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A Reliable Optimization Model for the Emergency Medical Service Systems

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Abstract In the context of Emergency Medical Service (EMS) systems, this paper addresses the dynamic and stochastic setting of the EMS systems. A dynamic location model is presented that tries to locate and relocate the vehicles in order to provide the best coverage of the emergency service demands. The dynamic aspect of the model controls the mouvements and the locations of the vehicles and the statistical parameters of the model provide stable vehicle locations for the simultaneous emergency service demand situations. Some numerical experiments have been carried out by using some real-world data sets that have been collected through the French emergency medical service system. The proposed model have been generalized to a stochastic one. The stochastic model has the ability of providing robust solutions in the stochastic setting of the emergency medical service systems.

Mathematics Subject Classification (2000)

Keywords Emergency medical service · Optimization · Location problems · Integer programming · Reliability.

1 Introduction

Due to the crucial role of the Emergency Medical Services (EMS) in saving the lives, numerous researches have been carried out in order to improve the quality of EMS systems. Different research directions have been taken into account. Some examples are adaptation

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of the service modes to the changes in the costumer needs (such as home care services), personnel scheduling in the medical centers, location of the service centers, etc. In any case, two main objectives are saving the lives (by reducing the mortality in the emergency cases) and reducing the costs.

In this context, the problem of locating the emergency service vehicles has attracted special attention during decades of research. The vehicle location problem in the context of EMS systems is dealing with locating the vehicles in some potential service sites in order to reduce the delay of covering the emergency service demands. The expected time limit depends on the country and can vary from nine minutes in Canada to about 15 minutes in some other countries.

Each emergency service vehicle is completely equipped to all emergency, so it is so expensive to buy any of this vehicles. Due to the fact that each emergency service has access to a limited number of the vehicles, it is important to optimally locate them in order to improve the responsiveness of the system.

Literature review

The earliest EMS models have been introduced in 70s by the article of Toregas et al. [35] and during decades the location problems of EMS vehicles became an active research area and numerous papers have been published on this topic. The published papers can be classified according to different types of criteria. One of the classifications is based on the objectives of the emergency services, that is their accessibility, adaptability, and availability [14].

The accessibility criteria concerns the ability of the patients (respectively, the emergency medical service provider) to reach a service center (respectively, the patient).

In the context of the adaptability criteria, the models try to take into account some kind of flexibility. The motivation of these models is based on the fluctuations that take place in service quality of the service centers. The fluctuations are due to the changes that occur in EMS systems, for example a vehicle can be found busy because of providing emergency service to a patient.

The models that are based on the availability criteria try to mixe the above models or to improve them by introducing some minor extensions [14].

Another classification can be made by distributing the models in three categories depending on their nature: static, dynamic, or stochastic models. An overlapping of the models can be found in this classification.

In what follows, we present a non comprehensive review of the models in chronological order. We start by the earliest static EMS models.

Historically speaking, the Location Set Covering (LSC) model of Toregas et al. is the earliest model of ambulance locating. The objective of the model is to minimize the number of the necessary ambulances for covering all demand points. The LSC model can penalize the users of the model by its expensive solutions; because it may provide a necessary number of ambulances that is so larger than it would be. Furthermore, the LSC model is so rather basic and it does not permit location of more than one ambulance in a service center.

Due to the limits of the LSC model, the Maximal Covering Location problem (MCLP) has been presented by Church et al. [9]. The MCLP model tries to maximize the covered population by taking into account a predefined number of ambulances.

The LSC and MCLP models are static models, in the sense that they do not take into account the possible fluctuations in the EMS system. In fact, when a call arrives to the call center of the EMS service center, it may need affectation of an ambulance. If such need is confirmed, an ambulance will be affected to cover the demand point. At this stage, the corresponding ambulance will be no longer available. Consequently, the static models must be solved from scratch for a less number of ambulances. This procedure is computationally expensive. At this point, one may use the dynamic models.

Another inconvenience of the LSC and MCLP models, is due to the problem of simultaneous emergency calls. That is, it is possible to receive more than one emergency service demand call at the same time or in a very short time delay. Each of the service demand points must be covered, consequently we may need to support the zones by more than one ambulances. The classical LSC and MCLP models are not able to provide such service.

In order to overcome the inconveniences of the static models, several approaches have been introduced in the literature. One approach consists of employing more than one ambulance to cover the simultaneous emergency demand calls. Multi-stage models are used too. These models are served to update the situation of ambulances according to the changes in the covered zones. Also, some dynamic models have been introduced in the literature that propose dynamic features. These models try to take into account any change of the system in order to present a better coverage of the emergency zones.

Daskin and Stern have proposed the *hierarchical objective set covering problem (HOSC)* and introduced the concept of multiple coverage [11]. The HOSC model tries to minimize the number of necessary ambulances for covering the demands and to maximize the multi-coverage of the demand zones. This model has some inconveniences, for example, the model may privilege the congestion of the ambulances.

In order to overcome the inconveniences of the HOSC, Hogan and ReVelle proposed the *Maximal Backup Coverage* models (1) and (2) (BACOP 1, BACOP 2) [23]. These models use two ambulances to cover the demands.

In the same context of multiple ambulances, the *Tandem Equipment Allocation Model (TEAM)* has been introduced by Schilling et al. [31]. The TEAM uses two kind of ambulances to cover the demands.

Storbeck and Slack [33] had the idea of using the goal programming approach. In their model that is called *maximal-multiple location covering problem (MMLCP)* model, they try to minimize the non-covered population and to maximize the multiple coverage by using a given number of ambulances. This model can be reformulated in form of the model BACOP 2. Similarly to the HOSC model, the MMLCP model may suffer from the congestion of the vehicles.

