Computational Geometry

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Computational Geometry
Chapter 1: Introduction

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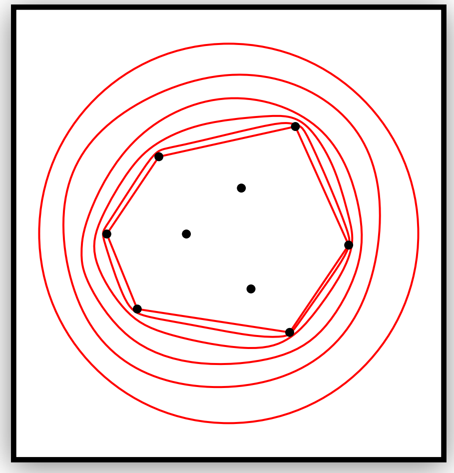
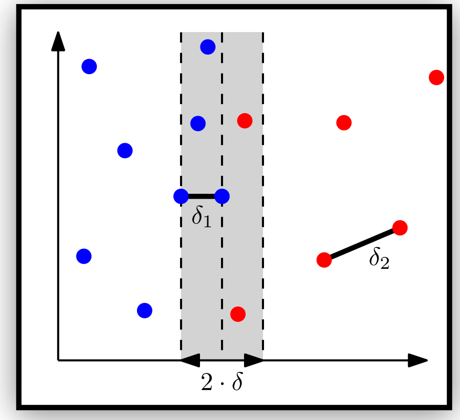
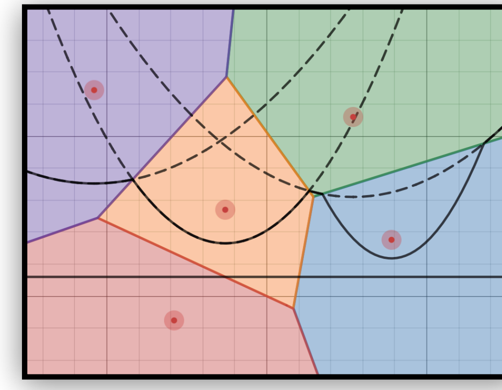
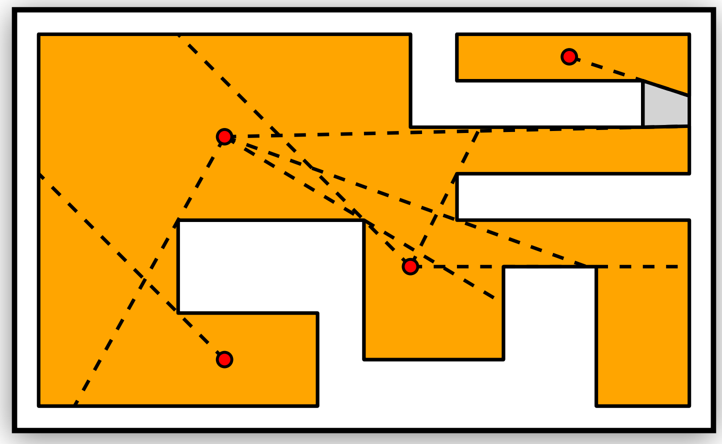
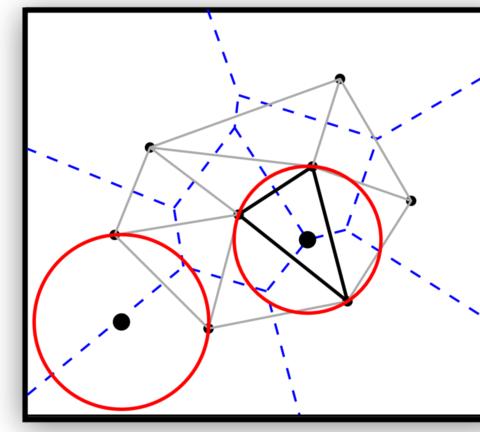
- Ch 1: Introduction
- Ch 2: Convex hulls
- Ch 3: Closest pairs
- Ch 4: Voronoi diagrams
- Ch 5: Polygon triangulation
- Ch 6: Point triangulation
- Ch 7: Location problems
- Ch 8: Tours and polygons

