Institut für Betriebssysteme
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Feedback based distributed adaptive transmit beamforming

Algorithmic considerations

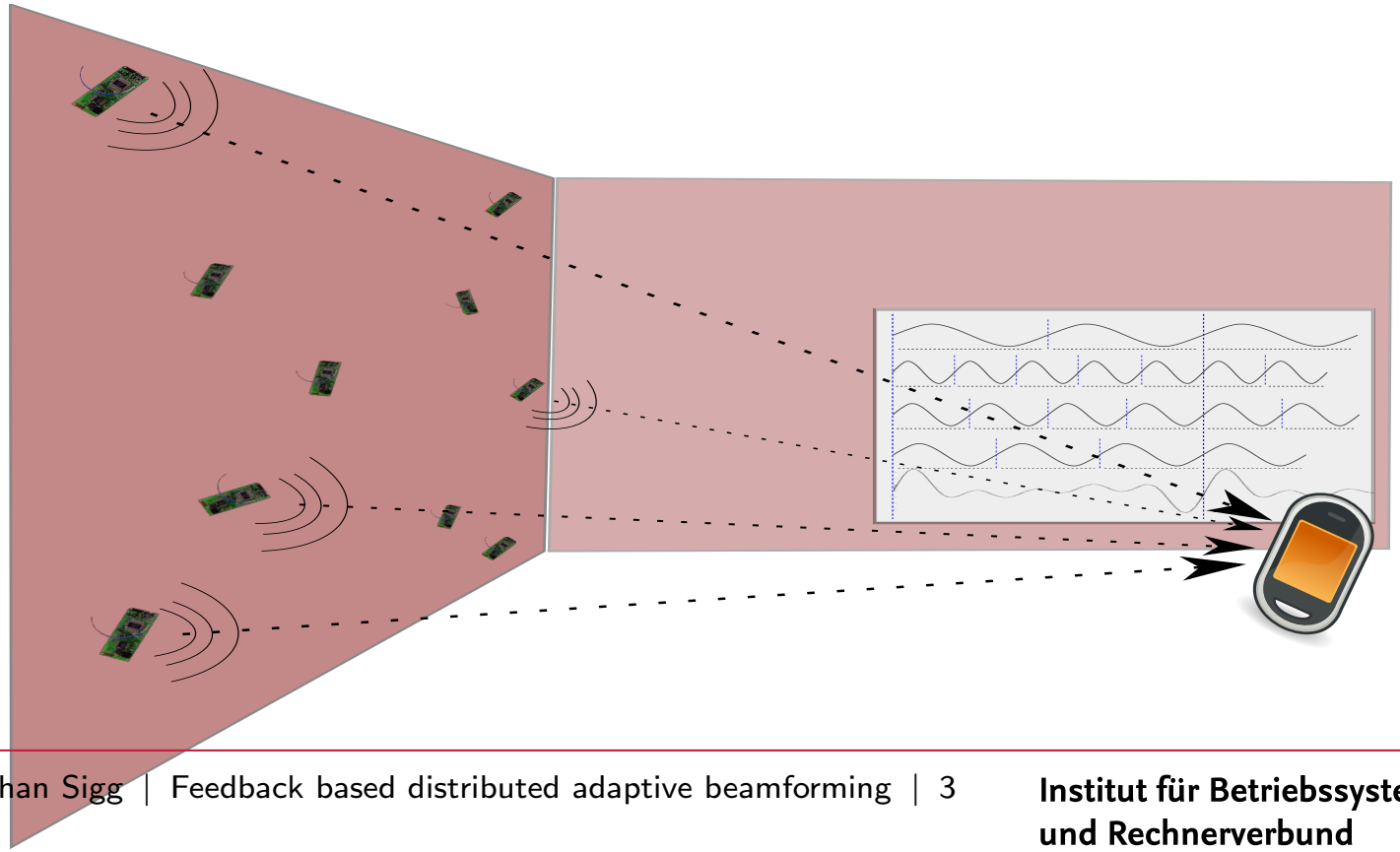
Stephan Sigg

Informatik Kolloquium, 31.01.2011, TU Braunschweig

Outline

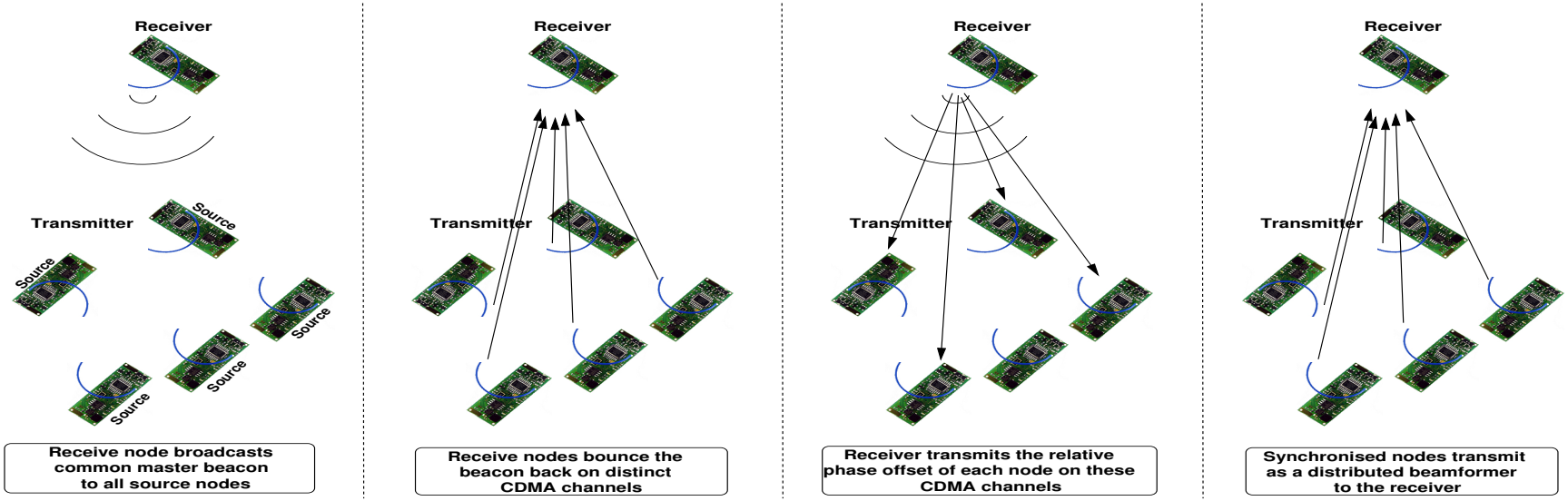
Introduction

- Distributed adaptive transmit beamforming
  - Distributed nodes synchronise the carrier frequency and phase offset of transmit signals
  - Low power and processing devices
  - Non-synchronised local oscillators



Introduction

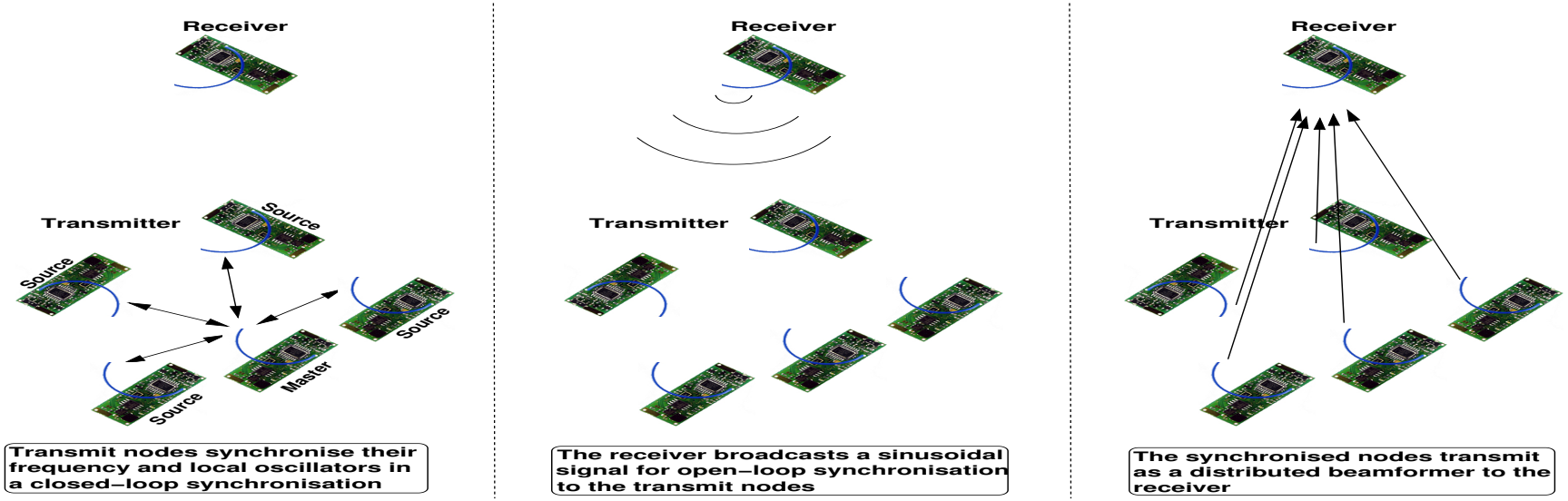
- Distributed synchronisation schemes
  - Closed loop carrier synchronisation[1]



[1] Y. Tu and G. Pottie, *Coherent Cooperative Transmission from Multiple Adjacent Antennas to a Distant Stationary Antenna Through AWGN Channels*, Proceedings of the IEEE VTC, 2002

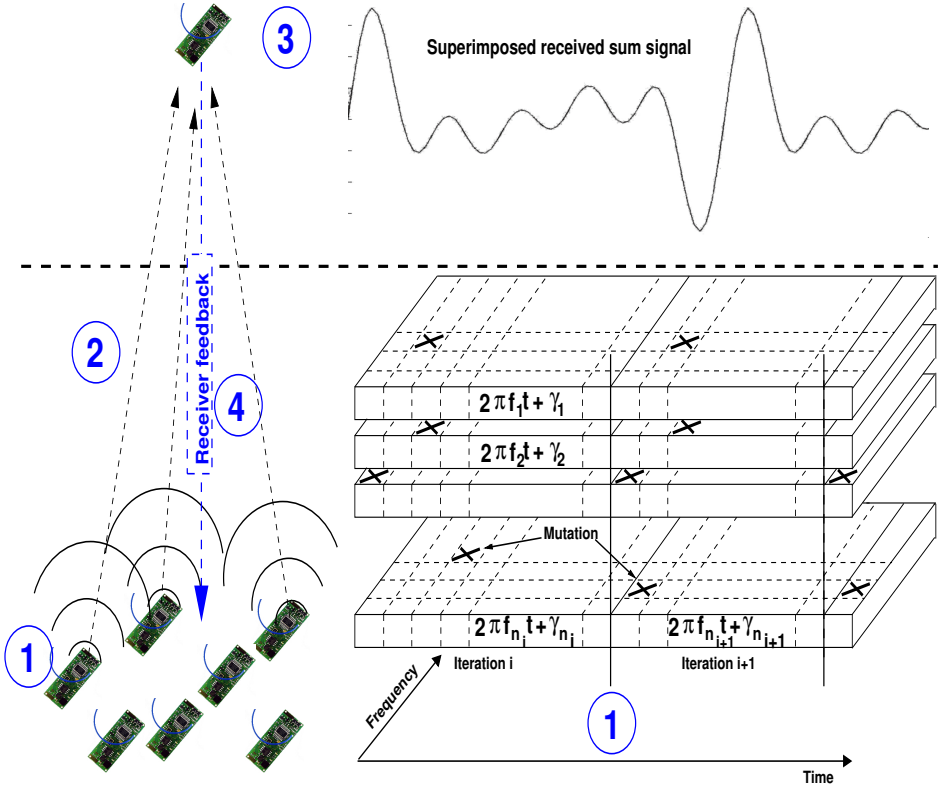
Introduction

- Distributed synchronisation schemes
  - Open loop carrier synchronisation[2]



