Computational Geometry

Tutorial #5 — Using Voronoi diagrams

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Organisation

Tutorials: Dates, Times, Topics

- Three tutorials (including today) remain.
- The final tutorial will be exam prep!
- If you have any topics you would like to repeat, send an email!

Date	Tutorial
16.11.2023	Tutorial #1
23.11.2023	Tutorial #2
07.12.2023	Tutorial #3
21.12.2023	Tutorial #4
11.01.2023	Tutorial #5
25.01.2023	Tutorial #6
08.02.2023	Tutorial #7

Organisation
Question Sheet #2

- Covers **Chapters III - VI**
- **Accessible on Course Website**
- Due on **Feb. 1st (in 3 weeks)**
  - Digital (properly formatted!)
  - Sketches where appropriate
  - **You may submit in pairs**
- 12 points, less “quizzy” than #1

Computational Geometry — Winter term 2023
TU Braunschweig, Algorithms Division

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Question Sheet 2

Submit in your solutions, in a single properly (digitally) formatted PDF file, to the folder at `https://nextcloud.ibr.cs.tu-bs.de/s/p5pNRkgYMJE9F5Z` by February 1, 2024. **You may submit this homework sheet in pairs. To do so, please clearly state the full name, field of study, and matriculation number of both partners at the top of the first page.** Name your submission as follows: `[your_matriculation_number].pdf`.

points lie on a common circle). Let $k \in \mathbb{N}$ be arbitrary but fixed.

a) For $k \geq 1$, what does the kth order Voronoi diagram represent?

b) Consider a region of the kth order Voronoi diagram. Argue into how many regions it will be split in the $(k+1)$th order Voronoi diagram.

c) Argue that for $n \geq 3$, the $(n-1)$th order Voronoi diagram forms a tree.

Question 2 (Polygons and triangulations): **(1+1 points)**

Given a simple polygon P with n vertices in general position (no three vertices are collinear).

a) Argue that every convex polygon permits a triangulation that has a dual graph with maximal vertex degree 2.

b) How can you decide in $\mathcal{O}(n \log n)$, if a given point p is inside of the polygon P?

Question 3 (Point triangulation): **(2+1 points)**

Given n points general position, i.e., no three input points are collinear and no four input points lie on a common circle.

a) Briefly, argue why the dual graph of a point sets Voronoi diagram is a Delauney Triangulation.

b) What does solving instances of the NP-hard problems Minimum-weight Triangulation and Max-Min Triangulation reveal?

Question 4 (Miscellaneous): **(2 points)**

Explain the components of a sweep-line algorithm for geometric problems. What are its components, what does it require? Name examples from the lecture!

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Organisation

Question Sheet #2

Question 1 (Voronoi diagram): **(1+1+2 points)**

Question 2 (Polygons and triangulations): **(1+2 points)**

Question 3 (Point triangulation): **(2+1 points)**

Question 4 (Miscellaneous): **(2 points)**

Explain the concept of sweep-line algorithms for geometric problems in your own words. What are its components and requirementments? Name examples from the lecture!

Common mistakes on Sheet #1

Convex Hull
Complexity and Algorithms

(1a) "What is the fastest feasible runtime guarantee of an algorithm which computes [the convex hull]?"

- What is the lower bound, and where does it come from?
- Which algorithms do we know, and what is their runtime?
- Which of these algorithms are (asymptotically) optimal?

Convex Hull
Complexity and Algorithms

Given n points in the plane:

- Lower bound: $\Omega(n \log n)$
- or: $\Omega(n \log h)$

… where h is the output size. We can argue this based on sorting.

Note that worst-case, $n = h$.

Algorithm	(Worst-case) runtime
Jarvis March	$\mathcal{O}(nh)$
Quickhull	$\mathcal{O}(n^2)$
Divide-and-Conquer	$\Theta(n \log n)$
Graham's Scan	$\Theta(n \log n)$
Kirkpatrick and Seidel	$\mathcal{O}(n \log h)$
Chan's Algorithm	$\mathcal{O}(n \log h)$

Runtime analysis

Median of medians + Closest pair

(2a) *“Explain the basic idea of the divide-and-conquer algorithm for computing the closest pair of a set of points.”*

(4c) *“How and how fast can we compute the median of a set of n integers?”*

- *Which algorithm paradigm do we apply with these two?*
- *How do we argue the runtime of such algorithms?*

Master Theorem

Theorem 3.3 (Master Theorem) *Let* $T : \mathbb{N} \to \mathbb{R}$ *with*

$$T(n) = \sum_{i=1}^{m} T(\alpha_i \cdot n) + \Theta(n^k),$$

where $\alpha_i \in \mathbb{R}$ *with* $0 < \alpha_i < 1$, $m \in \mathbb{N}$ *and* $k \in \mathbb{R}$. *Then*

$$T(n) \in \begin{cases} \Theta(n^k) & \textit{for } \sum_{i=1}^{m} \alpha_i^k < 1 \\ \Theta(n^k \log(n)) & \textit{for } \sum_{i=1}^{m} \alpha_i^k = 1 \\ \Theta(n^c) & \textit{with } \sum_{i=1}^{m} \alpha_i^c = 1 \textit{ for } \sum_{i=1}^{m} \alpha_i^k > 1 \end{cases}$$

Closest points pair

Divide-and-Conquer (Bentley and Shamos)

(2a) *"Explain the basic idea of the divide-and-conquer algorithm for computing the closest pair of a set of points."*

(2b) *"What is the key observation in the merging step of Bentley's and Shamos' algorithm?"*

- Which **are** the central steps of this algorithm?
- Why does the *merge* step not take $\Omega(n^2)$ time, but $\mathcal{O}(n)$?