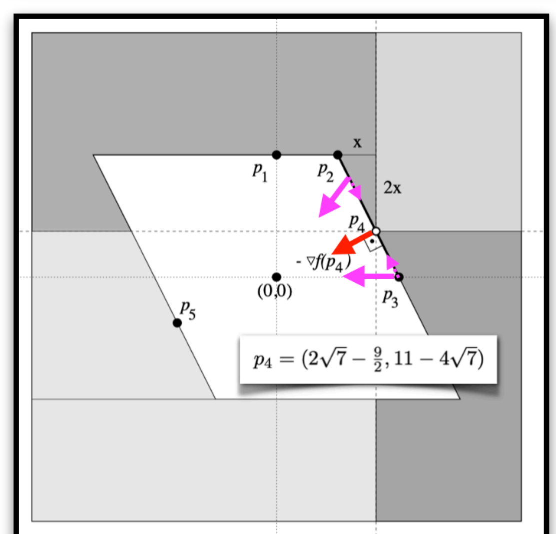


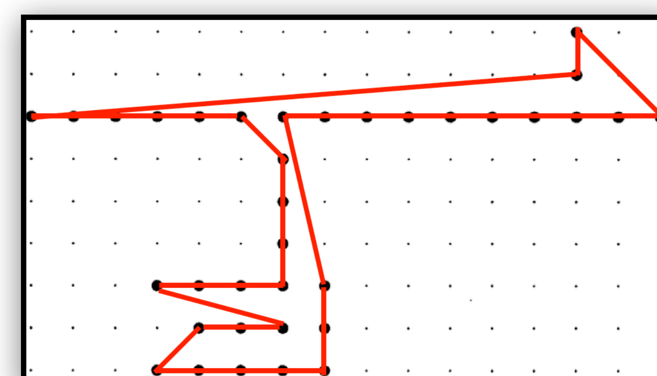










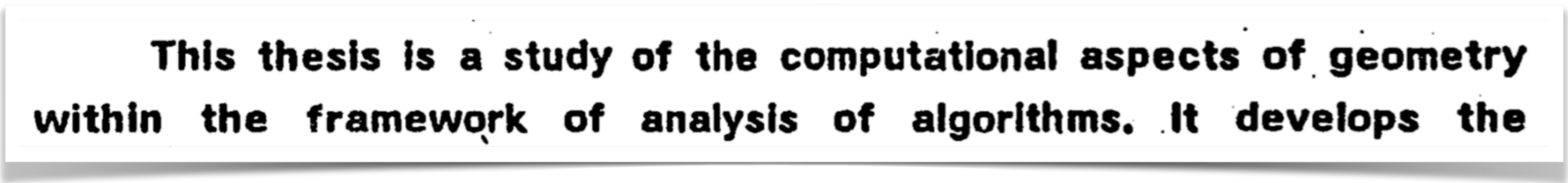


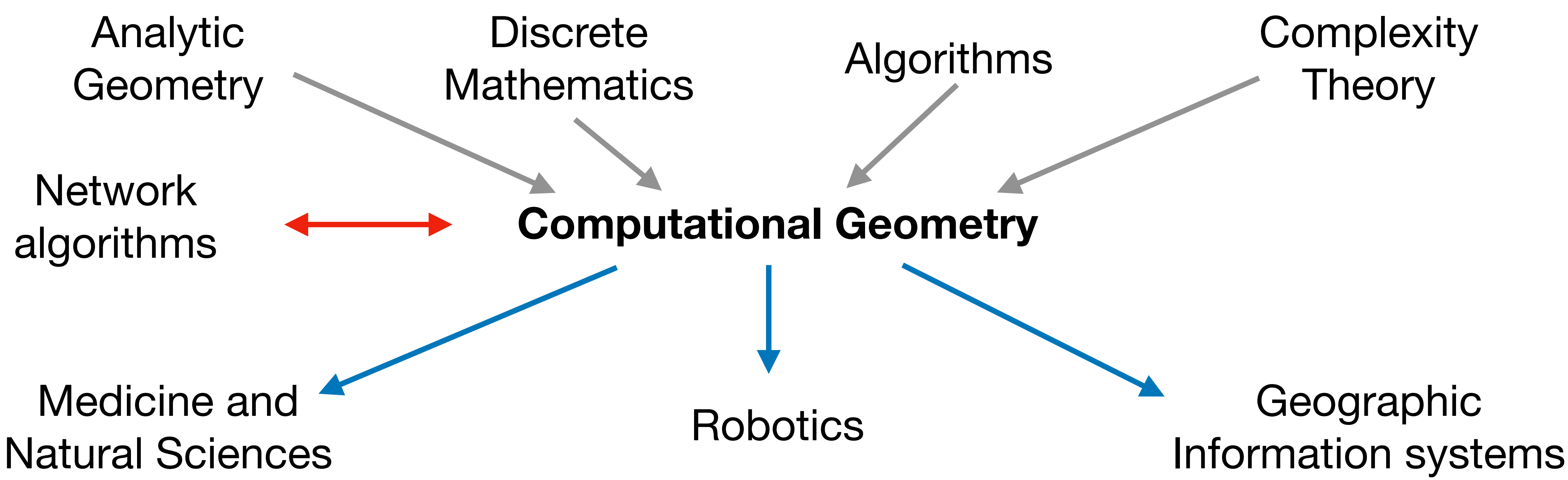
1. Introduction
2. Complexity
3. Basic algorithmic tools
4. Geometric primitives
5. Notation and abbreviations