[2] R. Mudumbai, G. Barriac and U. Madhow, *On the feasibility of distributed beamforming in wireless networks*, IEEE Transactions on Wireless Communications, Vol 6, May 2007

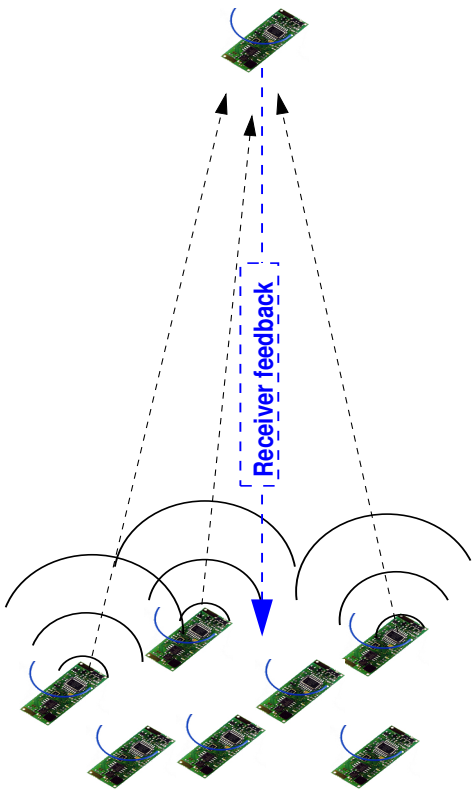
Introduction



- Distributed synchronisation schemes
  - Cosed loop feedback based carrier synchronisation[a]

[a] R. Mudumbai, J. Hespanha, U. Madhow, G. Barriac, *Distributed transmit beamforming using feedback control*, IEEE Transactions on Information Theory 56(1), volume 56, January 2010

Introduction



- Cosed loop feedback based carrier synchronisation
  - Algorithm always converges to the optimum [a]
  - Expected optimisation time $\mathcal{O}(n)$ when in each iteration the optimum Probability distribution is chosen [a]
  - Optimisation time can be improved by factor 2 when erroneous decisions are not discarded but inverted [b]
  - Phase and frequency synchronisation feasible[c]

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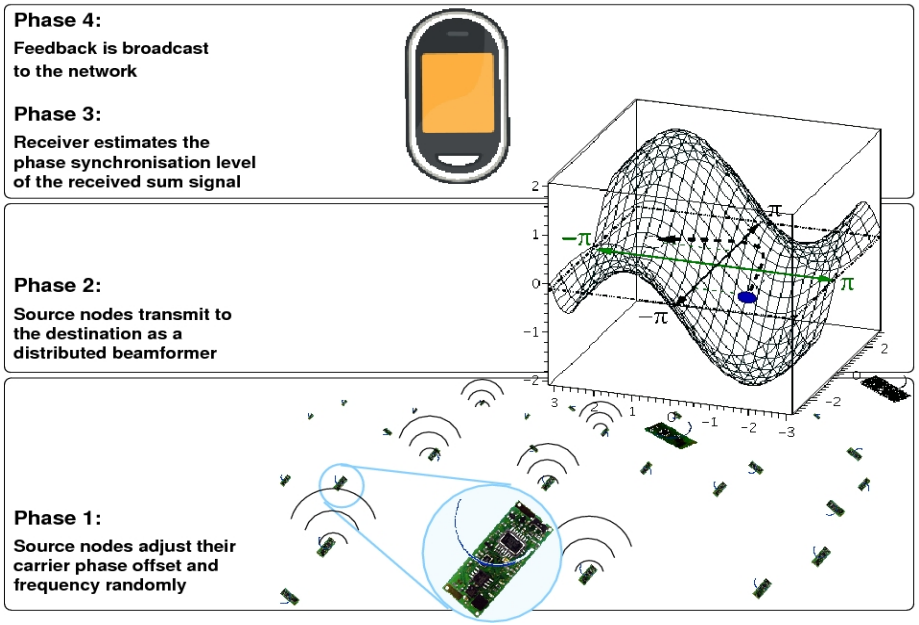
[a] R. Mudumbai, J. Hespanha, U. Madhow, G. Barriac, *Distributed transmit beamforming using feedback control*, IEEE Transactions on Information Theory 56(1), volume 56, January 2010

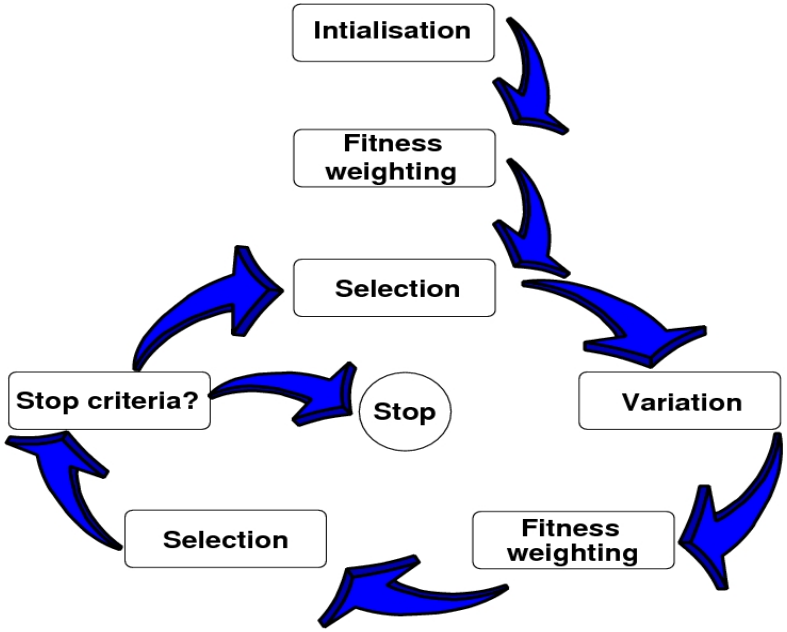
[b] J. Bucklew, W. Sethares, *Convergence of a class of decentralised beamforming algorithms*, IEEE Transactions on Signal Processing 56(6), volume 56, 2008

[c] M. Seo, M. Rodwell, U. Madhow, *A Feedback-Based Distributed phased array technique and its application to 60-GHz wireless sensor network*, IEEE MTT-S International Microwave Symposium Digest, 2008

Outline

An upper bound on the expected optimisation time





- Observations
  - Iterative approach similar to evolutionary random search
    - New search points are requested by altering the carrier phases
    - Fitness function implemented by receiver feedback
    - Selection of individuals based on feedback values
    - Population size and offspring population size: $\mu = \nu = 1$

An upper bound on the expected optimisation time

$$\mathcal{I} = \underbrace{0\ldots1}_{\log(k)\text{ bits}}^{\text{Phase offset of node 1}} \underbrace{0\ldots1}_{\log(\varphi)\text{ bits}}^{\text{Frequency offset for node 1}} \ldots \underbrace{0100\ldots1001}_{\substack{\log(k)\text{ bits encode } k \\ \text{distinct possible} \\ \text{phase offsets of node } i}}^{\text{Phase offset for node } i} \underbrace{0100\ldots1001}_{\substack{\log(\varphi)\text{ bits encode } \varphi \\ \text{distinct frequency} \\ \text{offsets of node } i}}^{\text{Frequency offset for node } i} \ldots \underbrace{0\ldots1}_{\log(k)\text{ bits}}^{\text{Phase offset for node } n} \underbrace{0\ldots1}_{\log(\varphi)\text{ bits}}^{\text{Frequency offset for node } n}$$

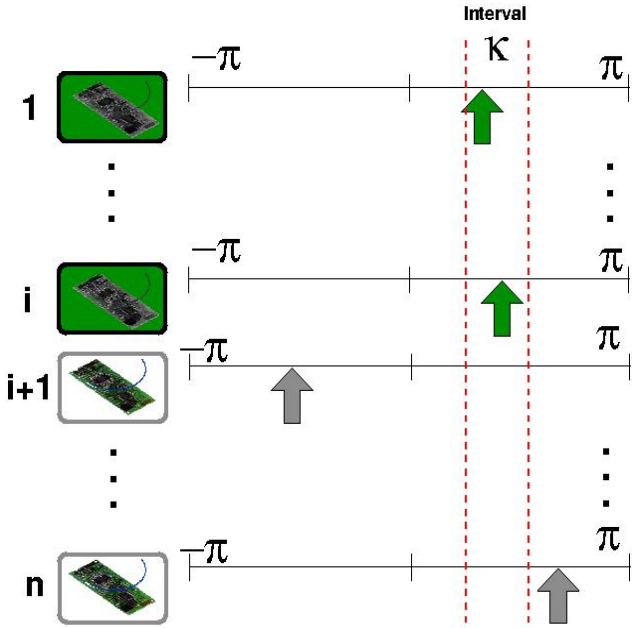
- Individual representation
  - Here: Binary representation of phase/frequency offsets
    - $\log(k)$ bits to represent k phase offsets
    - $\log(\varphi)$ bits to represent φ frequency offsets
    - Configurations for all nodes concatenated
  - Phase and frequency offsets enumerated in ascending order
  - Neighbourhood: Gray encoded bit sequence to respect neighbourhood similarities

An upper bound on the expected optimisation time

Assumptions :

- Network of n nodes
- Each node changes the phase of its carrier signal with probability $\frac{1}{n}$
- Carrier phase altered uniformly at random from $[0, 2\pi]$

