{3}{2}$-approximation algorithm.

(20 Punkte)

Exercise 3 (Input complexity): Give the input complexities for the problems TSP, Hamiltonian Circuit and Vertex Cover.

(10 Punkte)