- Computational Geometry:

  „Dealing with geometric problem with methods of Computer Science“

- Michael Ian Shamos: Computational Geometry, Diss. Yale, 1987.


This thesis is a study of the computational aspects of geometry within the framework of analysis of algorithms. It develops the



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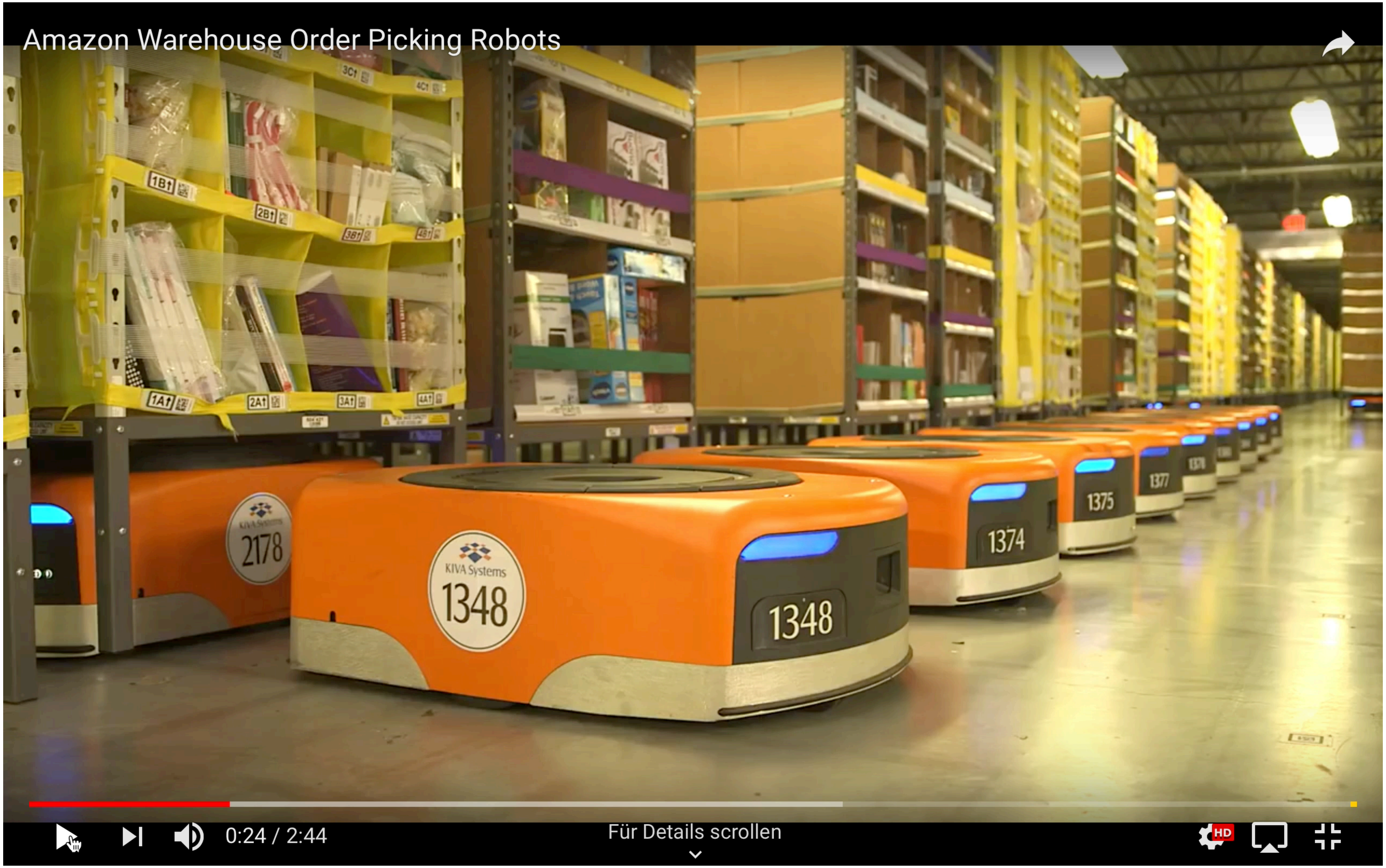