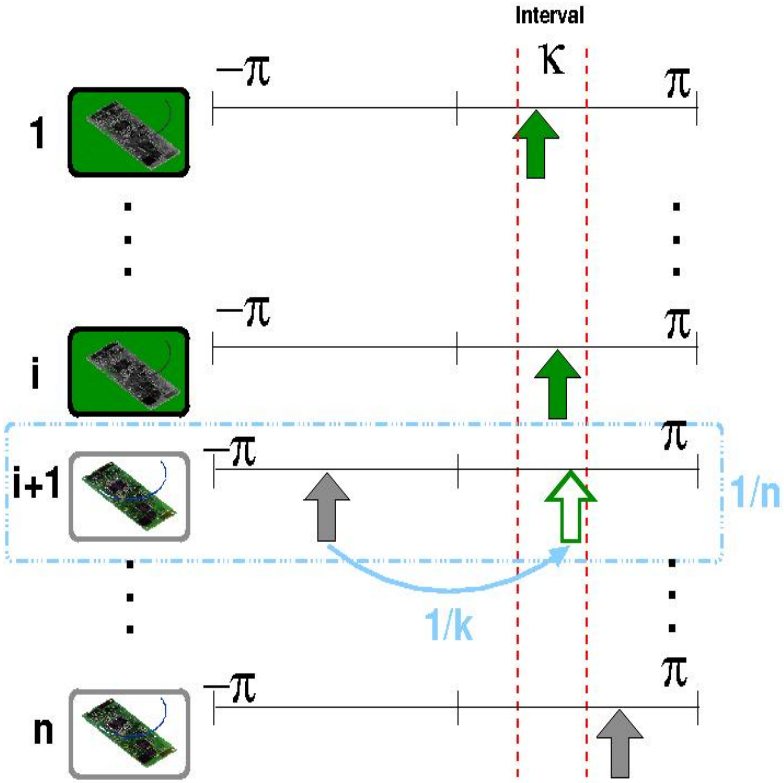
An upper bound on the expected optimisation time



Optimisation aim :

- Achieve maximum relative phase offset of $\frac{2\pi}{k}$
- Between any two carrier signals
- For arbitrary k
- Divide phase space into k intervals of width $\frac{2\pi}{k}$

An upper bound on the expected optimisation time

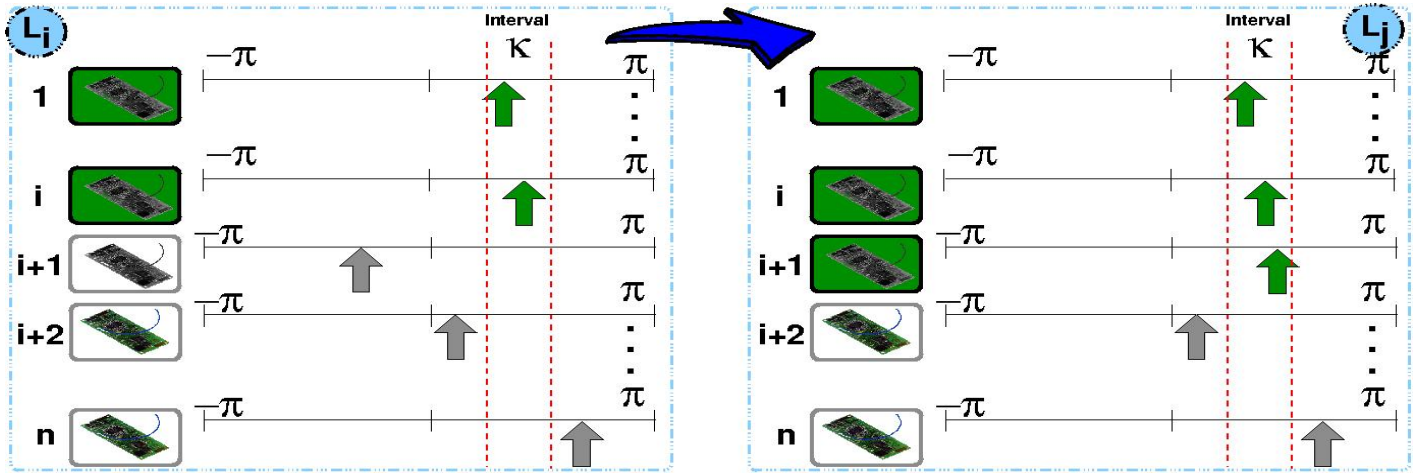


- Alter 1 carrier and keep $n-1$ signals
- This happens with probability

$$\binom{n-i}{1} \cdot \frac{1}{n} \cdot \frac{1}{k} \cdot \left(1-\frac{1}{n}\right)^{n-1}$$

$$= \left(\frac{n-i}{n \cdot k}\right) \cdot \left(1-\frac{1}{n}\right)^{n-1}$$

An upper bound on the expected optimisation time



- Since

$$\left(1-\frac{1}{n}\right)^{n} < \frac{1}{e} < \left(1-\frac{1}{n}\right)^{n-1}$$

- Probability that L_i is left for partition j, $j > i$:

$$P[L_i] \geq \frac{n-i}{n \cdot e \cdot k}$$

An upper bound on the expected optimisation time

- Expected number of iterations to change layer bounded from above by $P[L_i]^{-1}$:

$$
\begin{aligned}
E[T_{\mathcal{P}}] &\leq \sum_{i=0}^{n-1} \frac{e \cdot n \cdot k}{n-i} \\
&= e \cdot n \cdot k \cdot \sum_{i=1}^{n} \frac{1}{i} \\
&< e \cdot n \cdot k \cdot (\ln(n) + 1) \\
&= O\left(n \cdot k \cdot \log n\right)
\end{aligned}
$$

Outline

A lower bound on the expected optimisation time

$$\mathcal{I} = \underbrace{0\ldots1}_{\log(k)\text{ bits}}\ \underbrace{0\ldots1}_{\log(\varphi)\text{ bits}}\ \ldots\ \underbrace{0100\ldots1001}_{\log(k)\text{ bits encode }k\text{ distinct possible phase offsets of node }i}\ \underbrace{0100\ldots1001}_{\log(\varphi)\text{ bits encode }\varphi\text{ distinct frequency offsets of node }i}\ \ldots\ \underbrace{0\ldots1}_{\log(k)\text{ bits}}\ \underbrace{0\ldots1}_{\log(\varphi)\text{ bits}}$$

Phase offset of node 1 | Frequency offset for node 1 | Phase offset for node i | Frequency offset for node i | Phase offset for node n | Frequency offset for node n

- A lower bound on the synchronisation performance
  - We utilise the method of the expected progress
  - After initialisation, phases of carrier signals are identically and independently distributed.
  - Each bit in the binary representation of search point s_ζ has equal probability to be 1 or 0.

A lower bound on the expected optimisation time

																	$\sum$
$\mathcal{I}_i =$	1	0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	
$\mathcal{I}_{\text{opt}} =$	1	0	1	0	1	1	0	1	1	0	1	0	1	1	0	1	
$h(\mathcal{I}_i, \mathcal{I}_{\text{opt}}) =$	0	0	0	1	1	0	1	0	0	0	0	1	1	0	1	0	6

- Probability to start with hamming distance $h(s_{\text{opt}}, s_\zeta) \leq l$; $l \ll n \cdot \log(k)$ to global optima s_{opt} at most

$$
\begin{aligned}
P[h(s_{\text{opt}}, s_\zeta) \leq l] &= \sum_{i=0}^{l} \binom{n \cdot \log(k)}{n \cdot \log(k) - i} \cdot \frac{k}{2^{n \cdot \log(k) - i}} \\
&\leq \frac{(n \cdot \log(k))^{l+2}}{2^{n \cdot \log(k) - l}}
\end{aligned}
$$

A lower bound on the expected optimisation time

$$\mathcal{I} = \underbrace{\overset{\text{Phase offset of node 1}}{0\ldots1}}_{\log(k)\text{ bits}}\ \underbrace{\overset{\text{Frequency offset for node 1}}{0\ldots1}}_{\log(\varphi)\text{ bits}}\ \ldots\ \underbrace{\overset{\text{Phase offset for node } i}{0100\ldots1001}}_{\substack{\log(k)\text{ bits encode } k\\ \text{distinct possible}\\ \text{phase offsets of node } i}}\ \underbrace{\overset{\text{Frequency offset for node } i}{0100\ldots1001}}_{\substack{\log(\varphi)\text{ bits encode } \varphi\\ \text{distinct frequency}\\ \text{offsets of node } i}}\ \ldots\ \underbrace{\overset{\text{Phase offset for node } n}{0\ldots1}}_{\log(k)\text{ bits}}\ \underbrace{\overset{\text{Frequency offset for node } n}{0\ldots1}}_{\log(\varphi)\text{ bits}}$$

$$\begin{aligned} P[h(s_{\text{opt}}, s_\zeta) \leq l] &= \sum_{i=0}^{l} \binom{n \cdot \log(k)}{n \cdot \log(k) - i} \cdot \frac{k}{2^{n \cdot \log(k) - i}} \\ &\leq \frac{(n \cdot \log(k))^{l+2}}{2^{n \cdot \log(k) - l}} \end{aligned}$$

- This means that with high probability (w.h.p.) the hamming distance to the nearest global optimum is at least l.

A lower bound on the expected optimisation time

- Use method of expected progress to calculate lower bound:
- (s_ζ, t) denotes that s_ζ is achieved after t iterations
- Assume progress measure $\Lambda : \mathbb{B}^{n \cdot \log(k)} \to \mathbb{R}_0^+$
- $\Lambda(s_\zeta, t) < \Delta$: Global optimum not found in first t iterations
- For every $t \in \mathbb{N}$ we have

$$\begin{aligned} E[T_{\mathcal{P}}] &\geq t \cdot P[T_{\mathcal{P}} > t] \\ &= t \cdot P[\Lambda(s_\zeta, t) < \Delta] \\ &= t \cdot (1 - P[\Lambda(s_\zeta, t) \geq \Delta]) \end{aligned}$$

A lower bound on the expected optimisation time

$$E[T_{\mathcal{P}}] \quad \geq \quad t \cdot (1 - P[\Lambda(s_\zeta, t) \geq \Delta])$$

- With the help of the Markov-inequality we obtain

$$P[\Lambda(s_\zeta, t) \geq \Delta] \leq \frac{E[\Lambda(s_\zeta, t)]}{\Delta}$$

- and therefore

$$E[T_{\mathcal{P}}] \geq t \cdot \left(1 - \frac{E[\Lambda(s_\zeta, t)]}{\Delta}\right)$$

- Obtain lower bound by providing expected progress after t iterations

A lower bound on the expected optimisation time

- Probability for l bits to correctly flip at most

$$\left(1 - \frac{1}{n \cdot \log(k)}\right)^{n \cdot \log(k) - l} \cdot \left(\frac{1}{n \cdot \log(k)}\right)^{l} \leq \frac{1}{(n \cdot \log(k))^{l}}$$

- Expected progress in one iteration:

$$E[\Lambda(s_\zeta, t), \Lambda(s_{\zeta'}, t+1)] \leq \sum_{i=1}^{l} \frac{i}{(n \cdot \log(k))^{i}} < \frac{2}{n \cdot \log(k)}$$

- Expected progress in t iterations: $\leq \frac{2t}{n \cdot \log(k)}$

A lower bound on the expected optimisation time

- Choose $t = \frac{n \cdot \log(k) \cdot \Delta}{4} - 1$
- Double of expected progress still smaller than Δ.
- With Markov inequality: Progress not achieved with prob. $\frac{1}{2}$.
- Expected optimisation time bounded from below by

$$\begin{aligned} E[T_{\mathcal{P}}] &\geq t \cdot \left(1 - \frac{E[\Lambda(s_\zeta, t)]}{\Delta}\right) \\ &\geq \frac{n \cdot \log(k) \cdot \Delta}{4} \cdot \left(1 - \frac{\frac{2 \cdot n \cdot \log(k)}{4 \cdot n \cdot \log(k)} \cdot \Delta}{\Delta}\right) \\ &= \Omega(n \cdot \log(k) \cdot \Delta) \end{aligned}$$