The double standard model (DSM) [19] is another example of the models that use multiple ambulances in covering the demands. The DSM model is based on the standards of the North America; in which, all demands must be covered by an ambulance within r_2 minutes and a proportion α of the total demand must be covered within r_1 minutes. Obviously, $r_2 > r_1$.

The multi-coverage models try to handle the problem of the uncertainty of demands. Stochastic programming is another approach that is used to take into account the uncertainty. In spite of the multi-coverage models, the stochastic programming models try to cover the uncertainty in a more explicit way.

In the stochastic models, the origins of uncertainty in considered to be either the availability of the ambulances (vehicles) or the occurrence of the service requests at the demands points. The uncertainty of a vehicles comes back to the situation where the vehicle is occupied. Some studies rely the uncertainty on the occurrence of the service requests at the demands points, because no one knows where and when an emergency service demand will take place.

Concerning the literature, Daskin has studied the expected covering model and maximal expected location problem ([12, 13]). In these models the availability of the vehicles is the source of uncertainty. The maximal expected covering location problem (MEXCLP), tries to maximize the expected covering by using a given number of vehicles. The model uses an occupation rate parameter, to which the model has some level of sensitivity. Different versions of MEXCLP has been proposed by different authors (see [4, 25]).

There is an approach in stochastic programming in which an expected level of reliability is imposed to the model via some probabilistic constraints. This approach has found some applications in EMS context.

ReVelle and Hogan [30] proposed the *maximal availability location problem (MALP)* that maximizes the covered population for a given level of reliability. The model (MALP) uses a probabilistic constraint for assuring the reliability level.

Up to now, all of the presented models use the possible unavailability of the vehicles as the main source of uncertainty. In spite of these models, one can take into account the uncertainty by studying directly the random character of the emergency demands.

In this context, Ball and Lin [3] introduced a stochastic programming model (REL-P) that uses a probabilistic constraint in order to assure the emergency medical services for a given level of reliability. Beraldi et al. [5] introduced a robust model by using a probabilistic constraint. They present a location and sizing model that includes a chance constraint for assuring, under a certain probability, the presence of a sufficient number of vehicles.

The models that we presented are all single-period models that their solutions can be no more correct once a change occurs in the system. The multi-stage models and dynamic models try to handle this problem.

The multi-period models are different from the single-period models by the fact that the multi-period models try to take into account the changes in the system. For this aim, the they revisit the parameters of the model in some predefined time-points. At each revision, some adjustments are necessary in order to keep each part of the model in consistency with the other. Since the multi-period models do the revisions in some predefined time-points, any change occurring between two consecutive time-points makes the model inconsistent with the dynamic setting of the system. In order to overcome this inconvenience of multi-period models, the dynamic models have been introduced. There are some dynamic models in EMS literature.

Gendreau et al. has introduced a dynamic redeployment problem (RP^t) [20]. Their model maximizes the demand points that are covered two times and minimizes the costs associated to the mouvements. A penalty parameter is introduced in order to take all changes into account. The penalty parameter is adjusted at any revision of the model and penalizes:

- frequent mouvements,
- redeployments associated to long distances,
- round-trips.

The RP^t model has been successfully applied in the EMS system of Montreal. The model has also been used by Thirion [34] in optimization of EMS system of Wallonia, Belguim.

Another dynamic model has been proposed by Andersson and Varband [2]. Their model minimizes the maximal time accorded to the mouvements of the vehicles between different service centers. The authors present the notion of *preparedness* in order to have a dynamic setting in the model. The *preparedness* is defined as the capacity of the system in covering the present and future emergency service requests. It is used to measure the performance of the EMS system. The level of preparedness is verified regularly and the vehicles are redeployed, if and only if the level of preparedness goes down a prescribed threshold.

$i \in I := \{1,\ldots,n\}$	i is a demand point and I is the set of all potential demand points;
$j \in J := \{1,\ldots,m\}$	j is a service point (center or site) and J denotes the set of all service centers;
$k \in \{1,\ldots,\lvert K\rvert\}$	k is an ambulance and K is the set of all ambulances;
U_j	denotes the maximum number of the ambulances that can be located in the service center j;
d_i	denotes the mean density of the emergency demands at the point i;
r_1 and r_2	are the time thresholds to be respected in covering any demand point ($r_1 < r_2$);
γ_{ij}	a binary parameter that denotes whether a demand point i is accessible from the service center j in r_1 minutes;
δ_{ij}	a binary parameter that denotes whether a demand point i is accessible from the service center j in r_2 minutes;
$\alpha \in [0, 1.00]$	a real number indicating the proportion of all emergency service demands that must be covered in a given delay;
M^t_{jk}	a real valued parameter for controlling the relocations and mouvements of the vehicles at each period t;
$y_{jk} \in \{0,1\}$	to say whether the ambulance k is located in the service point j;
$x^\lambda_i \in \{0,1\}$	to say whether the demand point i is covered λ times (for $\lambda = 1$ or 2).

Table 1 Notation

In this paper, we are interested in adjustment of the RP^t [20] to the French EMS system. In this context, the demand points have no need to be covered by two ambulances but we introduce some new parameters in order to take into account the probability of simultaneous emergency service requests. The presented model is compared to the RP^t in terms of its capacity in covering the emergency service demands. We are then interested in introducing a stochastic programming model. To this aim, we use the presented deterministic model as the underlying model and then, we present our stochastic programming model. The stochastic model includes a chance constraint in order to guarantee the coverage of emergency service requests for a given level of reliability.