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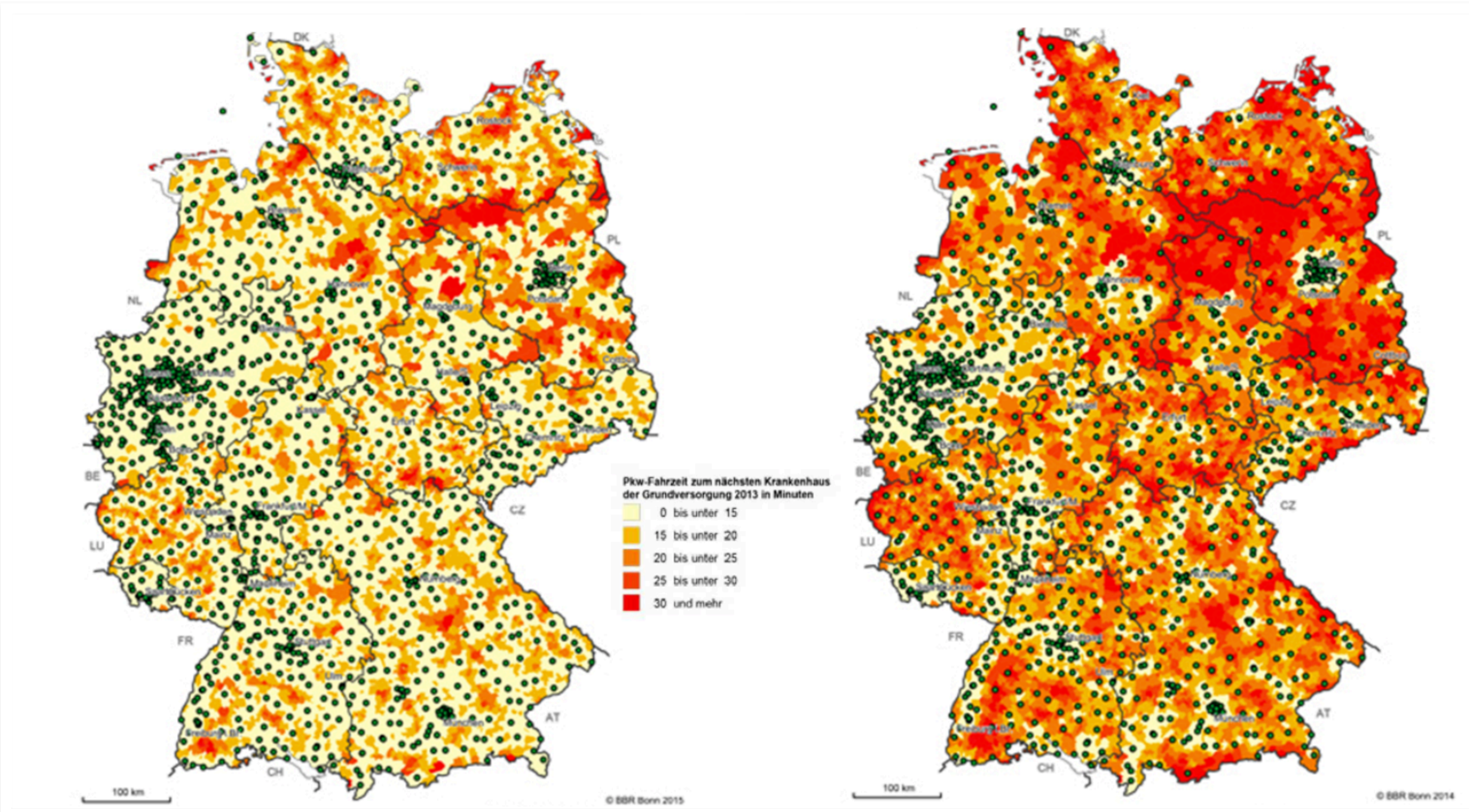
Context



