Feedback based distributed adaptive transmit beamforming
Algorithmic considerations

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Outline

Introduction

An upper bound on the expected optimisation time

A lower bound on the expected optimisation time

An asymptotically optimal optimisation scheme

An adaptive protocol for distributed adaptive beamforming

Conclusion
Introduction

- Distributed adaptive transmit beamforming
  - Distributed nodes synchronise the carrier frequency and phase offset of transmit signals
  - Low power and processing devices
  - Non-synchronised local oscillators
Introduction

- Distributed synchronisation schemes
  - Closed loop carrier synchronisation

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1. Y. Tu and G. Pottie, *Coherent Cooperative Transmission from Multiple Adjacent Antennas to a Distant Stationary Antenna Through AWGN Channels*, Proceedings of the IEEE VTC, 2002
Introduction

Distributed synchronisation schemes
- Open loop carrier synchronisation\(^2\)

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Introduction

Distributed synchronisation schemes

Closed loop feedback based carrier synchronisation

\[ f_2t + \gamma_2^2 \pi \]
\[ f_1t + \gamma_1^2 \pi \]
\[ i \]
\[ t + \gamma_n \]
\[ i+1 \]

\[ n \]

Introduction

- Closed loop feedback based carrier synchronisation
  - Algorithm always converges to the optimum $^{a}$
  - Expected optimisation time $O(n)$ when in each iteration the optimum Probability distribution is chosen $^{a}$
  - Optimisation time can be improved by factor 2 when erroneous decisions are not discarded but inverted $^{b}$
  - Phase and frequency synchronisation feasible $^{c}$

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An upper bound on the expected optimisation time

Observations
- Iterative approach similar to evolutionary random search
  - New search points are requested by altering the carrier phases
  - Fitness function implemented by receiver feedback
  - Selection of individuals based on feedback values
  - Population size and offspring population size: $\mu = \nu = 1$
An upper bound on the expected optimisation time

\[ \mathcal{I} = \begin{array}{ccccccc}
\text{Phase offset} & \text{Frequency} & \text{Phase offset for} & \text{Frequency} & \text{Phase offset for} & \text{Frequency} & \text{Phase offset for} \\
\text{of node 1} & \text{offset for node 1} & \text{node } i & \text{offset for node } i & \text{node } n & \text{offset for node } n \\
0 \ldots 1 & 0 \ldots 1 & \ldots & 0100 \ldots 1001 & 0100 \ldots 1001 & \ldots & 0 \ldots 1 \\
\log(k) \text{ bits} & \log(\varphi) \text{ bits} & \log(k) \text{ bits encode } k & \log(\varphi) \text{ bits encode } \varphi & \log(k) \text{ bits} & \log(\varphi) \text{ bits} \\
\end{array} \]

- **Individual representation**
  - Here: Binary representation of phase/frequency offsets
    - \( \log(k) \) bits to represent \( k \) phase offsets
    - \( \log(\varphi) \) bits to represent \( \varphi \) frequency offsets
    - Configurations for all nodes concatenated
  - Phase and frequency offsets enumerated in ascending order
  - Neighbourhood: Gray encoded bit sequence to respect neighbourhood similarities
An upper bound on the expected optimisation time

Assumptions:

- Network of \( n \) nodes
- Each node changes the phase of its carrier signal with probability \( \frac{1}{n} \)
- Carrier phase altered uniformly at random from \([0, 2\pi]\)
An upper bound on the expected optimisation time

Optimisation aim:
- Achieve maximum relative phase offset of $\frac{2\pi}{k}$
- Between any two carrier signals
- For arbitrary $k$
- Divide phase space into $k$ intervals of width $\frac{2\pi}{k}$
An upper bound on the expected optimisation time

- Alter 1 carrier and keep $n-1$ signals
- This happens with probability

\[
\left( \frac{n - i}{1} \right) \cdot \frac{1}{n} \cdot \frac{1}{k} \cdot \left(1 - \frac{1}{n} \right)^{n-1}
\]

\[
= \left( \frac{n - i}{n \cdot k} \right) \cdot \left(1 - \frac{1}{n} \right)^{n-1}
\]
An upper bound on the expected optimisation time