- With $\Delta = k \cdot \frac{\log(n)}{\log(k)}$: Same order as upper bound:

$$E[T_{\mathcal{P}}] = \Theta\left(n \cdot k \cdot \log(n)\right)$$

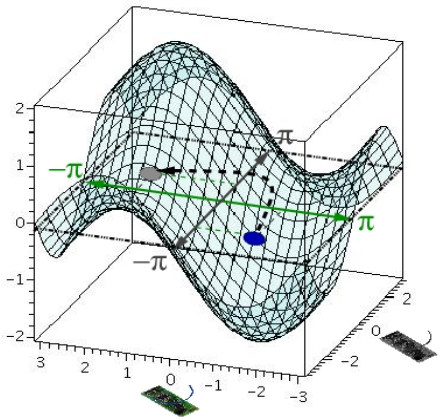
Outline

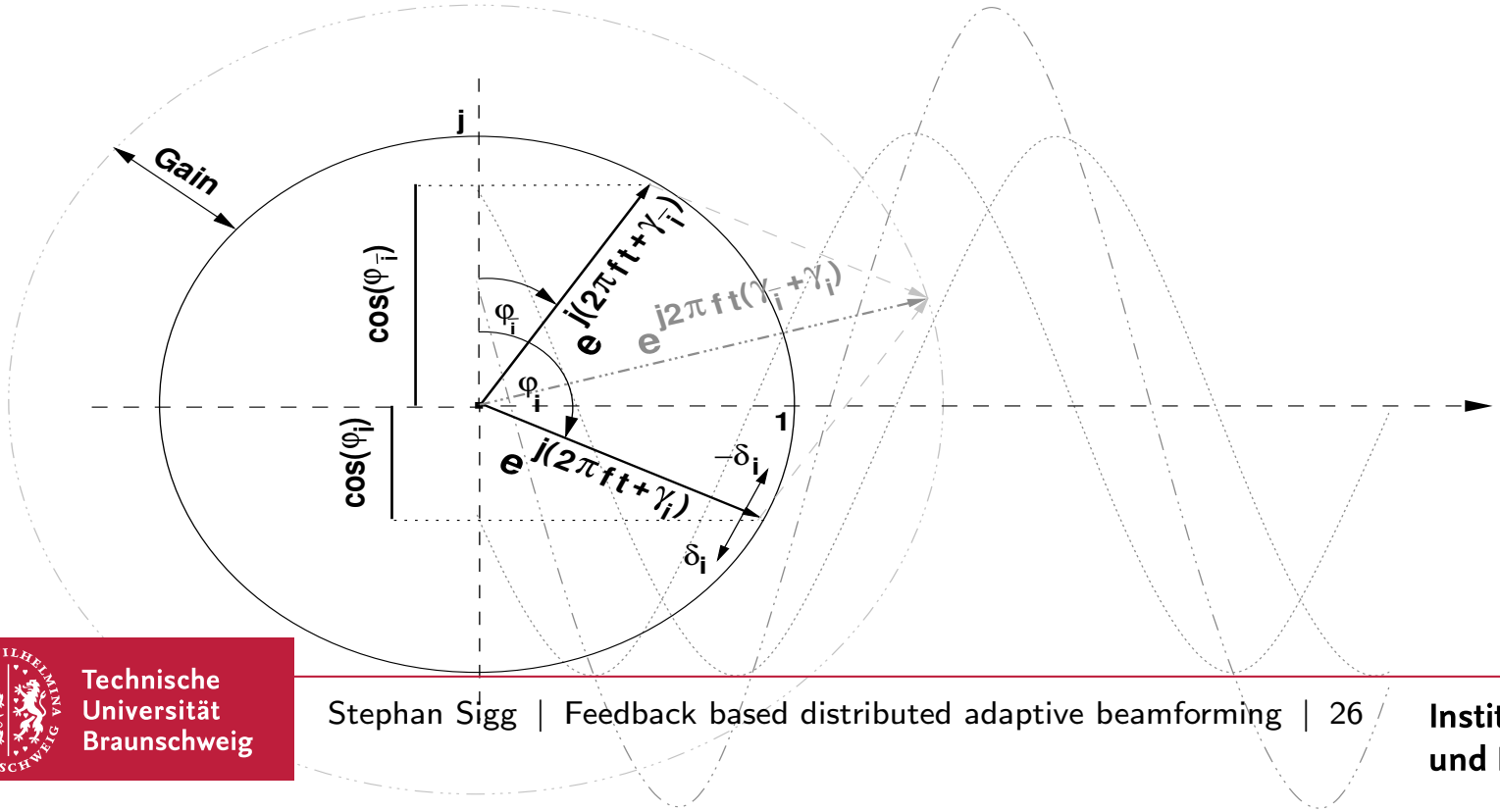
An asymptotically optimal optimisation scheme

- Reduce the amount of randomness in the optimisation
- Improve the synchronisation performance
- Improve the synchronisation quality

An asymptotically optimal optimisation scheme

- Global or local optima?
  - Weak multimodal fitness function

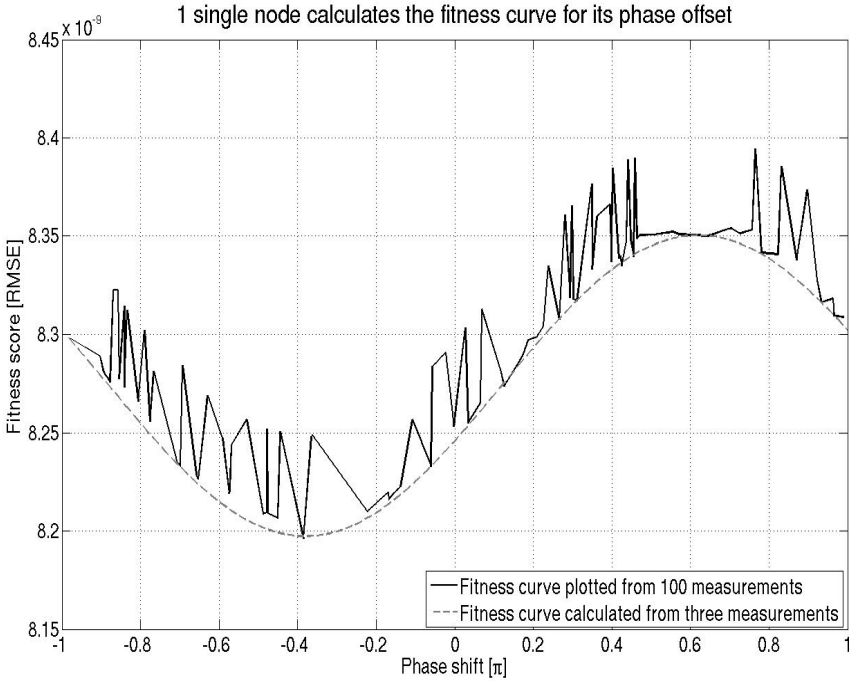




An asymptotically optimal optimisation scheme

- Fitness function observed by single node
- Constant carrier phase offset for $n-1$ nodes
- Fitness function:

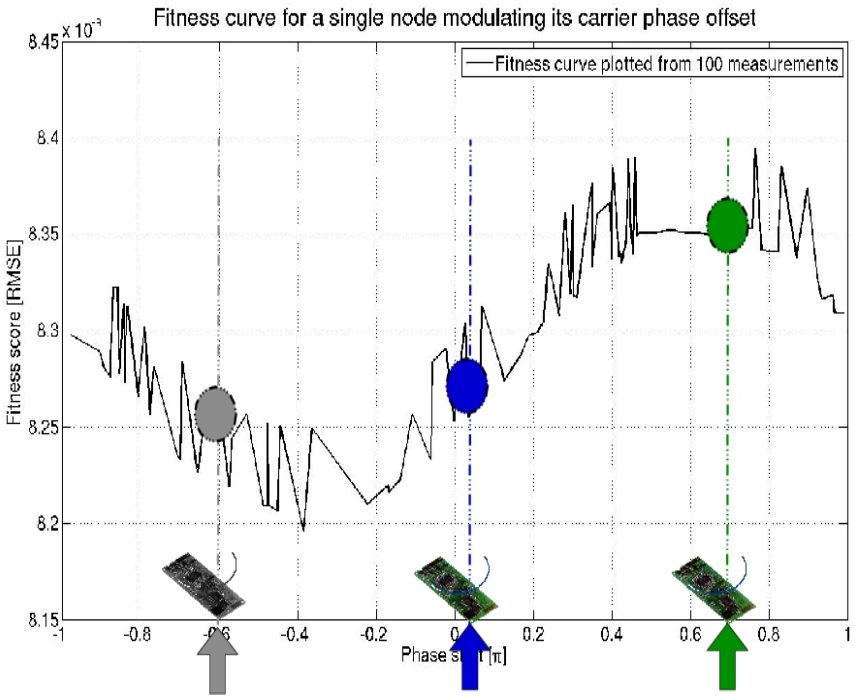
$$\mathcal{F}(\Phi_i) = A\sin(\Phi_i + \phi) + c$$



An asymptotically optimal optimisation scheme

Approach:

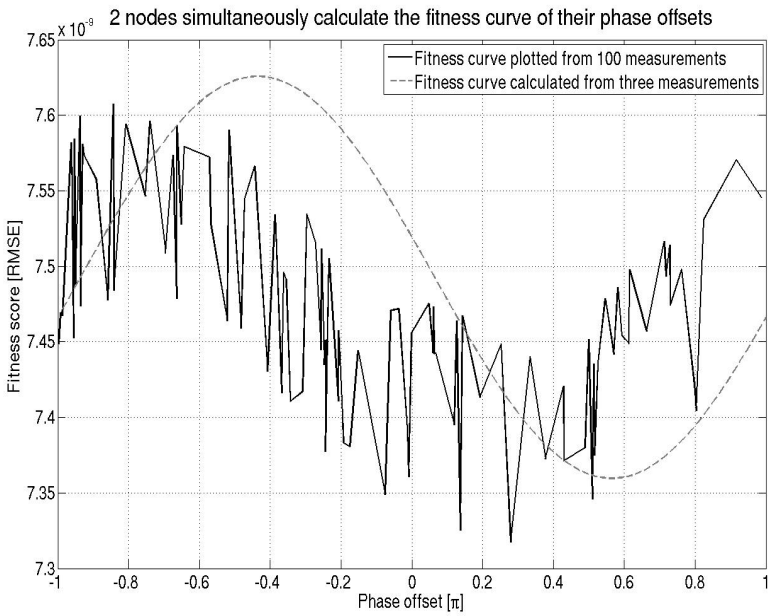
- Measure feedback at 3 points
- Solve multivariable equations
- Apply optimum phase offset calculated

