

## Online-Algorithms 4th Homework Assignment, 30. May 2011

Due on 27. June 2011 until 13:00 in the box opposite IZ 252 or in PK 11.1  
Don't forget to label each sheet with your name!

### Exercise 1 (Rendezvous on a Line):

Assume that there are two players on a road, with a (normalized) distance of 2 units between them. Assume furthermore that they cannot agree on some search strategy beforehand. Every person can move to the left or right with a speed of one distance unit per unit of time. Neither player knows in which direction the other player can be found. The goal is to rendezvous in the shortest amount of time.

Consider the strategy "1F2B": In every round each player chooses one of the directions left or right with 50% probability (and independently of the other player). This direction will be considered *forward*, "F", for the current round. Then the player moves one distance unit in the forward direction and two distance units in the other direction (*backward*, "B").

- In which positions can the two players be after the first round?
- What is the expected time until the rendezvous? You must prove your claim!

(10+20 points)

### Exercise 2 (Search on a Line):

*"Imagine a hungry cow, living on a line. If she knew that her hay is to the left, she could simply go there. But as she doesn't, her best strategy is to walk back and forth—each time doubling her search-distance—until she finds the hay."*

Assume the initial distance to the hay is at least one. The cow first walks one distance unit in one direction, then turns around and walks back to the start. From there she walks two distance units in the other direction, then back to the start, etc.

Prove that this strategy cannot have a competitive factor smaller than 9.

**Hint:** Use an example in which the distance of the hay to the starting point is slightly more than some square factor.

(30 points)