Since

\[
\left(1 - \frac{1}{n}\right)^n < \frac{1}{e} < \left(1 - \frac{1}{n}\right)^{n-1}
\]

- Probability that \( L_i \) is left for partition \( j, j > i \):

\[
P[L_i] \geq \frac{n - i}{n \cdot e \cdot k}
\]
An upper bound on the expected optimisation time

- Expected number of iterations to change layer bounded from above by \( P[L_i]^{-1} \):

\[
E[T_P] \leq \sum_{i=0}^{n-1} \frac{e \cdot n \cdot k}{n-i}
\]

\[
= e \cdot n \cdot k \cdot \sum_{i=1}^{n} \frac{1}{i}
\]

\[
< e \cdot n \cdot k \cdot (\ln(n) + 1)
\]

\[
= O(n \cdot k \cdot \log n)
\]
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A lower bound on the expected optimisation time

\[ I = \begin{array}{cccc}
\text{Phase offset} & \text{Frequency offset for node 1} & \text{Phase offset for node } i & \text{Frequency offset for node } n \\
\log(k) \text{ bits} & \log(\varphi) \text{ bits} & \log(k) \text{ bits encode } k \text{ distinct possible phase offsets of node } i & \log(\varphi) \text{ bits encode } \varphi \text{ distinct frequency offsets of node } i \\
0 \ldots 1 & 0 \ldots 1 & 0100 \ldots 1001 & 0100 \ldots 1001 \\
\end{array} \]

- A lower bound on the synchronisation performance
  - We utilise the method of the expected progress
  - After initialisation, phases of carrier signals are identically and independently distributed.
  - Each bit in the binary representation of search point \( s_\zeta \) has equal probability to be 1 or 0.
A lower bound on the expected optimisation time

<table>
<thead>
<tr>
<th>$\mathcal{I}_i$</th>
<th>$\mathcal{I}_{\text{opt}}$</th>
<th>$\sum$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 0 1 1 0 1 1 1 1 0 1 1 0 1 1 1</td>
<td>1 0 1 0 1 1 0 1 1 0 1 0 1 1 0 1</td>
<td>6</td>
</tr>
</tbody>
</table>

- Probability to start with hamming distance $h(s_{\text{opt}}, s_\zeta) \leq l$;
  
  $l \ll n \cdot \log(k)$ to global optima $s_{\text{opt}}$ at most

\[
P[h(s_{\text{opt}}, s_\zeta) \leq l] = \sum_{i=0}^{l} \left( \frac{n \cdot \log(k)}{n \cdot \log(k) - i} \right) \cdot \frac{k}{2^{n \cdot \log(k) - i}}
\]

\[
\leq \frac{(n \cdot \log(k))^{l+2}}{2^{n \cdot \log(k) - l}}
\]
A lower bound on the expected optimisation time

\[ P[h(s_{\text{opt}}, s_\zeta) \leq l] = \sum_{i=0}^{l} \left( \frac{n \cdot \log(k)}{n \cdot \log(k) - i} \right) \cdot \frac{k}{2^n \cdot \log(k) - i} \leq \frac{(n \cdot \log(k))^{l+2}}{2^n \cdot \log(k) - l} \]

This means that with high probability (w.h.p.) the hamming distance to the nearest global optimum is at least \( l \).
A lower bound on the expected optimisation time

- Use method of expected progress to calculate lower bound:
- \((s_\zeta, t)\) denotes that \(s_\zeta\) is achieved after \(t\) iterations
- Assume progress measure \(\Lambda : \mathbb{B}^{n \cdot \log(k)} \rightarrow \mathbb{R}_0^+\)
- \(\Lambda(s_\zeta, t) < \Delta\): Global optimum not found in first \(t\) iterations
- For every \(t \in \mathbb{N}\) we have

\[
E[T_P] \geq t \cdot P[T_P > t] \\
= t \cdot P[\Lambda(s_\zeta, t) < \Delta] \\
= t \cdot (1 - P[\Lambda(s_\zeta, t) \geq \Delta])
\]
A lower bound on the expected optimisation time

\[ E[T_P] \geq t \cdot (1 - P[\Lambda(s_\zeta, t) \geq \Delta]) \]