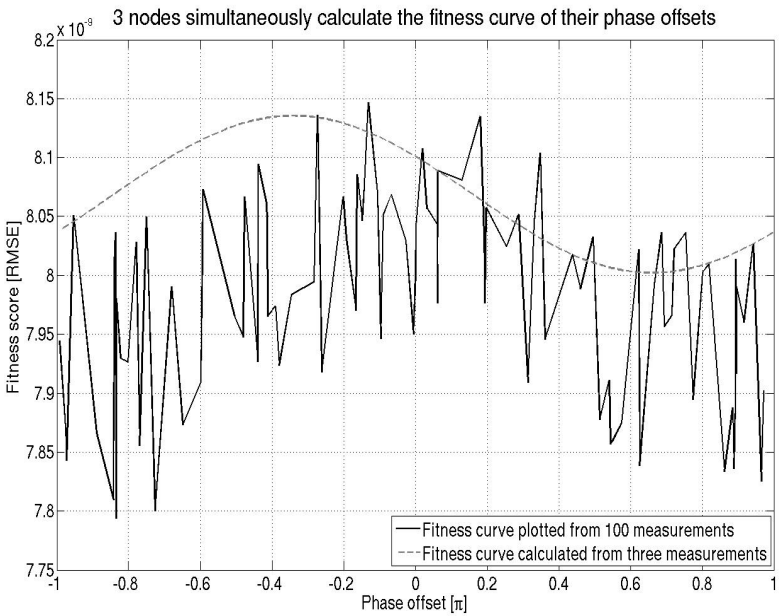


$$\mathcal{F}(\Phi_i) = A\sin(\Phi_i + \phi) + c$$

An asymptotically optimal optimisation scheme

- Problem:
  - Calculation not accurate when two or more nodes alter the phase of their transmit signals





An asymptotically optimal optimisation scheme

- Node estimates the quality of the function estimation itself
- Transmit with optimum phase offset and measure channel again
- When Expected fitness deviates significantly from measured fitness, discard altered phase offset
- Deviation:

1 node:	$\approx 0.6\%$
2 nodes:	$\approx 1.5\%$
3 nodes:	$> 3\%$

An asymptotically optimal optimisation scheme

1. Transmit with phase offsets $\gamma_1 \neq \gamma_2 \neq \gamma_3$; measure feedback
2. Estimate feedback function and calculate γ_i^*
3. Transmit with $\gamma_4 = \gamma_i^*$
4. If deviation smaller 1% finished, otherwise discard γ_i^*

An asymptotically optimal optimisation scheme

- Asymptotic synchronisation time:

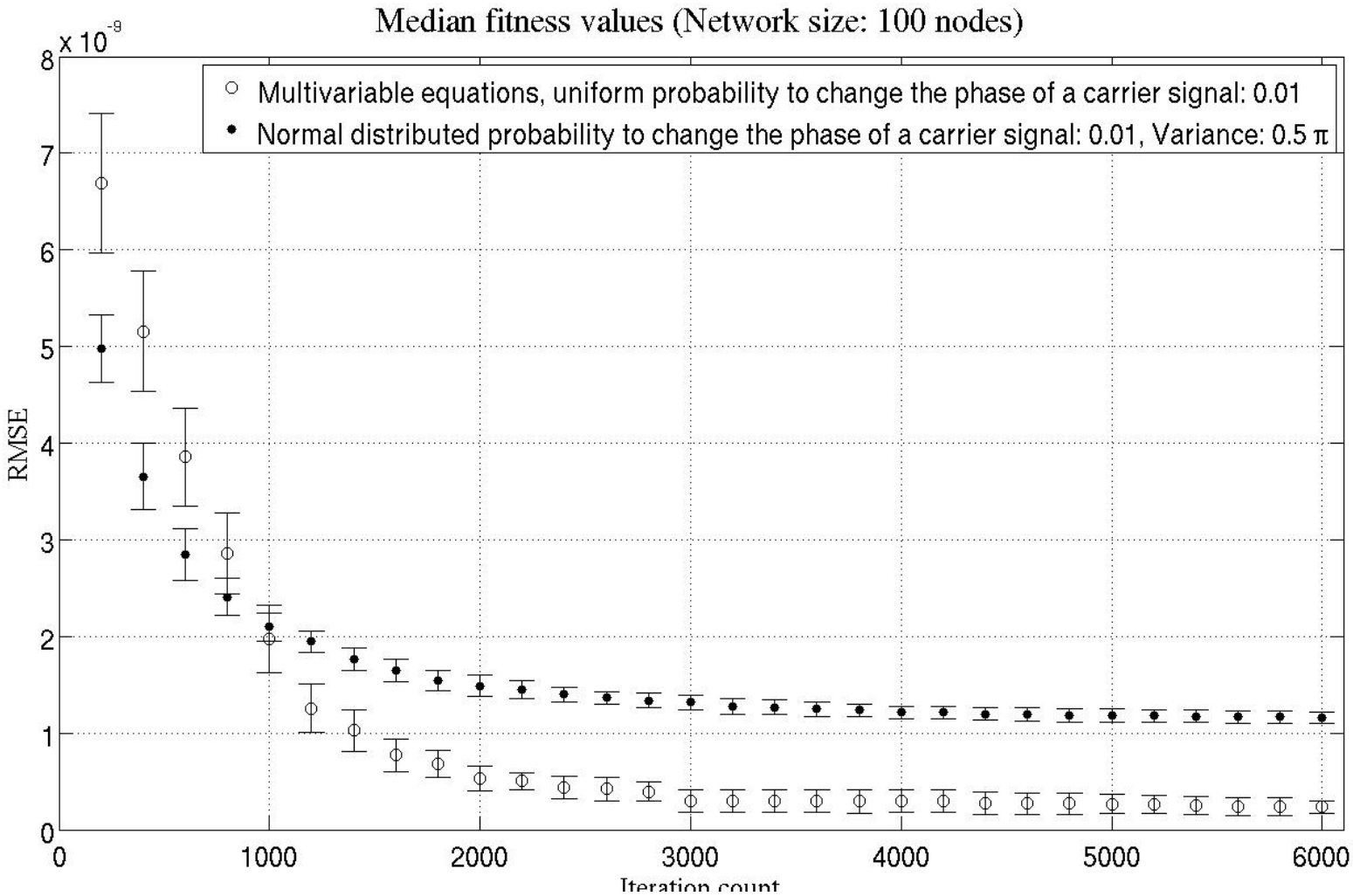
$$\mathcal{O}(n)$$

- Classic approach:[3]

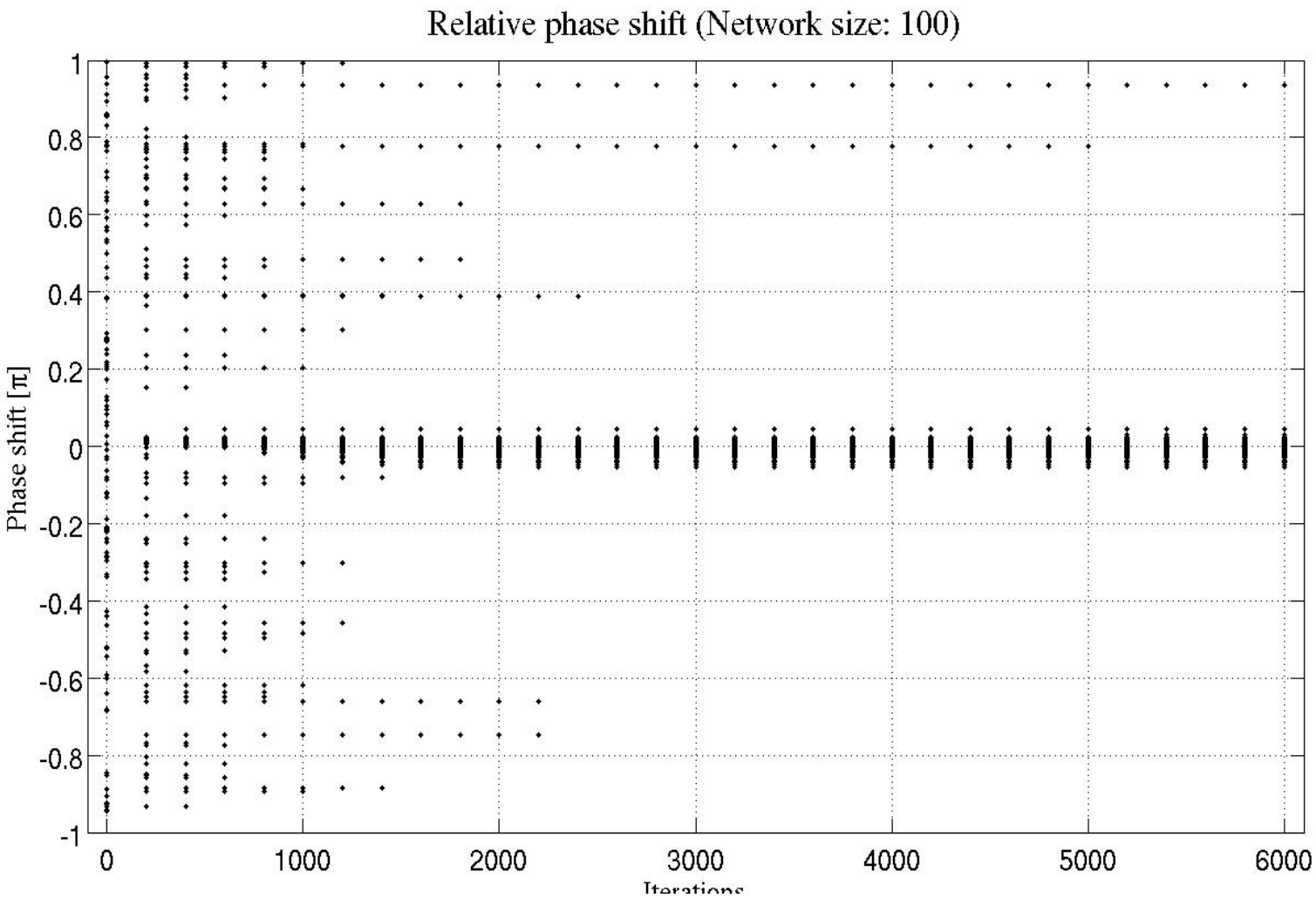
$$\Theta(n \cdot k \cdot \log(n))$$

[3] Sigg, El Masri and Beigl, A sharp asymptotic bound for feedback based closed-loop distributed adaptive beamforming in wireless networks (Accepted for Transactions on Mobile Computing)