Voronoi diagrams

… and the convex hull

(3c) *“Is there a relationship between the convex hull of a point set and its Voronoi diagram?”*

- What is *duality*? Are the problems of computing the hull and the Voronoi diagram dual problems?
- What can we say about extremal points of a point set in the Voronoi diagram of that set?

Exam preparation!

Once more:

- The final tutorial will be exam prep
- If you have specific topics or questions that you would like to repeat, send an email ahead of time!

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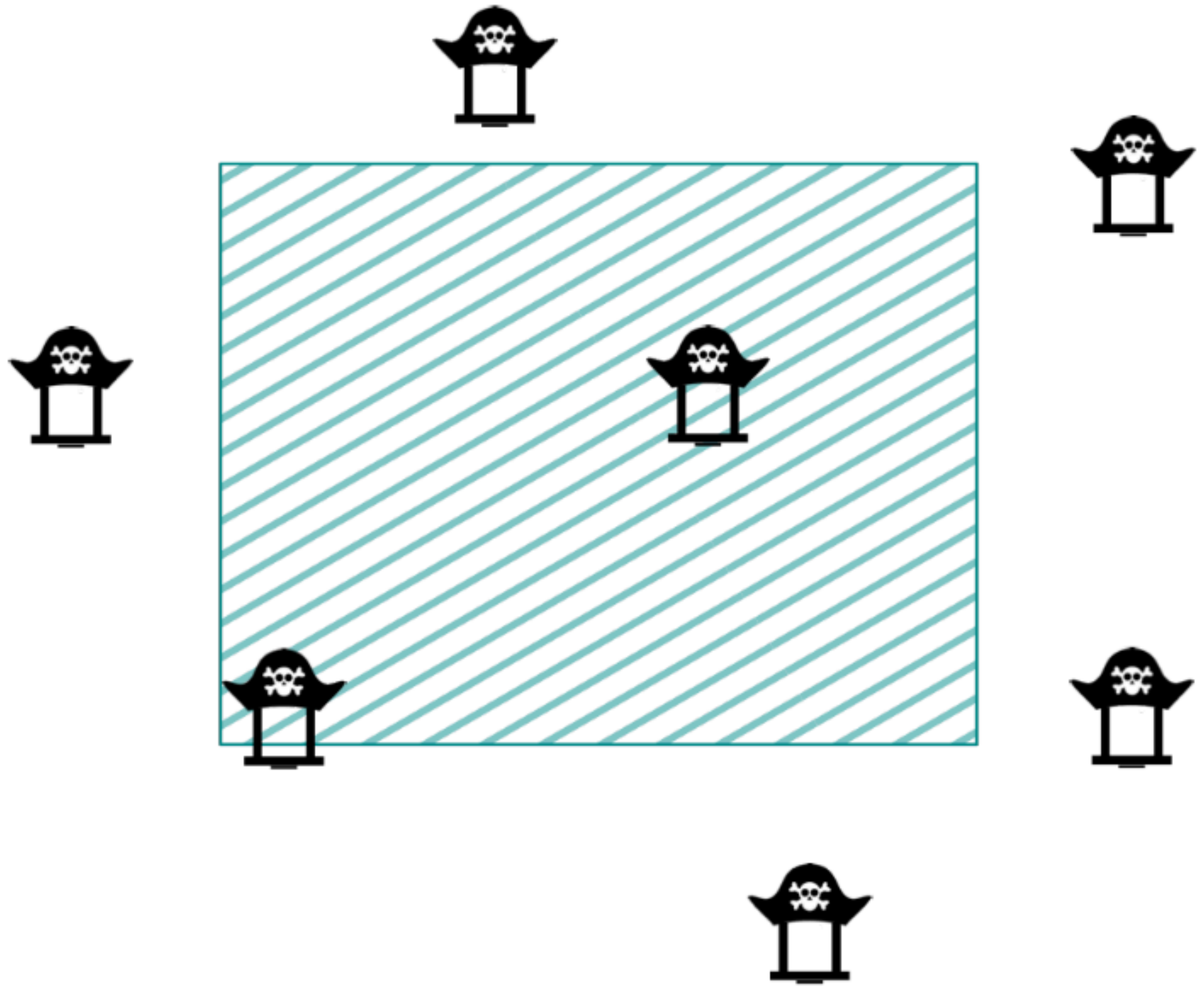
More location problems!

Maximizing Distances to sites in a bounded area

- Provided a set of sites P in the plane and a rectangle R, find a point inside of R with maximal distance to the nearest $p \in P$.



Voronoi diagrams

Maximizing Distances to sites in a bounded area

- Provided a set of sites P in the plane and a rectangle R, find a point inside of R with maximal distance to the nearest $p \in P$.

- Assume that $Vor(P)$ is given.