- With the help of the Markov-inequality we obtain

\[ P[\Lambda(s_\zeta, t) \geq \Delta] \leq \frac{E[\Lambda(s_\zeta, t)]}{\Delta} \]

- and therefore

\[ E[T_P] \geq t \cdot \left(1 - \frac{E[\Lambda(s_\zeta, t)]}{\Delta}\right) \]

- Obtain lower bound by providing expected progress after \( t \) iterations
A lower bound on the expected optimisation time

- Probability for $l$ bits to correctly flip at most
  \[
  \left( 1 - \frac{1}{n \cdot \log(k)} \right)^{n \cdot \log(k) - l} \cdot \left( \frac{1}{n \cdot \log(k)} \right)^l \leq \left( \frac{1}{n \cdot \log(k)} \right)^l
  \]

- Expected progress in one iteration:
  \[
  E[\Lambda(s_\zeta, t), \Lambda(s_\zeta', t+1)] \leq \sum_{i=1}^l \frac{i}{(n \cdot \log(k))^i} < \frac{2}{n \cdot \log(k)}
  \]

- Expected progress in $t$ iterations:
  \[
  \leq \frac{2t}{n \cdot \log(k)}
  \]
A lower bound on the expected optimisation time

- Choose \( t = \frac{n \cdot \log(k) \cdot \Delta}{4} - 1 \)
- Double of expected progress still smaller than \( \Delta \).
- With Markov inequality: Progress not achieved with prob. \( \frac{1}{2} \).
- Expected optimisation time bounded from below by
  \[
  E[T_P] \geq t \cdot \left(1 - \frac{E[\Lambda(s_\zeta, t)]}{\Delta}\right)
  \]
  \[
  \geq \frac{n \cdot \log(k) \cdot \Delta}{4} \cdot \left(1 - \frac{2 \cdot n \cdot \log(k) \cdot \Delta}{4 \cdot n \cdot \log(k) \cdot \Delta}\right)
  \]
  \[
  = \Omega(n \cdot \log(k) \cdot \Delta)
  \]
- With \( \Delta = k \cdot \frac{\log(n)}{\log(k)} \): Same order as upper bound:
  \[
  E[T_P] = \Theta(n \cdot k \cdot \log(n))
  \]
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An asymptotically optimal optimisation scheme

- Reduce the amount of randomness in the optimisation
- Improve the synchronisation performance
- Improve the synchronisation quality
An asymptotically optimal optimisation scheme

- Global or local optima?
  - Weak multimodal fitness function
An asymptotically optimal optimisation scheme

- Fitness function observed by single node
- Constant carrier phase offset for $n - 1$ nodes
- Fitness function:

$$\mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c$$
An asymptotically optimal optimisation scheme

Approach:
- Measure feedback at 3 points
- Solve multivariable equations
- Apply optimum phase offset calculated

\[ \mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c \]
An asymptotically optimal optimisation scheme

**Problem:**
- Calculation not accurate when two or more nodes alter the phase of their transmit signals
An asymptotically optimal optimisation scheme

- Node estimates the quality of the function estimation itself
- Transmit with optimum phase offset and measure channel again
- When Expected fitness deviates significantly from measured fitness, discard altered phase offset
- Deviation:
  - 1 node: ≈ 0.6%
  - 2 nodes: ≈ 1.5%
  - 3 nodes: > 3%
An asymptotically optimal optimisation scheme

1. Transmit with phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$; measure feedback
2. Estimate feedback function and calculate $\gamma_i^*$
3. Transmit with $\gamma_4 = \gamma_i^*$
4. If deviation smaller 1% finished, otherwise discard $\gamma_i^*$
An asymptotically optimal optimisation scheme