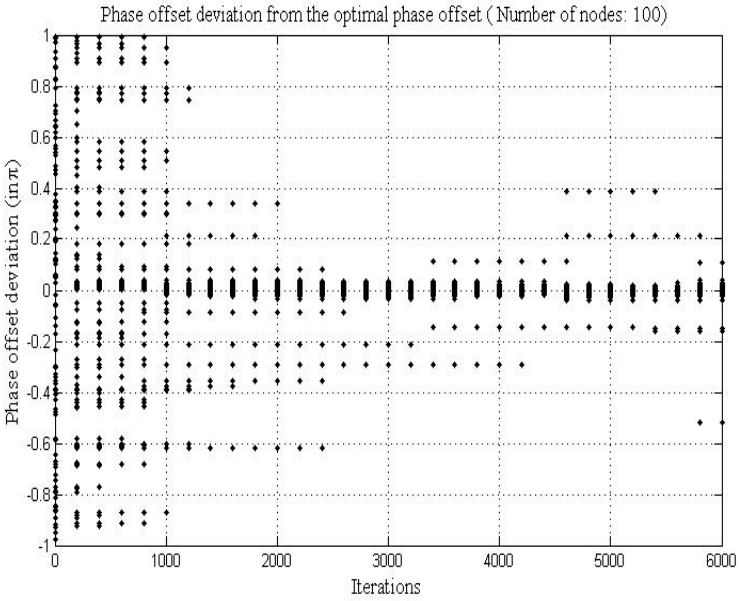
An asymptotically optimal optimisation scheme

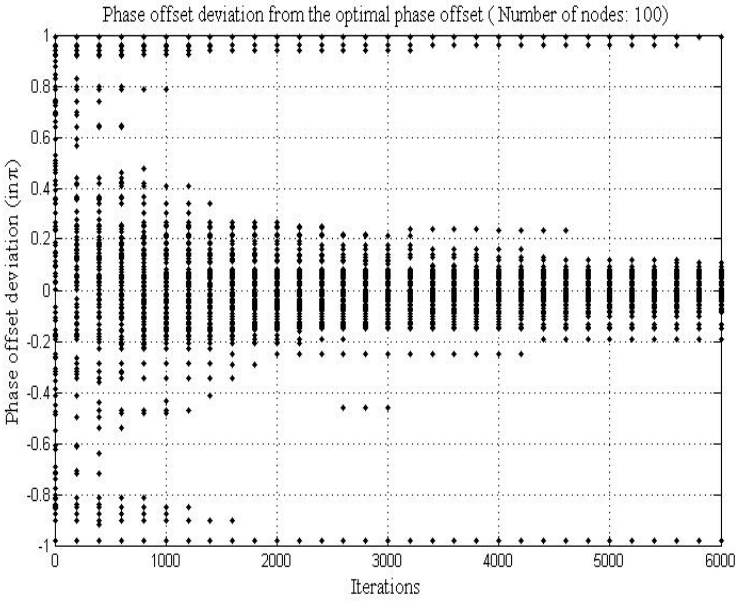


An asymptotically optimal optimisation scheme



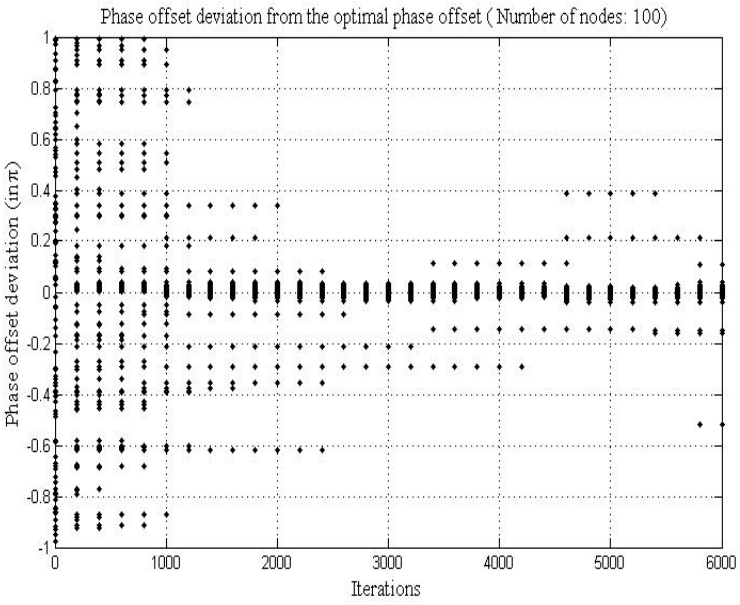
An asymptotically optimal optimisation scheme

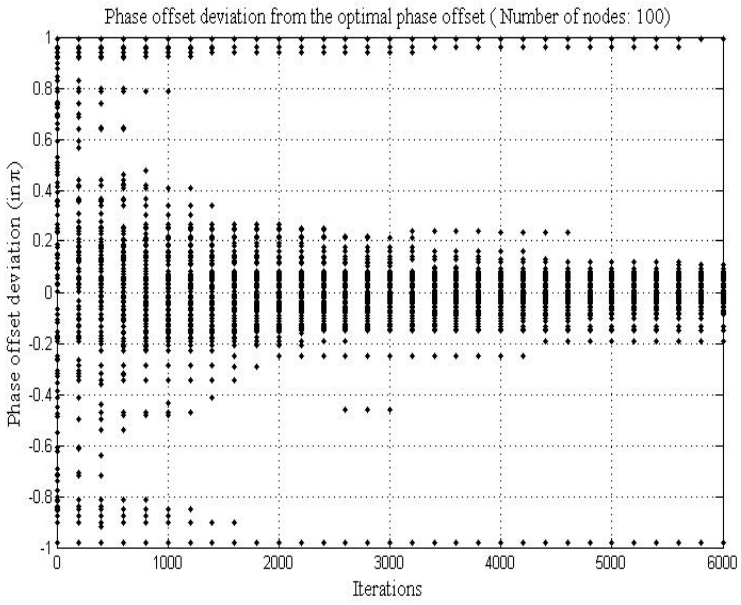




- Phase offset of distinct nodes is within $+/-0.05\pi$ for up to 99% of all nodes.

An asymptotically optimal optimisation scheme

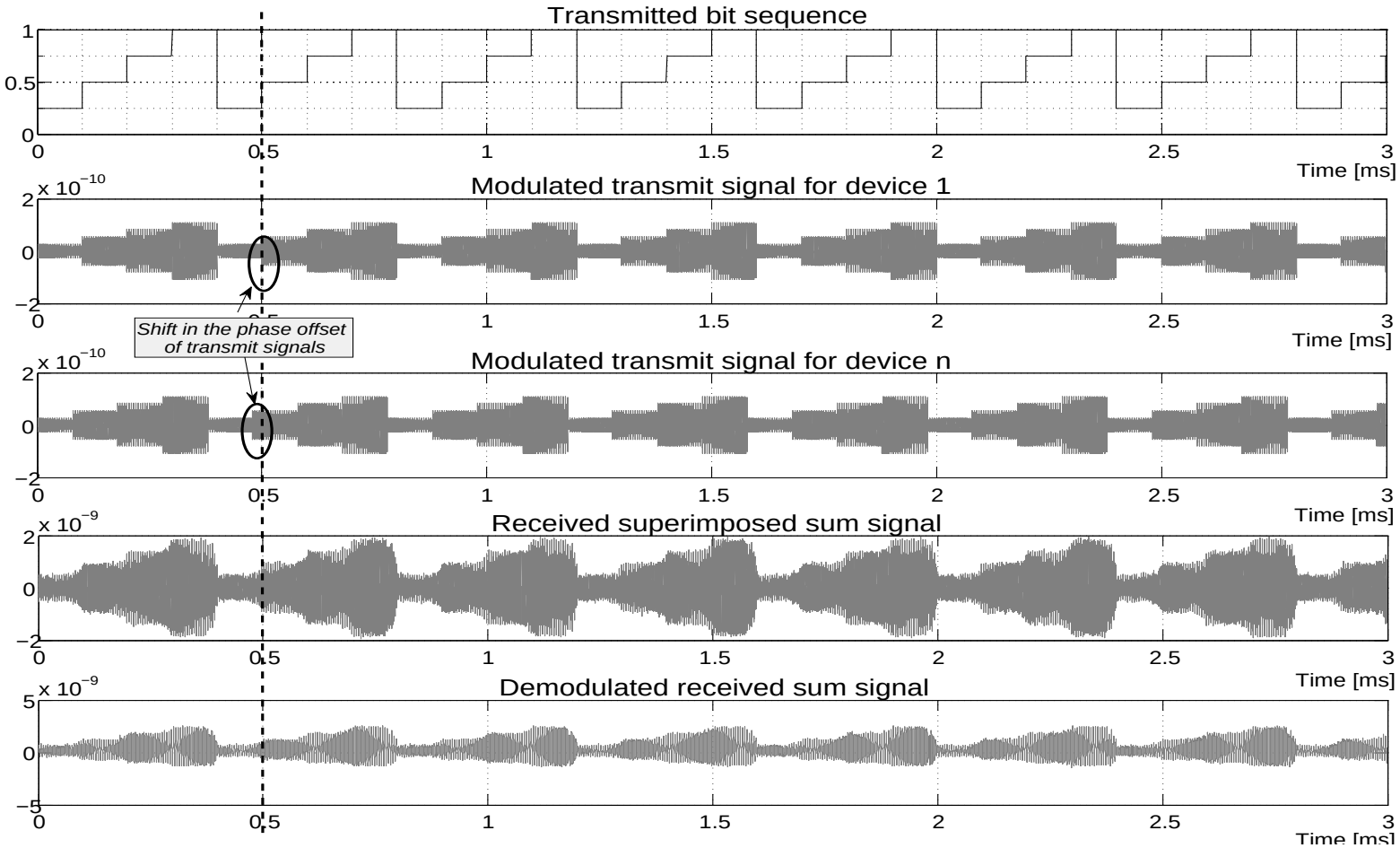




- Asymptotically optimal synchronisation time
- Simulations: $\approx 12n$
- Further improvement:
  - 3 iterations per turn
  - Utilise last transmission from previous iteration