Pkw-Fahrzeit zum nächsten Krankenhaus der Grundversorgung 2013 in Minuten
0 bis unter 15
15 bis unter 20
20 bis unter 25
25 bis unter 30
30 und mehr
DK
NL
PL
BE
LU
CZ
FR
CH
AT
100 km
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Status quo:

- Polynomial-time algorithms for 2D problems

  „Let P be a set of points in the plane …"

Trends:

- Interaction with application fields

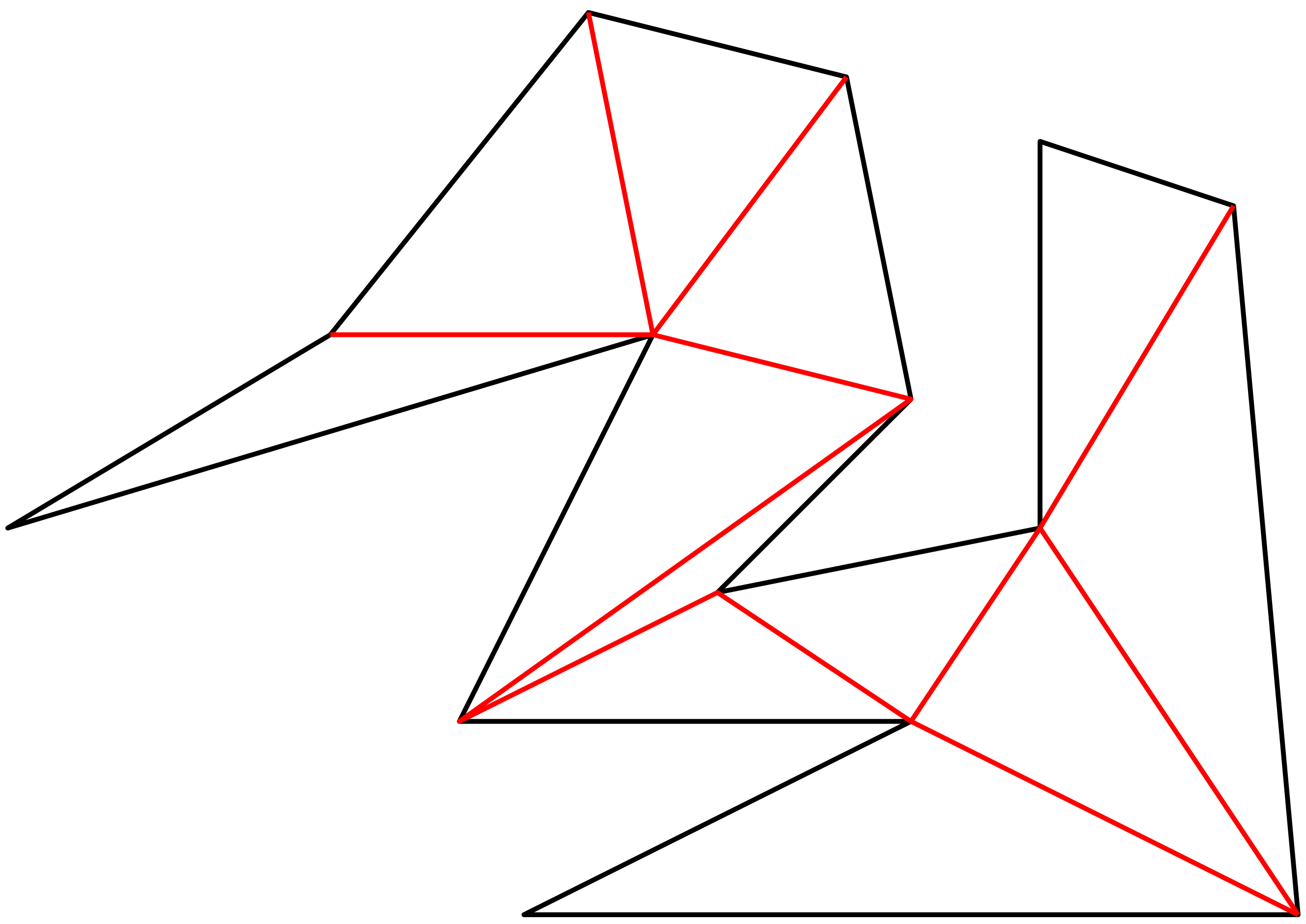
  -> Potential: new challenges.