- Asymptotic synchronisation time:
  \[ \mathcal{O}(n) \]

- Classic approach:\(^3\)
  \[ \Theta(n \cdot k \cdot \log(n)) \]

\(^3\) Sigg, El Masri and Beigl, A sharp asymptotic bound for feedback based closed-loop distributed adaptive beamforming in wireless sensor networks (Accepted for Transactions on Mobile Computing)
An asymptotically optimal optimisation scheme

Median fitness values (Network size: 100 nodes)

- Multivariable equations, uniform probability to change the phase of a carrier signal: 0.01
- Normal distributed probability to change the phase of a carrier signal: 0.01, Variance: 0.5 π
An asymptotically optimal optimisation scheme

Relative phase shift (Network size: 100)
An asymptotically optimal optimisation scheme

- Phase offset of distinct nodes is within $+/-0.05\pi$ for up to 99% of all nodes.
An asymptotically optimal optimisation scheme

- Asymptotically optimal synchronisation time
- Simulations: $\approx 12n$
- Further improvement:
  - 3 iterations per turn
  - Utilise last transmission from previous iteration
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An adaptive protocol for distributed adaptive beamforming
An adaptive protocol for distributed adaptive beamforming

Transmitted bit sequence

Modulated transmit signal for device 1

Modulated transmit signal for device n

Received superimposed sum signal

Demodulated received sum signal
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BER for various modulation schemes

- **Amplitude modulation (1 bit per symbol)**
- **Amplitude modulation (2 bits per symbol)**
- **Amplitude modulation (3 bits per symbol)**

Distance [meters]

Median BER

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An adaptive protocol for distributed adaptive beamforming

BER (80 m distance)

- 100 nodes, 10 kbps, 80m distance
- 60 nodes, 10 kbps, 80m distance
- 20 nodes, 10 kbps, 80m distance

Probability to alter the phase offset for one node
An adaptive protocol for distributed adaptive beamforming

```
Nr | prob | RMSE
---|------|------
1  | 0.5  | 1.230101e-09
2  | 0.25 | 1.438299e-09
3  | 0.75 | 1.198927e-09
4  | 0.875| 1.139293e-09
5  | 0.9375| 1.155027e-09
6  | 0.8375| 1.191585e-09
7  | 0.90625| 1.151049e-09
8  | 0.85938| 1.182819e-09
9  | 0.89062| 1.209551e-09
```
An adaptive protocol for distributed adaptive beamforming

RMSE for various mutation probabilities
(50m distance, 100 transmit nodes, uniform distribution)
### An adaptive protocol for distributed adaptive beamforming

<table>
<thead>
<tr>
<th>Situation</th>
<th>mean</th>
<th>median</th>
<th>$\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Door state (opened/closed)</td>
<td>0.952</td>
<td>0.9513</td>
<td>0.0099</td>
</tr>
<tr>
<td>Presence of individual</td>
<td>0.817</td>
<td>0.8238</td>
<td>0.0455</td>
</tr>
<tr>
<td>Phone call (gsm)</td>
<td>0.9</td>
<td>1.0</td>
<td>0.32</td>
</tr>
<tr>
<td>Door opened (cond.: Empty room)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Door closed (cond.: Empty room)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Door closed (cond.: Room occupied)</td>
<td>0.832</td>
<td>0.83</td>
<td>0.041</td>
</tr>
<tr>
<td>Door opened (cond.: Room occupied)</td>
<td>0.976</td>
<td>0.98</td>
<td>0.0184</td>
</tr>
<tr>
<td>Room occupied (cond.: Door closed)</td>
<td>0.673</td>
<td>0.66</td>
<td>0.1143</td>
</tr>
<tr>
<td>Room occupied (cond.: Door open)</td>
<td>0.595</td>
<td>0.54</td>
<td>0.1247</td>
</tr>
<tr>
<td>Empty room (cond.: Door closed)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Empty room (cond.: Door open)</td>
<td>1.0</td>
<td>1.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>
Questions?

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