Abteilung Algorithmik
Institut für Betriebssysteme und Rechnerverbund **SS 11**
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Online-Algorithms
1st Homework Assignment, 21. April 2011

Due on 2. May 2011 until 13:00 in the box in IZ 262 or in PK 11.1
Don't forget to label each sheet with your name!

Exercise 1 (Ski Rental Problem):
Let R be the ratio of the purchase price p and the lending fee ℓ ($R = \frac{p}{\ell}$).

a) Design a strategy that reaches a competitive ratio of $2 - \frac{1}{R}$. Prove the correctness of your strategy.

b) In the following, let $p = 100, \ell = 20$.

i) What is the competitive ratio of the strategy *rent for 3 days, buy on the 4th*?

ii) What is the competitive ratio of the optimal strategy?

iii) Design a strategy with competitive ratio 3.

(5 + 10 points)

Exercise 2 (Marking Algorithms):
Given: $k = 4$ pages, numbered 1,2,3,4, and a cache of size 3. In the beginning, the pages 1,2,3 are residing in the cache. Consider the sequence $\sigma = (41234)$ of requests.

Wanted: Compare for the marking algorithm the strategies FIFO (first in - first out) and LFU (least frequently used).

- Which strategy generates more page faults for the sequence σ given above?
- How many page faults occur for FIFO and LFU, respectively?
- For each step, indicate which pages are residing in the cache.

(15 points)

Exercise 3 (Bahncard Problem):
We presented the online algorithm SUM for the *Bahncard Problem* during the tutorials. This algorithm has a competitive ratio of $2 - \beta$. Construct a sequence σ that reaches this competitive ratio (i.e., a worst-case example).

(15 points)

Exercise 4 (Paging):
Prove the following statement:

Let ALG be any marking algorithm as presented in the lecture with a cache of size k, and let OPT be an optimal offline-algorithm with a cache of size $h \leq k$. Then ALG is $\frac{k}{k-h+1}$-competitive.

Hint: Analogous to the problem in the lecture, consider a decomposition of a sequence σ into phases of length k.

(15 points)