The structure of the paper is as follows. The RP^t model [20] is reviewed and our dynamic model is presented in the section 2. The models are tested on real-world data sets collected from the French EMS system. The computational experiments are reported in the section 3. The appendix **A** contains an stochastic formulation of the introduced dynamic model and the appendix **B** is devoted to a worst-case model in the EMS context. Finally, last section includes some conclusions and future work.

2 Dynamic location and relocation model

In this section we present our dynamic location and relocation model. The proposed model is an extension of the RP^t model introduced by Gendreau et al. [20]. For the sake of completeness, we first give a presentation of the RP^t model. In order to present this model, we need some notations that are presented in Table 1. The RP^t model of Gendreau et al. [20] is tailored to the EMS systems of North America. According to this context, all ambulances must be located in the service centers in a way to cover all emergency service demands in r_2 minutes. Furthermore, there is a relative coverage demand constraint. According to this constraint, a proportion α of total demands must be covered in r_1 minutes (where $r_1 < r_2$).

The RP^t model of Gendreau et al. [20] is a dynamic location and relocation model that tries to maximize the backup coverage demand, i.e., the model maximizes the proportion of the demands that are covered by at least two ambulances within a radius r_1, minus the cost associated to the relocation of the vehicles.

The RP^t model is presented as follows:

(RP^t) :

$$\max \quad \sum_{i=1}^{n} d_i x_i^2 - \sum_{j=1}^{m} \sum_{k=1}^{|K|} M_{jk}^t y_{jk} \tag{1}$$

$s.t.$

$$\sum_{j=1}^{m} \sum_{k=1}^{|K|} \delta_{ij} y_{jk} \geq 1 \qquad : i = 1,\ldots,n, \tag{2}$$

$$\sum_{i=1}^{n} d_i x_i^1 \geq \alpha \sum_{i=1}^{n} d_i, \tag{3}$$

$$\sum_{j=1}^{m} \sum_{k=1}^{|K|} \gamma_{ij} y_{jk} \geq x_i^1 + x_i^2 \qquad : i = 1,\ldots,n, \tag{4}$$

$$x_i^2 \leq x_i^1 \qquad : i = 1,\ldots,n, \tag{5}$$

$$\sum_{j=1}^{m} y_{jk} = 1 \qquad : k = 1,\ldots,|K|, \tag{6}$$

$$\sum_{k=1}^{|K|} y_{jk} \leq U_j \qquad : j = 1,\ldots,m, \tag{7}$$

$$x_i^1, x_i^2 \in \{0,1\} \qquad : i = 1,\ldots,n, \tag{8}$$

$$y_{jk} \in \{0,1\} \qquad : \forall j \text{ and } \forall k. \tag{9}$$

In this model, the objective is to maximize the demand points that are satisfied two times and to minimize the costs related to the relocation of the vehicles. The constraint (2) ensures the absolute coverage of the demands and the constraint (3) ensures the coverage by a single vehicle. The set of the constraints (3) and (4) specify the proportional coverage (in α percentage) of the demands. Furthermore, the constraint (4) ensures a sufficient number of vehicles for a single or double coverage. While x_i^1 is equal to 1, the demand i is covered by an ambulance, hence x_i^2 can be equal to 1. In other words, the demand point i can be covered twice, if it is already covered at least once. This is summarized in the constraint (5). According to the constraint (6), each ambulance must be assigned to a potential service center. Finally, an upper bound is defined, by the constraints (7), on the number of the ambulances that can be assigned to a service point.

2.1 Improved dynamic redeployment model

To our knowledge, the RP^t model is an ideal formulation for solving a double coverage problem in a dynamic setting. The double coverage will help to overcome, even partially, the randomness of the demands. Despite this fact, the RP^t model can be changed for being adapted to new situations or new EMS systems. The following contributions have been made for this aim:

(i) The model has been changed in order to be casted into the French EMS system. In fact, unlike the North America, the French EMS system imposes no kind of obligation in multi-coverage of the demands. Furthermore, we believe that the new model has some advantages in comparison to the RP^t model. In fact, some new parameters have been introduced into the model. The new parameters help to take into account different kinds of coverage. According to the demand point, one may need to give more importance to the double coverage in the demand point in comparison to another one. This fact is included explicitly in the new model.
(ii) The improved model has been used as an underlying deterministic model for an extension to a stochastic one. The stochastic model includes a chance constraint that gives a more robust design for locating the ambulances.

In order to present our model, we need to define two new parameters. We will use the parameters d_i^1 and d_i^2 for specifying, respectively, the mean occurrence number of the all service requests and the mean occurrence number of the simultaneous emergency demands at the point i. The other notations are the same as the RP^t model. Our dynamic redeployment problem at time t, i.e., DRP^t, is formalized as follows:

(DPR^t) :

$$\max \quad \sum_{i=1}^{n}(d_i^1 x_i^1 + d_i^2 x_i^2) - \sum_{j=1}^{m}\sum_{k=1}^{|K|} M_{jk}^t y_{jk} \tag{10}$$