Outline

An adaptive protocol for distributed adaptive beamforming



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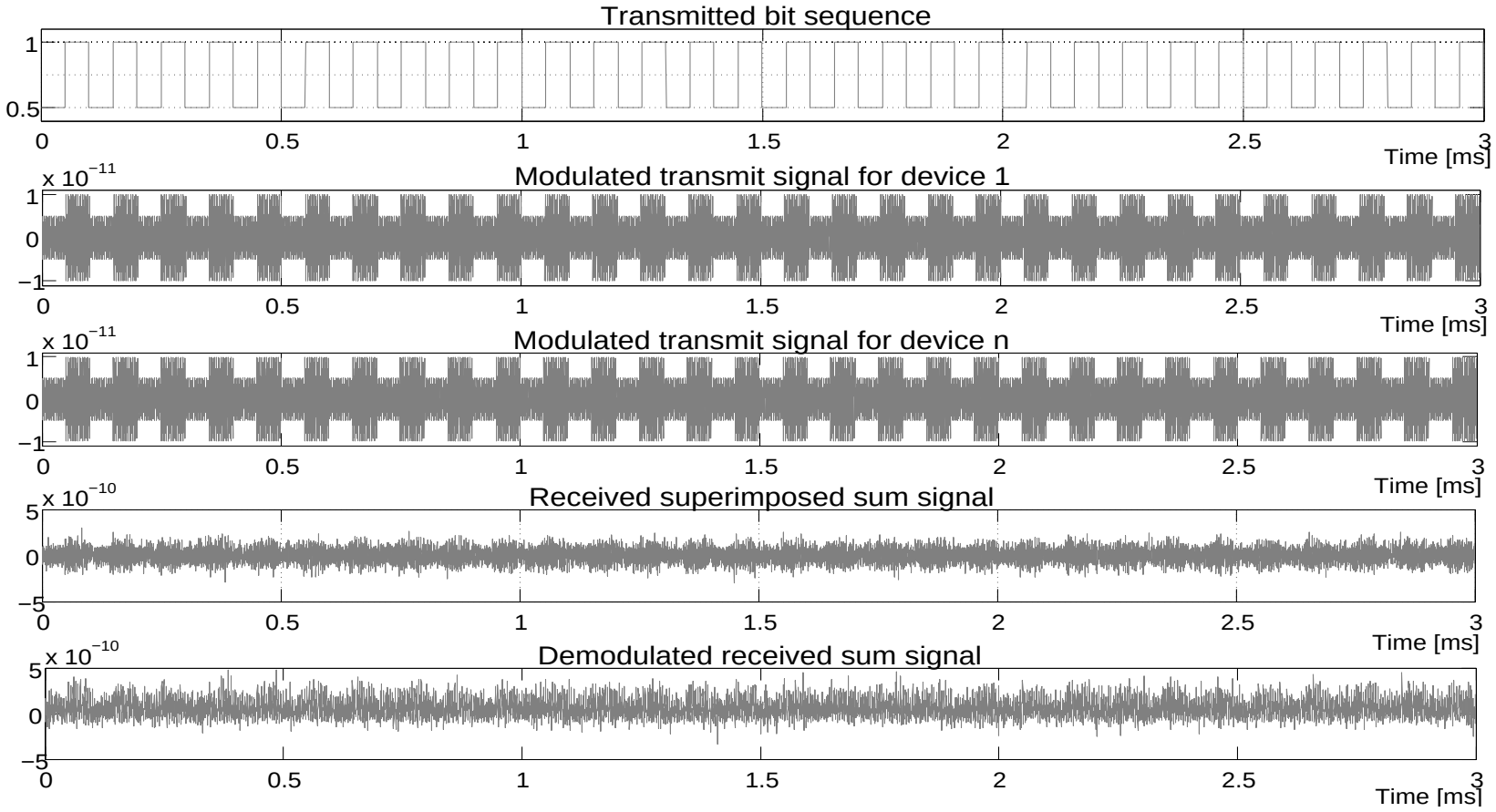

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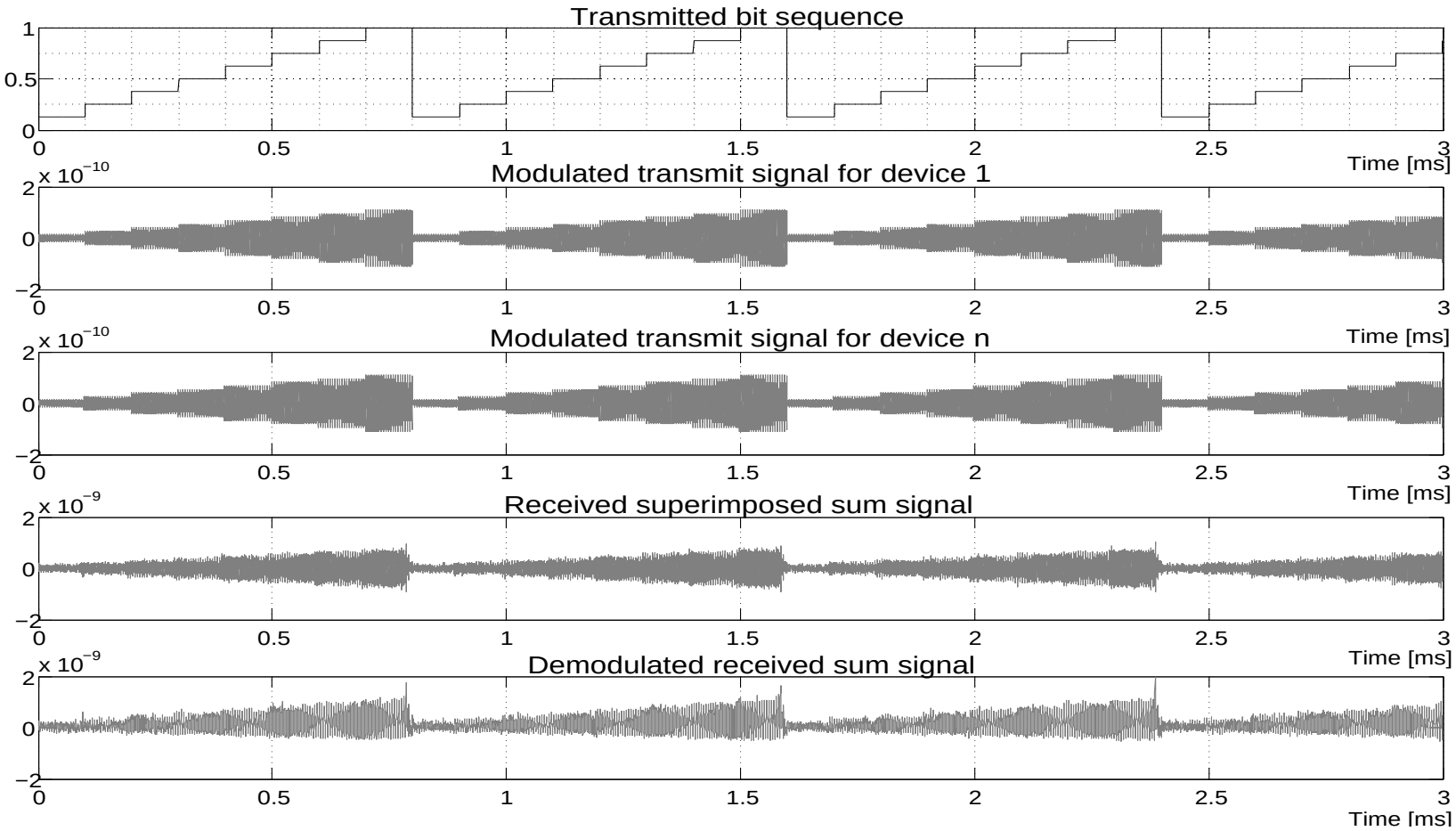

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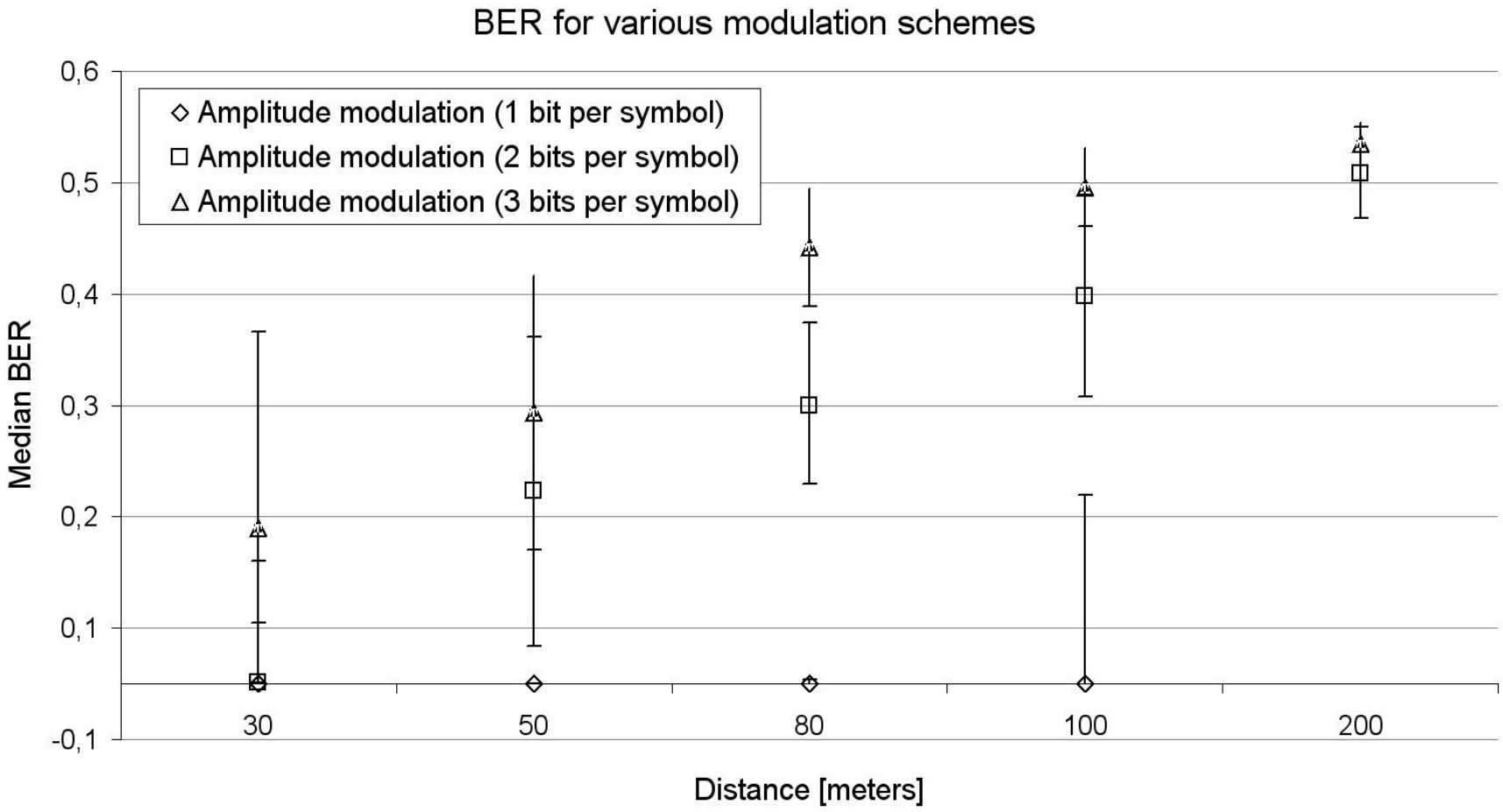

