Abteilung Algorithmik
Institut für Betriebssysteme und Rechnerverbund **SS 11**
TU Braunschweig

Prof. Dr. Sándor Fekete
Dr. Iris Reinbacher

Online-Algorithms
3rd Homework Assignment, 16. May 20111

Due on 30. May 2011 until 13:00 in the box opposite IZ 252 or in PK 11.1
Don't forget to label each sheet with your name!

Exercise 1 (Heuristic for Robot Navigation):
Consider the robot navigation problem "looking around a corner", where we want to find the length of each step on the trajectory for the robot. The robot follows a polygonal path which is inscribed in a semicircle of diameter d (spanned by the starting point of the trajectory and the corner). We want to determine the points where the robot will stop to perform the scan. For this, we can use the following equation presented in the lecture:

$$x_{i+1} = c \cdot (1 + d_i) - (1 + i) - \sum_{j=1}^{i} x_j. \tag{1}$$

In the jth step the robot moves along a chord of length x_j. This chord is visible from the corner at an angle of $\varphi_j = \arcsin \frac{x_j}{d}$. The chord connecting the starting point with the ith position of the robot has a length of

$$d_i = d \cdot \sin \sum_{j=1}^{i} \varphi_j. \tag{2}$$

Using the equation 2 for d_i in equation 1, the result is a recursion for the step length. For a given $c > 1$, the first value is chosen as $x_1 = c - 1$. As soon as the computed length of the last step is smaller than zero, i.e., $x_j < 0$, the computation can be aborted.

a) Why is it necessary that the robot moves all the way to the corner?

b) Why is the strategy *"move directly to the corner"* not competitive?

c) How is it possible to decide whether the strategy with the computed step length will arrive at the other side of the semicircle?

d) Consider a diameter of $d = 40$ and for the constant c the two values 2.0015 and 2.0016.
Is it possible to achieve a competive factor of c in either case?

For your convenience, we provide the following intermediate results:
For $c = 2.0015$ we have $\sum_{j=1}^{19} x_j = 61.7568$ and $\sum_{j=1}^{19} \varphi_j = 1.5541$.
For $c = 2.0016$ we have $\sum_{j=1}^{18} x_j = 61.2617$ and $\sum_{j=1}^{18} \varphi_j = 1.5432$.

(5+10+10+35 points)