Collaborative transmission in wireless sensor networks

Alternative algorithmic approaches

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June 27, 2010
Overview and Structure

- Introduction to context aware computing
- Wireless sensor networks
- Wireless communications
- Basics of probability theory
- Randomised search approaches
- Cooperative transmission schemes
- Distributed adaptive beamforming
  - Feedback based approaches
  - Asymptotic bounds on the synchronisation time
  - Alternative algorithmic approaches
  - Alternative Optimisation environments
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  - Alternative Optimisation environments
Outline

Alternative beamforming approaches

1. Hierarchical clustering
2. Local random search
3. An asymptotically optimal algorithm
4. Environmental changes
   - Velocity of nodes
   - Multiple receiver nodes
   - Increased population size
   - Receive beamforming
Alternative algorithmic approaches
Hierarchical clustering

- For feedback based distributed adaptive transmit beamforming:
  - $RSS_{\text{sum}}$ changes linear with the network size $n$.
  - Bound on the synchronisation time is more than linear in $n$. 
Alternative algorithmic approaches
Hierarchical clustering

\[ E[T_P] = \Theta (n \cdot k \cdot \log(n)) \]
Alternative algorithmic approaches
Hierarchical clustering

1. Determine clusters
2. Synchronise clusters successively (with possibly increased transmit power for nodes)
3. Build and synchronise overlay-cluster of representative nodes from all clusters.
4. Nodes alter carrier phase by phase offset experienced by representative node:
   - \( \zeta_i = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t(\gamma_i + \phi_i + \psi_i)} \right) \) (before)
   - \( \zeta_i' = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t(\gamma_i' + \phi_i + \psi_i)} \right) \) (after)
5. Node \( h \) from same cluster alters carrier signal
   - \( \zeta_h = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t(\gamma_h + \phi_h + \psi_h)} \right) \) to
   - \( \zeta_h' = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t(\gamma_h + \phi_h + \psi_h + \gamma_i - \gamma_i')} \right) \)

Ideal conditions: All nodes should now in phase

Final synchronisation among all nodes
Alternative algorithmic approaches

Hierarchical clustering
Alternative algorithmic approaches
Hierarchical clustering

Potential problem: Phase noise
- Only one cluster synchronised at a time
- Due to practical properties of oscillators, phases of nodes in the inactive clusters experience phase noise and start drifting out of phase
- Sufficient synchronisation possible in the order of milliseconds

Positive:
- No inter-node communication required

Open Issue:
- More than one hierarchy stage might be optimal for optimisation time
- for energy consumption
- Optimum hierarchy depth and cluster size derived by integer programming in time $O(n^2)$
Alternative algorithmic approaches

Hierarchical clustering

Determine optimum cluster size and hierarchy depth:

- Expected optimisation time:
  \[ E[T_{P_n}] = c \cdot k \cdot n \cdot \log(n) \]

- Expected energy consumption:
  \[ E[E_{P_n}] = c \cdot k \cdot n \cdot \log(n) \cdot E_{P_n} \]

Hierarchy and cluster structure that minimises these formulae optimal
Alternative algorithmic approaches
Hierarchical clustering

Opt. cluster size and hierarchy depth (integer programming) :

- For a cluster size of $m$:

$$E[T_Pn] = E[T_{P\frac{n}{m}}] \cdot \frac{n}{m} \cdot E[T_{Pm}]$$
$$E[E_Pn] = E[E_{P\frac{n}{m}}] \cdot \frac{n}{m} \cdot E[E_{Pm}]$$

- Define recursion by

$$E_{opt}[T_Pn] = \min_m \left[ E_{opt}[T_{P\frac{n}{m}}] \cdot \frac{n}{m} \cdot E_{opt}[T_{Pm}] \right]$$
$$E_{opt}[E_Pn] = \min_m \left[ E_{opt}[E_{P\frac{n}{m}}] \cdot \frac{n}{m} \cdot E_{opt}[E_{Pm}] \right]$$

- Start of recursion ($\eta$ min feasible cluster size):

  - $E_{opt}[T_{P\eta}]$
  - $E_{opt}[E_{P\eta}]$
Alternative algorithmic approaches
Hierarchical clustering

Opt. cluster size and hierarchy depth (integer programming) :