$s.t.$

$$\sum_{j=1}^{m}\sum_{k=1}^{|K|} \delta_{ij} y_{jk} \geq 1 \qquad : i = 1,\dots,n, \tag{11}$$

$$\sum_{i=1}^{n} d_i^1 x_i^1 \geq \alpha \sum_{i=1}^{n} d_i^1, \tag{12}$$

$$\sum_{j=1}^{m}\sum_{k=1}^{|K|} \gamma_{ij} y_{jk} \geq x_i^1 + x_i^2 \qquad : i = 1,\dots,n, \tag{13}$$

$$x_i^2 \leq x_i^1 \qquad : i = 1,\dots,n, \tag{14}$$

$$\sum_{j=1}^{m} y_{jk} = 1 \qquad : k = 1,\dots,|K|, \tag{15}$$

$$\sum_{k=1}^{|K|} y_{jk} \leq U_j \qquad : j = 1,\dots,m, \tag{16}$$

$$x_i^1, x_i^2 \in \{0,1\} \qquad : i = 1,\dots,n, \tag{17}$$

$$y_{jk} \in \{0,1\} \qquad : \forall j \text{ and } \forall k. \tag{18}$$

The differences between the models RP^t and DRP^t are in the objective functions and the proportional coverage constraints. Similarly to the RP^t model, the DRP^t maximizes the coverage of the emergency demands but the DRP^t model uses two parameters. The first one, i.e., d_i^1, is associated to the mean occurrence number of the demands at the point i. Due to the randomness of the demands, there may happen situations while some simultaneous demands occur. By enforcing double coverage of the demand points, we can, even partially, cover this kind of situations. To this aim, a second parameter, i.e., d_i^2 is used. The value of these parameters depends on the mean occurrence number of all demands and simultaneous demands (during a time unit) at the demand point i. These values can be calculated by using the historical data. It is clear that $d_i^1 \geq d_i^2$, furthermore d_i^1 may be significantly larger than the values of d_i^2. This is due to fact that the simultaneous demands at a point are rarely happen in comparison to the number of all demands. Consequently, a single ambulance is, in general, sufficient to cover a service request at a demand point i. The objective of the model DRP^t is to cover the demands by taking into account the weights associated to the two categories of demands. Finally, there is a difference (between models RP^t and DRP^t) concerning the proportional coverage constraints (3) and (12). In fact, the weight d_i^1 is used in place of the parameter d_i. The substitution of d_i by d_i^1 in the constraint (12) can be considered as a generalization of the constraint (3). In fact, d_i^1 can be interpreted as the mean occurrence number of all non-simultaneous demands.

3 Computational Experiments

3.1 Data description

The models were used for the EMS system in Val-de-Marne, a county in Paris (France). The population of the county amounts to approximately 1.3 million inhabitants and it is divided to three parts and 47 quarters (see Fig. 1).

The EMS call center of the county of Val-de-Marne receives more than 1000 calls per day, but just a small part of the calls requires a coverage by an ambulance. The number of the covered calls are, in general, between 20 and 30 calls per day. Each of these calls must be covered in less than 10 minutes. 8 ambulances are in use in the county and they cover not only the emergency demands in the 47 quarters of the county, but also some emergency demands coming from other quarters. These belong to the counties that are in the common frontiers with the county of Val-de-Marne. There are 30 potential service centers in the county, but only 12 centers are in daily use (see Fig. 2). The objective of this case study was to improve the EMS service of the county, in order to cover a given percentage of all emergency service demands. For our computational experiments we used some recently collected data from the EMS system of the county. Data collection has been made possible by means of a new GPS localization system. It has been installed in the Hospital Henri Mondor, that is located in the county of Val-de-Marne. The system has been installed in the late 2010. Since the system was newly installed, the collection and treatment of data was not straightforward. Some treatments were made manually. The treatment of the data continues and we use the exploited data of some early months of 2011.

The standard solver IBM Cplex (version 12.2) has been used to solve the mathematical optimization models corresponding to the case study.

3.2 Results

In order to emphasize the introduction of new parameters in the model DRP^t, we solved the models RP^t and DRP^t under same conditions and compared the results. The results that are obtained reflect the behavior of the introduced model.

The figures show the experiments that have been carried out on two consecutive time periods. The figures show the coverage proposed by the models RP^t (shown on the figures by RP) and DRP^t (shown on the figures by DRP). Different proportional coverage percentages have been taken into account. They vary from 90 to 100. For some of these values, the

Fig. 1 The county of Val-de-Marne is divided to 47 quarters.

Fig. 2 The quarters of Val-de-Marne, numbered from 1 to 47, and the 12 service centers that are in use.

corresponding optimization model was not feasible, and consequently no result is shown on the figures that corresponds to the infeasible models.

The Figures 3, 4, and 5 show the results for the starting time period. At this period the values of M^t_{jk} are all initialized by zero (for all j and k). The Fig. 3 correspond to the demand calls that can be covered by single vehicle. In a similar way, the Fig. 4 correspond to the demand calls that need a double coverage due to simultaneous demands. Finally, the Fig. 5 corresponds to the sum of the single coverage and double coverage.

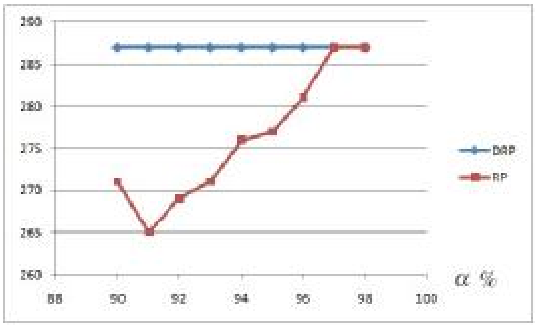

Fig. 3 The number of the single covered demands for different proportions of the total demands (starting period).

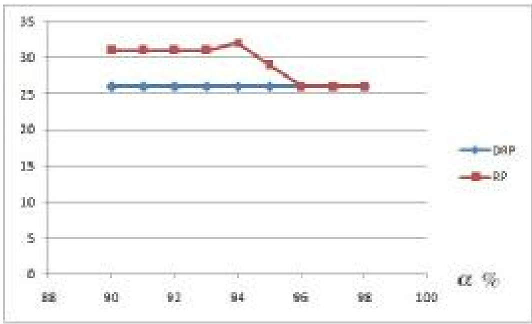

Fig. 4 The number of the double covered demands for different proportions of the total demands (starting period).