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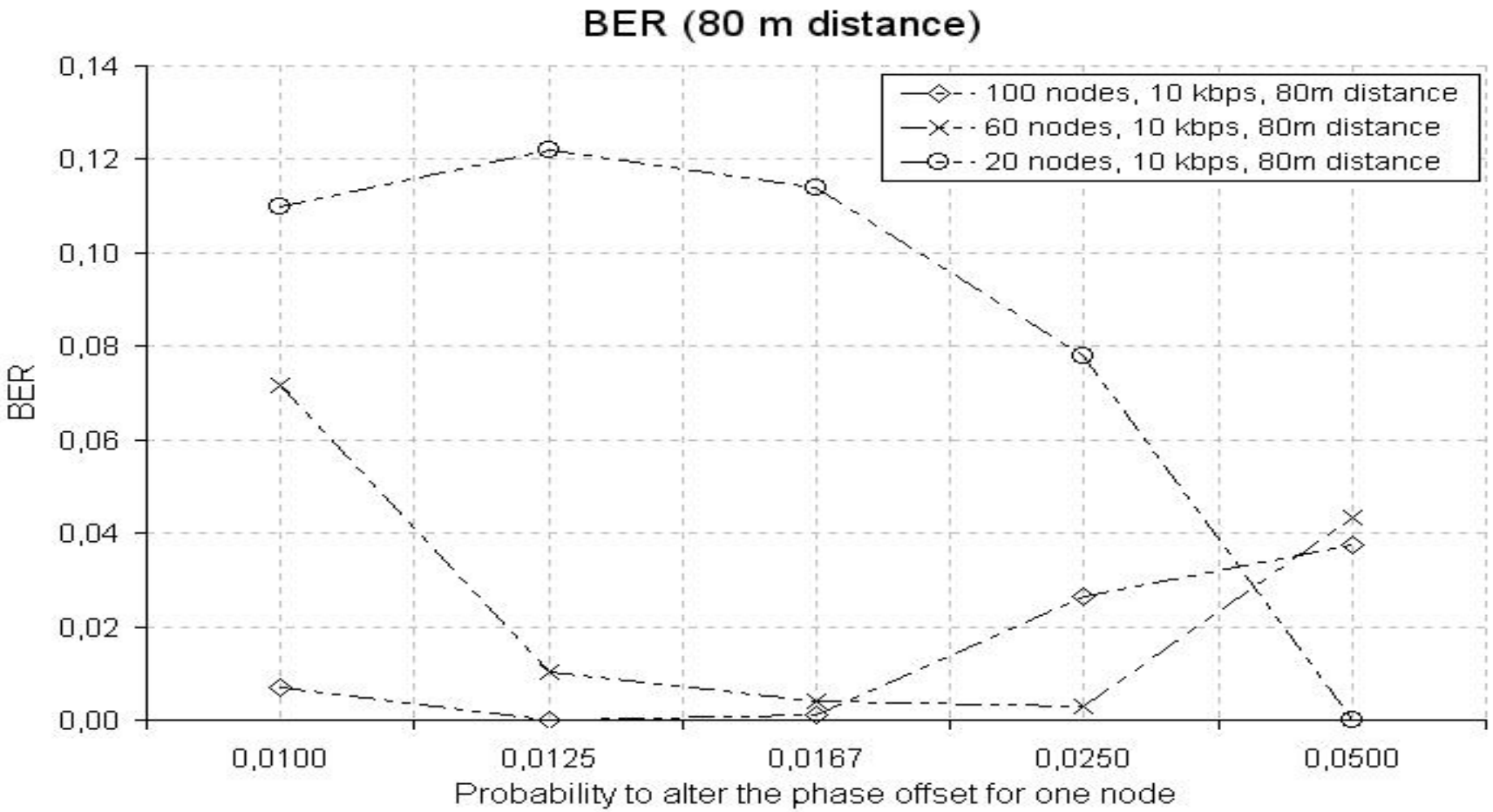

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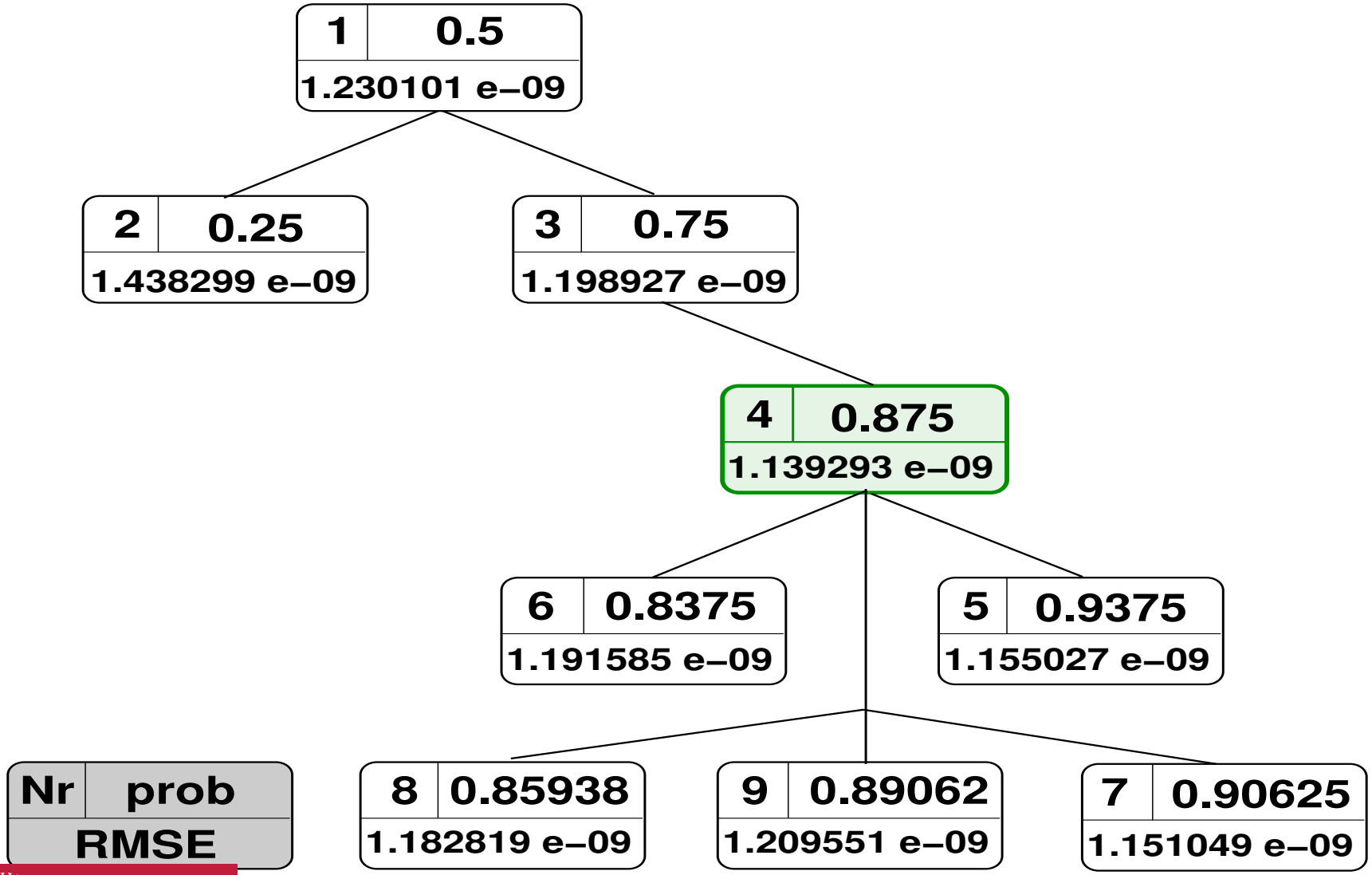
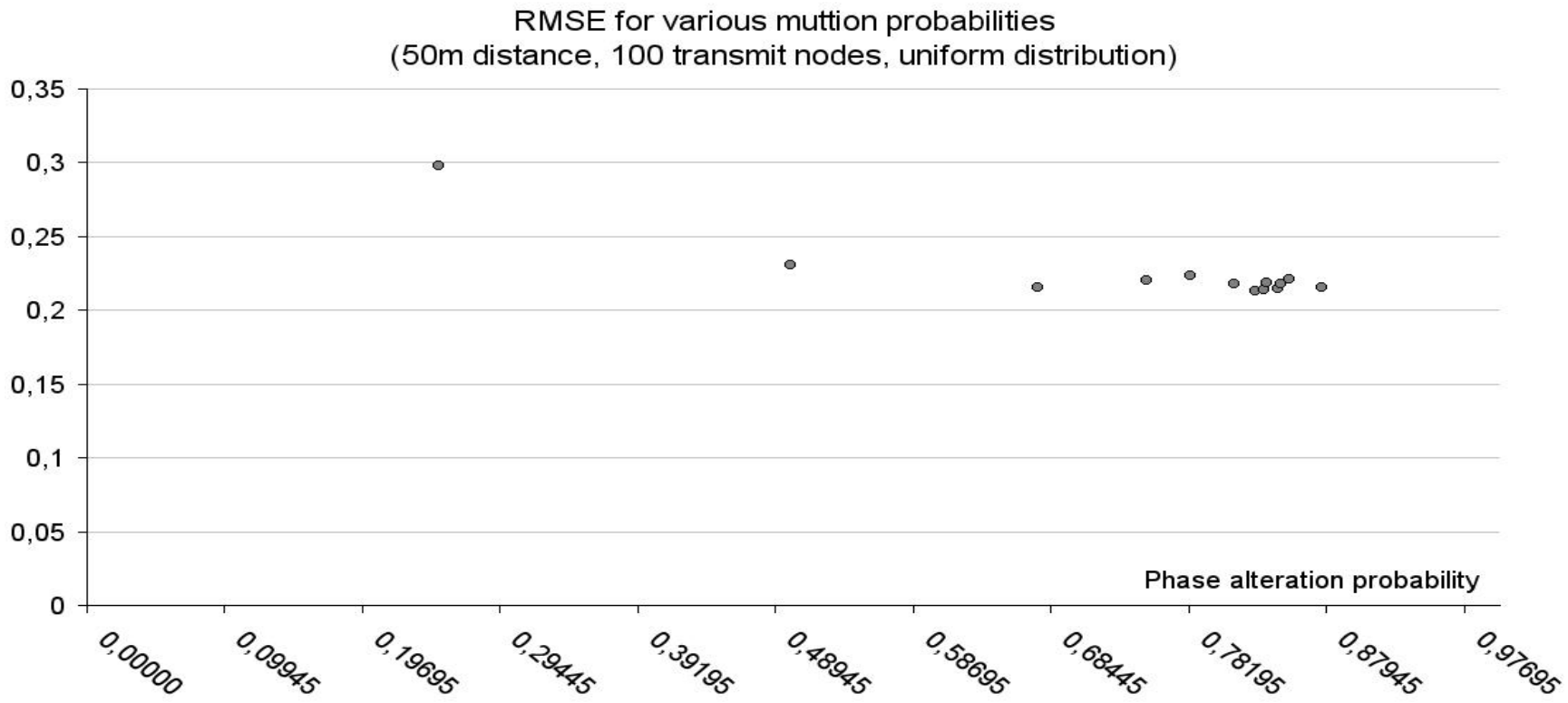


An adaptive protocol for distributed adaptive beamforming



An adaptive protocol for distributed adaptive beamforming

Situation	mean	median	σ
Door state (opened/closed)	0.952	0.9513	0.0099
Presence of individual	0.817	0.8238	0.0455
Phone call (gsm)	0.9	1.0	0.32
Door opened (cond.: Empty room)	1.0	1.0	0.0
Door closed (cond.: Empty room)	1.0	1.0	0.0
Door closed (cond.: Room occupied)	0.832	0.83	0.041
Door opened (cond.: Room occupied)	0.976	0.98	0.0184
Room occupied (cond.: Door closed)	0.673	0.66	0.1143
Room occupied (cond.: Door open)	0.595	0.54	0.1247
Empty room (cond.: Door closed)	1.0	1.0	0.0
Empty room (cond.: Door open)	1.0	1.0	0.0

Questions?

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