- Time required for calculation is quadratic.
  - With a network of $n$ nodes, at most $n^2$ distinct terms
    - $E_{\text{opt}}[T_\mathcal{P}i]$
    - $E_{\text{opt}}[E_\mathcal{P}i]$
- Start calculation at
  - $E_{\text{opt}}[E_\mathcal{P}\eta]$
  - $E_{\text{opt}}[T_\mathcal{P}\eta]$
- All other values by table loop-up in time $O(n^2)$ according to
  - $E_{\text{opt}}[T_\mathcal{P}n]$
  - $E_{\text{opt}}[E_\mathcal{P}n]$ in time $O(n^2)$
Alternative algorithmic approaches
Hierarchical clustering

- Reduction of synchronisation time and transmission power
- Calculation of optimum cluster size and depth in $\mathcal{O}(n^2)$
Outline
Alternative beamforming approaches

1. Hierarchical clustering
2. Local random search
3. An asymptotically optimal algorithm
4. Environmental changes
   - Velocity of nodes
   - Multiple receiver nodes
   - Increased population size
   - Receive beamforming
Scenario analysis and algorithmic improvement

Local random search

- Global random search:
  - Synchronisation performance might deteriorate when the optimum is near
- With small local search space:
  - Majority of worse points excluded
Local random search
An upper bound on the synchronisation performance

Assumptions:

- Mutation probability: \( n^{-1} \)
- Uniform phase alteration
- Initial distance to the optimum: \( \geq \frac{n \cdot \log(k)}{2} \) (Chernoff)

Technical assumption:

- Fitness space divided in \( k \) slices of identical width
Local random search
An upper bound on the synchronisation performance

Analysis in two phases for the synchronisation process

Phase 1: Optimum outside search neighbourhood for at least one node

Phase 2: Optimum within search neighbourhood for all nodes
Local random search
An upper bound on the synchronisation performance

Phase 1: Optimum is outside the neighbourhood
- Reach search point with improved fitness: $\geq \frac{1}{2}$
Local random search
An upper bound on the synchronisation performance

When \( i \) signals synchronised:

- Improve \( n - i \) non-optimal signals
- \( i \) already optimal ones unchanged:

\[
(n - i) \cdot \frac{1}{n} \cdot \frac{1}{2} \cdot (1 - \frac{1}{n})^i
\]

\[
= \frac{n-i}{2n} \cdot (1 - \frac{1}{n})^i
\]

since \( (1 - \frac{1}{n})^n < e < (1 - \frac{1}{n})^{n-1} \)

\[
s_i \geq \frac{n - i}{2en}
\]

- Expected number of mutations to increase fitness bounded by \( s_i^{-1} \).
Local random search

An upper bound on the synchronisation performance

- Time until optimum is within the neighbourhood?
  - Constant time to leave slice
  - $k$ distinct slices

\[
E[T_P] \leq c \cdot \sum_{i=0}^{k} \frac{2en}{n-i} = 2cen \cdot \sum_{i=1}^{k+1} i^{-1} < 2cen \cdot \ln(k+1) = \mathcal{O}(n \cdot \log(k))
\]
Local random search
An upper bound on the synchronisation performance

Phase 2: Optimum within search neighbourhood
- Worst case: Increase fitness with probability $\frac{1}{N}$
- Similar to consideration above:

$$O(N \cdot n \cdot \log(k))$$

Overall synchronisation time:

$$O(N \cdot n \cdot \log(k)).$$
A lower bound on the synchronisation time:

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size $n$ by neighbourhood size $N$
Scenario analysis and algorithmic improvement

Local random search

A lower bound on the synchronisation time:

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size $n$ by neighbourhood size $N$
  - Probability to alter individual bit
    \[
    \frac{1}{N \cdot \log(k)}
    \]
  - Instead of
    \[
    \frac{1}{n \cdot \log(k)}
    \]
A lower bound on the synchronisation time:

- With similar arguments as for global random search, lower bound

\[ \Omega(N \cdot \log(k) \cdot \Delta) \]
Mathematical simulation environment

Impact of the node choice

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Node distribution area</td>
<td>(30m \times 30m)</td>
</tr>
<tr>
<td>Location of the receiver</td>
<td>((15m, 15m, 30m))</td>
</tr>
<tr>
<td>Mobility</td>
<td>stationary nodes</td>
</tr>
<tr>
<td>Base band frequency</td>
<td>(f_{\text{base}} = 2.4) (\text{GHz})</td>
</tr>
<tr>
<td>Transmission power of nodes</td>
<td>(P_{\text{tx}} = 1) (\text{mW})</td>
</tr>
<tr>
<td>Gain of the transmit antenna</td>
<td>(G_{\text{tx}} = 0) (\text{dB})</td>
</tr>
<tr>
<td>Gain of the receive antenna</td>
<td>(G_{\text{rx}} = 0) (\text{dB})</td>
</tr>
<tr>
<td>Iterations per simulations</td>
<td>6000</td>
</tr>
<tr>
<td>Identical simulation runs</td>
<td>10</td>
</tr>
<tr>
<td>Random noise power [46]</td>
<td>(-103) (\text{dBm})</td>
</tr>
<tr>
<td>Pathloss calculation ((P_{rr}))</td>
<td>(P_{\text{tx}} \left(\frac{\lambda}{2\pi d}\right)^2 G_{\text{tx}} G_{\text{rx}})</td>
</tr>
</tbody>
</table>

Fitness measure:

\[
RMSE = \sqrt{\sum_{t=0}^{\tau} \frac{\left(\sum_{i=1}^{n} s_i + s_{\text{noise}}(i) - s^*\right)^2}{n}}.
\]
Scenario analysis and algorithmic improvement

Local random search

Median fitness values (Network size: 100 nodes)

- Hillclimber, mutation probability: 0.01, Neighbourhood size: 0.6 \pi
- Uniform distribution, Mutation probability: 0.01

RMSE vs Iteration count
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Multivariable equations

Received sum signal

- Reduce the amount of randomness in the optimisation
- Improve the synchronisation performance
- Improve the synchronisation quality
Scenario analysis and algorithmic improvements

Search space

- Search space:
  - Spanned by all Configurations of carrier phase offsets $\gamma_i$
- Search point / Configuration:
  - One possible configuration of carrier phase offsets
Multivariable equations

Received sum signal

- Fitness function observed by single node
- Constant carrier phase offset for $n - 1$ nodes
- Fitness function:

$$\mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c$$
Multivariable equations

Received sum signal

Approach:
- Measure feedback at 3 points
- Solve multivariable equations
- Apply optimum phase offset calculated

\[ F(\Phi_i) = A \sin(\Phi_i + \phi) + c \]
Problem:
- Calculation not accurate when two or more nodes alter the phase of their transmit signals
An active node will:
1. Transmit with three distinct phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$ and measure feedback.
2. From these three feedback values and phase offsets, estimate feedback function and optimum phase offset $\gamma_i^*$. 
3. Transmit a fourth time with $\gamma_4 = \gamma_i^*$. 
4. If the deviation is less than 1% save $\gamma_i^*$ as optimal phase offset, otherwise discard it.

A passive node will:
1. Transmit 4 times with identical phase offset $\gamma_i$. 

Multivariable equations

Solution

- Node estimates the quality of the function estimation itself
- Transmit with optimum phase offset and measure channel again
- When Expected fitness deviates significantly from measured fitness, discard altered phase offset
- Deviation:
  1 node:  \( \approx 0.6\% \)
  2 nodes:  \( \approx 1.5\% \)
  3 nodes:  \( > 3\% \)
Multivariable equations
Synchronisation process

1. Transmit with phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$; measure feedback
2. Estimate feedback function and calculate $\gamma_i^*$
3. Transmit with $\gamma_4 = \gamma_i^*$
4. If deviation smaller 1% finished, otherwise discard $\gamma_i^*$
Multivariable equations

Received sum signal

- Asymptotic synchronisation time:

\[ O(n) \]

- Classic approach:¹

\[ \Theta(n \cdot k \cdot \log(n)) \]

¹Sigg, El Masri and Beigl, A sharp asymptotic bound for feedback based closed-loop distributed adaptive beamforming in wireless sensor networks (submitted to Transactions on Mobile Computing)
Multivariable equations
Performance estimation

Median fitness values (Network size: 100 nodes)

- Multivariable equations, uniform probability to change the phase of a carrier signal: 0.01
- Normal distributed probability to change the phase of a carrier signal: 0.01, Variance: 0.5 π

RMSE

Iteration count

0 1000 2000 3000 4000 5000 6000
Multivariable equations
Performance estimation

Relative phase shift (Network size: 100)
Phase offset of distinct nodes is within $\pm 0.05\pi$ for up to 99% of all nodes.
Asymptotically optimal synchronisation time