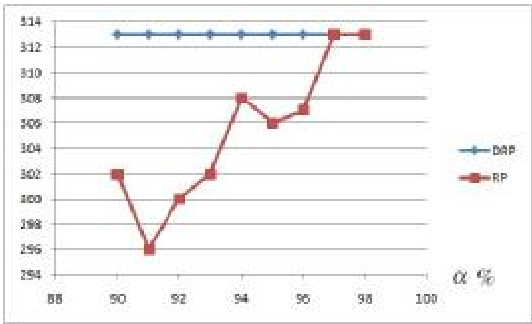

Fig. 5 The sum of the single and double covered demands for different proportions of the total demands (starting period).

Some observations on the results of the first period are as follows:

– For a given proportion of demand to be satisfied, the behavior of the two models is significantly different. The value of the covered demands remains stable in the DRP^t model, but this value may decrease or increase in the RP^t model. This observation can be justified by reviewing the structure of the objective functions. The variables x_i^1 and x_i^2) (weighted by d_i^1 and d_i^2, respectively) are present in the objective function the DRP^t model, but this is case of the RP^t model. Due to this fact, the curves of the DRP^t model are monotone.

- According to the Fig. 3, the DRP^t model present a deployment policy of the ambulances with a better coverage in comparison to the RP^t model. The coverage includes all types of the call, i.e., simultaneous demands as well as the non-simultaneous ones. In contrary to the Fig. 3, the Fig. 4 presents a better coverage provided by the RP^t model. The difference is always justified by the structure of the objective functions in the DRP^t model and the RP^t model. The RP^t model includes only the x_i^2) (that is weighted by d_i^1), which privileges the double coverage.
- The Fig. 5 concerns the sum of the single covered and double covered demands. The curves present a kind of superiority of the DRP^t model in comparison to the RP^t model concerning the coverage of the demands. When the proportional of the covered demands increases, there is a convergence of the curves toward common values. The gap between the curves is in direct relation with the difference between the mean number of the single covered demands and double covered demands.

In order to pass to the second period, we need to adjust the values of M_{jk}^t. The main issue was to reduce the mouvements of the vehicles. Based on this policy, the distance between the service centers j and k has been considered as the value of M_{jk}^t (for all j and k). Under the same policy, if $y_{jk} = 1$ then the corresponding penalty parameter is fixed to zero, i.e., $M_{jk}^t = 0$.

Furthermore, we suppose that in the second period one of the vehicles is busy because of a mission. Hence we must solve the optimization models with one vehicle less than the previous period, i.e., $K = 7$. The results are depicted in the Figures 6, 7, and 8.

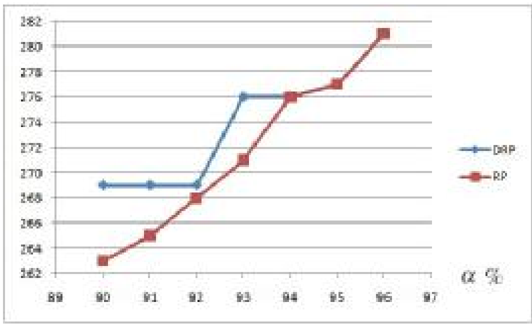

Fig. 6 The number of the single covered demands for different proportions of the total demands (second period).

Some observations on the results of the second period are as follows:

- The results of the second period are significantly different from the results of the first period. Neither the curves of the DRP^t model are monotone nor the curves of the RP^t model. We remember that the values of M_{jk}^t are adjusted in a way to reduce the total

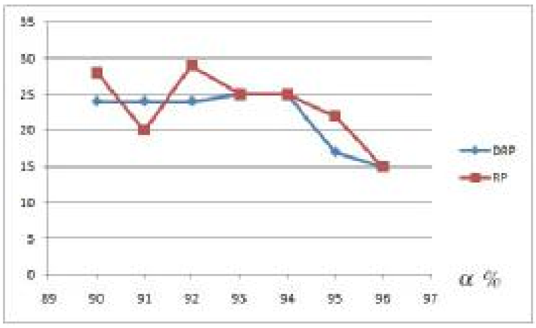

Fig. 7 The number of the double covered demands for different proportions of the total demands (second period).

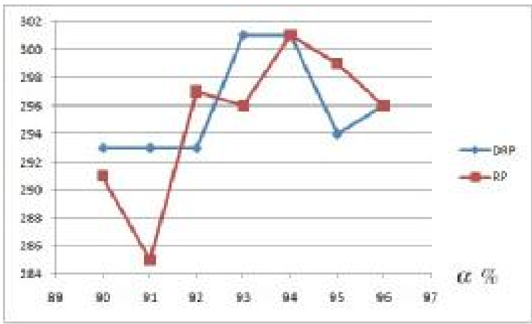

Fig. 8 The sum of the single and double covered demands for different proportions of the total demands (second period).

mouvements of the ambulances. Due to this fact, when we consider two DRP^t models corresponding to two different values of α, the corresponding objective functions of the models may be different. Any similarity in the solutions of the first period may provide similar models for the second period.

- Similarly to the first period, we observe that the DRP^t model provides solutions for which we have a better coverage of the demands. Furthermore, there is no more absolute superiority in the quantity of the double covered demands by the RP^t model in comparison to the DRP^t model.
- In all of the cases, there is a convergence to a common value when the value of α tends to 100 %.

Appendix A

In this appendix we are interested in introducing a reliable optimization model. We address the reliability issue by introducing a model in the stochastic programming framework. The stochastic model includes a probabilistic constraint in order to provide robust EMS solutions. The DRP^t is used as the underlying deterministic model to establish the stochastic programming model.