Simulations: $\approx 12n$

Further improvement:
- 3 iterations per turn
- Utilise last transmission from previous iteration
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Environmental changes

Introduction

- Velocity of nodes
- Multiple receiver nodes
- Increased population size
- Receive beamforming
Environmental changes

Velocity of nodes

Moving receiver:
- Straight line
- Random walk

Moving transmitter:
- Straight line
- Random walk
Environmental changes
Velocity of nodes

Moving receiver:
- Straight line
- Random walk

Aspects:
- Only one moving node
- Simple case
- Also applicable when all transmitters move identically
Environmental changes

Velocity of nodes

Moving transmit nodes:
- Straight line
- Random walk

Aspects:
- Multiple nodes moving
- Hard case
Environmental changes

Velocity of nodes

Phase offset deviation from the optimal phase offset (Number of nodes: 100)

Random walk – receiver:

- Maximum velocity for classic algorithm: 5m/sec
Environmental changes
Velocity of nodes

Random walk – receiver:
- Max. velocity for Multivariable equations: 5m/sec easily supported
Environmental changes

Velocity of nodes

Random walk – transmitter:

- Maximum velocity for classic algorithm: 2m/sec
Environmental changes

Velocity of nodes

Random walk – transmitter:

- Max. velocity for Multivariable equations: 5m/sec supported
Environmental changes
Velocity of nodes

straight line – maximum relative speed :

- Maximum velocity for classic algorithm: 30m/sec
- Regardless if transmitter or receiver move
Environmental changes
Velocity of nodes

- Maximum velocity for Multivariable equations algorithm: 60m/sec
- Regardless if transmitter or receiver move
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Environmental changes

Multiple receiver nodes

Runde 1:
Erstes Netzwerk optimiert sein Trägersignal, zweites Netzwerk gibt Feedback.

(Summensignal)

Phasenanpassung wie beim Szenario mit digitalem Feedback.
Feedback wird durch Vorhandensein von einem Signal gegeben.

(Einzelsignal)

Runde 2:
Zweites Netzwerk optimiert sein Trägersignal, erstes Netzwerk gibt Feedback.

(Summensignal)
### Environmental changes

#### Multiple receiver nodes

<table>
<thead>
<tr>
<th></th>
<th>3m</th>
<th>12m</th>
<th>24m</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Knoten</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gain zur Anfangsamplitude (Median) [dB]</td>
<td>0,96 2,39 1,40 1,46 1,10</td>
<td>1,24 0,63 1,39 2,06 1,47</td>
<td>1,12 2,33 2,76 3,61 1,67</td>
</tr>
<tr>
<td>Gain zu einem Knoten (Median) [dB]</td>
<td>2,33 2,32 2,37 3,50 4,05</td>
<td>2,53 1,09 2,00 2,74 4,18</td>
<td>1,2 2,54 2,03 5,15 3,76</td>
</tr>
<tr>
<td>Amplitude nach Synchronisation [%]</td>
<td>92,4 51,4 65,3 91,0 90,7</td>
<td>57,1 92,0 86,5 86,4 86,6</td>
<td>94,2 80,0 61,4 95,8 97,9</td>
</tr>
</tbody>
</table>
Environmental changes

Multiple receiver nodes

3m
Environmental changes

Multiple receiver nodes

12m
Environmental changes

Multiple receiver nodes

Multiple receiver nodes – issues:
- Only binary feedback value
  - Therefore only classic optimisation approach
- Distance between transmit and receive nodes relative to spatial diversity of nodes in one network
  - Better synchronisation when nodes in one network in spatial proximity
  - When nodes in one network communicate: No issue
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Increased population size – Discussion:

How to achieve population size greater than one?
- Separate transmit times
- WCDMA
- Distinct frequencies simultaneously

Only separate transmit times feasible for WSN

More time for each iteration
- Initial solution: Random search
- Not clear if performance improvement possible by crossover
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Environmental changes
Receive beamforming

Receive beamforming – Discussion:

- Transmit node transmits only once
- Receiver nodes combine received signal fragments in the network
- Tradeoff:
  - Transmission power for in-network communication
  - Transmission over several iterations with receiver node
- More complex computation of transmit nodes
Questions?

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