The main motivation in establishing a stochastic model is the presence of the uncertainty in EMS context. Different sources of uncertainty are present in this context. The data of the models may contain some errors due to the changes in the system. Furthermore, any EMS system has a dynamic setting and the changes in the system are not known with certainty. In order to overcome the uncertainty issue different approaches may be used. One of them consist of establishing reliable models. In what follows, we address the reliability by using the probabilistic constraints. By using the probabilistic constraints, we ensure the robustness of the provided solutions with a desired level of probability. In a similar way to [5], we assume that the main source of uncertainty is related to the randomness the requests at the demand points. Hence, we measure the reliability of the system by its ability in coverage of the random service demands with a prescribed level of probability. We denote this probability by α and use the integer-valued random variables ξ_i to say whether and how many random service requests are generated at the demand point i, for $i = 1,\dots,n$. Using these notations, any positive value of the random variable ξ_i means that a service request has taken place at the demand point i. In order to cover the demand point i, the variable x_i^1 must be equal to 1. We are interested in establishing a emergency service by which any demand point has the chance of being covered under the prescribed probability α. Mathematically speaking, the condition is translated as follows:

$$p(x_i^1 \geq min(1,\xi_i)) \geq \alpha \quad : i = 1,\dots,n. \tag{19}$$

The random variables ξ_i are supposed to be independent. This is a common assumption in the EMS literature.

Replacing the constraint (12) by the constraints (19), will give us the following chance constrained optimization model:

$(P - DRP^t)$:

$$\max \quad \sum_{i=1}^{n}(d_i^1 x_i^1 + d_i^2 x_i^2) - \sum_{j=1}^{m}\sum_{k=1}^{|K|} M_{jk}^t y_{jk} \tag{20}$$

$s.t.$

$$\sum_{j=1}^{m}\sum_{k=1}^{|K|} \delta_{ij} y_{jk} \geq 1 \quad : i = 1,\dots,n, \tag{21}$$

$$p(x_i^1 \geq min(1,\xi_i)) \geq \alpha \quad : i = 1,\dots,n, \tag{22}$$

$$\sum_{j=1}^{m}\sum_{k=1}^{|K|} \gamma_{ij} y_{jk} \geq x_i^1 + x_i^2 \quad : i = 1,\dots,n, \tag{23}$$

$$x_i^2 \leq x_i^1 \quad : i = 1,\dots,n, \tag{24}$$

$$\sum_{j=1}^{m} y_{jk} = 1 \quad : k = 1,\dots,|K|, \tag{25}$$

$$\sum_{k=1}^{|K|} y_{jk} \leq U_j \quad : j = 1,\dots,m, \tag{26}$$

$$x_i^1, x_i^2 \in \{0,1\} \quad : i = 1,\dots,n, \tag{27}$$

$$y_{jk} \in \{0,1\} \quad : \forall j \text{ and } \forall k. \tag{28}$$

In order to solve the $P-DRP^t$, we start by reformulating the model. By using an approach similar to [5] and [15], the $P-DRP^t$ model can be reformulated as an equivalent deterministic form.

First, we note that ξ_i are supposed to be independent, hence the constraint (22) can be rewritten as follows:

$$F_i(x_i^1) \geq \alpha \quad : i = 1, \dots, n, \tag{29}$$

(see [5,15]), where $F_i(.)$ stands for the marginal probability distribution of the random variables $\eta_i := min(1, \xi_i)$ (for $i = 1, \dots, n$). Hereafter, we focus on the model $P-DRP^t$ in which the constraints (22) are replaced by the constraints (29).

Denote by $G_i(.)$ the marginal probability distribution of the random integer-valued variables ξ_i. According to the constraint (29), we can write

$$x_i^1 \geq l_i \quad : i = 1, \dots, n, \tag{30}$$

where $l_i = F_i^{-1}(\alpha)$ stands for the α-quantile of the marginal distribution F_i ([5,15]). In other words, l_i is the smallest integer value such that $F_i(z_i) \geq \alpha$. Under assumption of log-concave marginal distribution, one can write:

$$x_i^1 = l_i + z_i \quad : i = 1, \dots, n, \tag{31}$$

where z_i is a binary variable. We note that 0 and 1 are the only values that l_i can take. Define

$$a_i = F_i(l_i + 1) - F_i(l_i) \quad : i = 1, \dots, n, \tag{32}$$

and

$$\beta = \alpha - F(l), \tag{33}$$

where F is the joint probability distribution function of the random variables η_i for $i = 1, \dots, n$. Using these definitions, we can rewrite constraint (29) as follows:

$$a_i z_i \geq \beta. \tag{34}$$

We note that since $\eta_i := min(1, \xi_i))$ and $G_i(.)$ is the marginal probability distribution of ξ_i, hence the value of the parameters a_i depends indirectly on $G_i(.)$.

Through these transformations, we obtain the following deterministic equivalent formulation of $P-DRP^t$:

$$
\begin{aligned}
&(Det-DRP^t): \\
&\max \quad \sum_{i=1}^{n}(d_i^1 x_i^1 + d_i^2 x_i^2) - \sum_{j=1}^{m}\sum_{k=1}^{|K|} M_{jk}^t y_{jk} && (35)\\
&s.t. \\
&\sum_{j=1}^{m}\sum_{k=1}^{|K|} \delta_{ij} y_{jk} \geq 1 \qquad : i = 1,\ldots,n, && (36)\\
&a_i z_i \geq \beta \qquad : i = 1,\ldots,n, && (37)\\
&x_i^1 = l_i + z_i \qquad : i = 1,\ldots,n, && (38)\\
&\sum_{j=1}^{m}\sum_{k=1}^{|K|} \gamma_{ij} y_{jk} \geq x_i^1 + x_i^2 \qquad : i = 1,\ldots,n, && (39)\\
&x_i^2 \leq x_i^1 \qquad : i = 1,\ldots,n, && (40)\\
&\sum_{j=1}^{m} y_{jk} = 1 \qquad : k = 1,\ldots,|K|, && (41)\\
&\sum_{k=1}^{|K|} y_{jk} \leq U_j \qquad : j = 1,\ldots,m, && (42)\\
&x_i^1, x_i^2 \in \{0,1\} \qquad : i = 1,\ldots,n, && (43)\\
&y_{jk} \in \{0,1\} \qquad : \forall j \text{ and } \forall k. && (44)\\
&z_i \in \{0,1\} \qquad : i = 1,\ldots,n. && (45)
\end{aligned}
$$

Solving the model $Det-DRP^t$ provides α-efficient points for the $P-DRP^t$ model. The random variables have been contributed implicitly by calculation of the parameters a_i and β through the marginal probability distribution functions of ξ_i and η_i.

Appendix B

In the EMS context, the source of uncertainties is related to either the demands or to the vehicles. The stochastic model (that we presented in the **Appendix A**) suppose that the demand points are the main source of uncertainty. In this appendix, we change our mind in order to attribute the uncertainty to the vehicles, more precisely, we are interested in introducing a model that can provide robust solutions in versus the uncertainties in the availability of the ambulances. In order to introduce the model and in addition to the notations 1, we define a new set of variables. Let x_{ki} be the rate of use associated to the vehicle k that can be used for covering the demands at the point i. Clearly, x_{ki} is a nonnegative real number. Consider the following model:

$$
\begin{aligned}
\min \quad \max_{k \in K} \quad & \sum_{i=1}^{n} x_{ki} && (46)\\
s.t. \quad & && \\
& \sum_{j=1}^{m} y_{jk} = 1 \quad && : k = 1, \ldots, |K|, \quad (47)\\
& \sum_{k=1}^{|K|} x_{ki} = d_i \quad && : i = 1, \ldots, n, \quad (48)\\
& \sum_{j=1}^{m} \delta_{ij} y_{jk} \geq \frac{1}{d_i} x_{ki} \quad && : i = 1, \ldots, n; k = 1, \ldots, |K|, \quad (49)\\
& y_{jk} \in \{0, 1\} \quad && : j = 1, \ldots, m; k = 1, \ldots, |K|, \quad (50)\\
& x_{ki} \in \mathbb{R} \quad && : i = 1, \ldots, n; k = 1, \ldots, |K|. \quad (51)
\end{aligned}
$$

By introducing the real valued variable t, the above model can be transformed to the following equivalent form:

$$
\begin{aligned}
& (Min-Max): && \\
& \min \quad t && (52)\\
& s.t. && \\
& t \geq \sum_{i=1}^{n} x_{ki} \quad && : k = 1, \ldots, |K|, \quad (53)\\
& \sum_{j=1}^{m} y_{jk} = 1 \quad && : k = 1, \ldots, |K|, \quad (54)\\
& \sum_{k=1}^{|K|} x_{ki} = d_i \quad && : i = 1, \ldots, n, \quad (55)\\
& \sum_{j=1}^{m} \delta_{ij} y_{jk} \geq \frac{1}{d_i} x_{ki} \quad && : i = 1, \ldots, n; k = 1, \ldots, |K|, \quad (56)\\
& y_{jk} \in \{0, 1\} \quad && : j = 1, \ldots, m; k = 1, \ldots, |K|, \quad (57)\\
& x_{ki} \in \mathbb{R} \quad && : i = 1, \ldots, n; k = 1, \ldots, |K|, \quad (58)\\
& t \in \mathbb{R}. && (59)
\end{aligned}
$$

In the model ***Min-Max***, the objective is to locate the vehicles in the service centers such that the maximum occupation of the vehicles be minimized. The constraint (54) ensures that each vehicle finds a place in a service center. The constraint (55) states that all demands are covered. The constraint (56) ensures a sufficient number of vehicles to cover the emergency service demands.

The model (Min-Max) can become infeasible in most of the situations. In our numerical experiments, we ofetn encountered this situation. This is due to the total demand satisfaction constraint, i.e., (55) and also it can be due to insufficient number of vehicles to satisfy the (56).

4 Conclusion

In this paper, we present a new dynamic location and relocation model in the context of the Emergency Medical Services. Furthermore, we develop a dynamic integer programming model in order to provide some more flexible location and relocation EMS policies. The models are tested and verified on real-world recently collected data. The models are solve efficiently for the studied cases. But it will need some more performant approches for large scale cases. The numerical results show improvements in the coverage of the demands by using the introduced models in comparison to the existing models. Finally two new models have been introduced in this paper. The first one concerns the stochastic formulation of the proposed dynamic model. We postpone the numerical experiments on the stochastic model for another article. The second model concerns the worst-case study in the EMS context. To this aim, we proposed a model based on the busy rate of ambulances.

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References

1. Alsalloum, O. I. et G. K. Rand, Extensions to emergency vehicle location model, *Computers & Operations Research*, vol. 33, pp. 2725-2743, (2006).
2. Andersson, T. et P. Varband, Decision support tools for ambulance dispatch and relocation, *Journal of the Operational Research Society*, vol. 58, pp. 195-201, (2007).
3. Ball, M. O. et L. F. Lin, A reliability model applied to emergency service vehicle location, *Operations Research*, vol. 41, pp. 18-36, (1993).
4. Batta, R., J. M. Dolan et N. N. Krishnamurty, The maximal expected covering location problem : Revisited, *Transportation Science*, vol. 23, pp. 277-287, (1989).
5. Beraldi P., M.E. Bruni, Conforti D., Designing robust emergency medical service via stochastic programming, *European journal of Operational Research*, 158, 183–193, (2004).
6. Beraldi P., M.E. Bruni, A probabilistic model applied to emergency service vehicle location, *European journal of Operational Research*, 196, 323–331, (2009).
7. Brotcorne L., Laporte G., Semet F., Ambulance location and relocation models, *European journal of Operational Research*, 147, 451–461, (2003).
8. Carpentier, G. La conception et la gestion d'un rseau de service ambulancier. Master's thesis, Universit Laval, (2006).
9. Church, R. L. et C. S. ReVelle, The maximal covering location problem, *Papers of Regional Science Association*, vol. 32, pp. 101-118, (1974).
10. Coskun N., Erol R., An Optimization Model for Locating and Sizing Emergency. Medical Service Stations, *Journal of Medical Systems*, 34, 43–49, (2010).
11. Daskin, M. S., A hierarchical objective set covering model for emergency medical service vehicle deployment, *Transportation Science*, vol. 15, pp. 137-152, (1981).
12. Daskin, M. S., Application of an expected covering model to emergency medical service design, *Decision Sciences*, vol. 13, pp. 416-439, (1982).
13. Daskin, M. S., A maximum expected location problem : Formulation, properties and heuristic solution, *Transportation Science*, vol. 17, pp. 416-439, (1983).
14. Daskin M.S., L.K. Dean, Location of health care facilities, In Brandeau, Margaret L.; Sainfort, Francois; Pierskalla, William P. (Eds.) *Operations Research and Health Care: A Handbook of Methods and Applications*, Springer, (2004).
15. Dentcheva D., Prekopa A., Ruszczynski A., On stochastic integer programming under probabilistic constraints, RUTCOR-Rutgers Center for Operations Research RRR 29–98, Rutgers University, 640 Bartholomew Road Piscataway, New Jersey 08854-8003, USA, (1998).

16. Dentcheva D., Prekopa A., Ruszczynski A., Concavity and efficient points of discrete distribution in probabilistic programming, *Mathematical Programming*, 89 55–77, (2000).
17. Eaton, D. J., H. M. U Sanchez, R. R. Lantigua et J. Morgan, Determining ambulance deployment in Santo Domingo, Dominican Republic, *Journal of the Operational Research Society*, vol. 37, pp. 113-126, (1986).
18. Erkut, E., A. Ingolsson, T. Sim et G. Erdogan, Computational comparison of five maximal covering models for locating ambulances, *Geographical Analysis*, vol. 41, pp. 43-65, (2009).
19. Gendreau, M., G. Laporte et F. Semet, Solving an ambulance location model by tabu search, *Location Science*, vol. 5, pp. 75-88, (1997).
20. Gendreau, M., G. Laporte et F. Semet, A dynamic model and parallel tabu search heuristic for real-time ambulance relocation, *Parallel Computing*, vol. 27, pp. 1641-1653, (2001).
21. Gendreau, M., G. Laporte et F. Semet, The maximal expected relocation problem for emergency vehicles, *Journal of the Operational Research Society*, vol. 57, pp. 22-28, (2006).
22. Hakimi, S.L. Optimum locations of switching centers and the absolute centers and medians of a graph. *Operations Research*, 12, 450-459, (1964).
23. Hogan, K. et C. S. ReVelle, Concepts and application of backup coverage, *Management Science*, vol. 34, pp. 1434-1444, (1986).
24. Hongzhong J., Fernando Ordonez, Maged Dessouky, A modeling framework for facility location of medical services for large-scale emergencies, *IIE Transactions*, 39, 41-55 (2007).
25. Mandell, M. B., Covering models for two-tiered emergency medical services system, *Location Science*, vol. 6, pp. 355-368, (1988).
26. McLay Laura A., A maximum expected covering location model with two types of servers, *IIE Transactions*, 41, 730–741, (2009).
27. Rajagopalan, H. K., C. Saydam et J. Xiao, A multiperiod set covering location model for dynamic redeployment of ambulances, *Computers* & *Operations Research*, vol. 35, pp. 814-826, (2008).
28. Repede, J. F. et J. J. Bernardo, Developping and validating a decision support system for location emergency medical vehicles in Louisville, Kentucky, *European Journal of Operational Research*, vol. 75, pp. 567-581, (1994).
29. ReVelle, C. S. et K. Hogan, A reliability constrained siting model with local estimates of busy fractions, *Environment and Planning B*, vol. 15, pp. 143-152, (1988).
30. ReVelle, C. S. et K. Hogan, The maximum availability location problem, *Transportation Science*, vol. 23, pp. 192-200, (1989).
31. Schilling, D.A., Elzinga, D.J., Cohon, J., Church, R.L., ReVelle, C.S., The TEAM/FLEET models for simultaneous facility and equipment sitting. *Transportation Science*, vol. 13, 163-175, (1979).
32. Schmid V., Doerner K.F.,Ambulance location and relocation problems with time-dependent travel times, *European Journal of Operational Research*, 207, 1293-1303, (2010).
33. Storbeck, J., Slack, natural slack and location covering, *Socio-Economic Planning Sciences*, vol. 16, pp. 99-105, (1982).
34. Thirion, A., Modèles de localisation et de réallocation d'ambulances. Application aux communes en provinces de Namur et Brabant Wallon. Ph.D. Thesis, Facultés Universitaires Notre-Dame de la Paix, Namur, Belgium, (2006).
35. Toregas, C., R. Swain, C. S. ReVelle et L. Bergman, The location of emergency service facilities, Operations Research, vol. 19, pp. 1363-1373, (1971).