Geometric problems:

- Easily stated
- Easy to visualize

Example: Polygon triangulation

- Given: Polygonal floorplan
- Wanted: Partition into triangles



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Reminder:

- Runtime as a function of „input size“ n
- n „sufficiently large“

Details:

- Input size: #basic components (points, lines, circles, …)
- Runtime := #basic operations (comparing numbers, computing distances, ….)

Asymptotic bounds

- $\mathcal{O}(\cdot) \leftrightarrow$ Upper bound (Runtime of algorithms, complexity of objects, …).
- $\Omega(\cdot) \leftrightarrow$ Lower bound (Solvability of problems, runtime of algorithms, …).
- Master theorem

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- Binary search in $\mathcal{O}(\log n)$, sorting in $\mathcal{O}(n \log n)$
- Median and Rank-k in $\mathcal{O}(n)$
- AVL trees and priority queues
  - $\mathcal{O}(\log n)$ query time
  - $\mathcal{O}(\log n)$ update time
  - $\mathcal{O}(n)$ memory consumption
- Hashing
- Perfect dynamic Hashing [Dietzfelbinger et al., 1994, Pagh und Rodler, 2004]
  - $\mathcal{O}(1)$ query time
  - $\mathcal{O}(1)$ expected update time
  - $\mathcal{O}(n)$ expected memory usage

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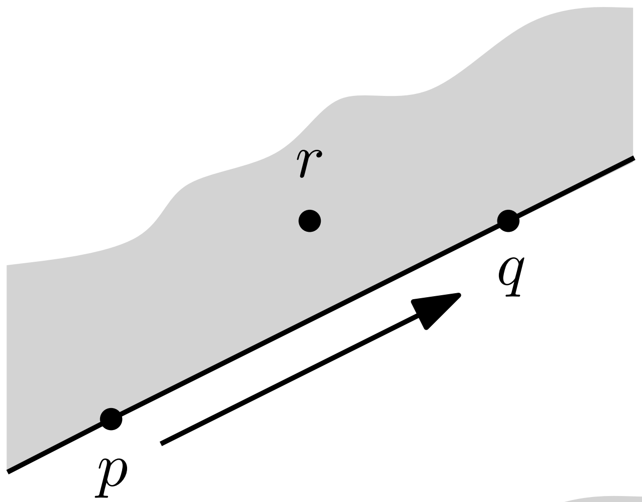
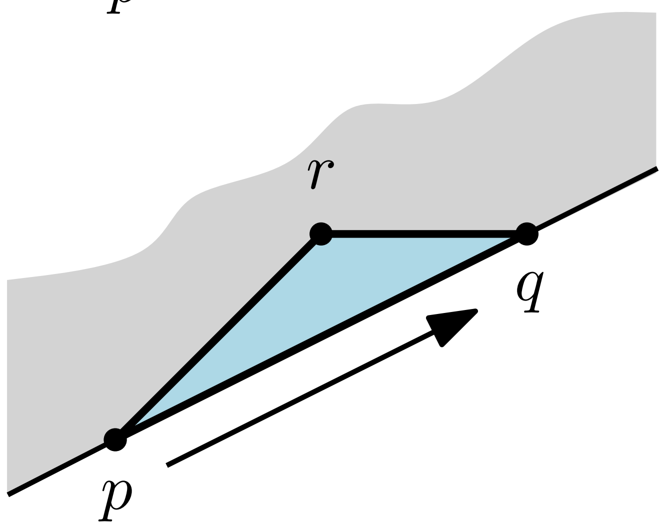
Point-line test:

- Does point $r \in \mathbb{R}^2$ lie on directed line through $p, q \in \mathbb{R}^2$?
  $\rightarrow$ If not: Does it lie left or right of the line?



- Oriented area A of $\triangle(p, q, r)$

$$A(\triangle(p,q,r)) = \frac{1}{2}\begin{vmatrix} 1 & p.x & p.y \\ 1 & q.x & q.y \\ 1 & r.x & r.y \end{vmatrix}$$



Observe:

$A(\triangle(p,q,r)) > 0 \Leftrightarrow p, q, r$ oriented in counterclockwise (CCW) order

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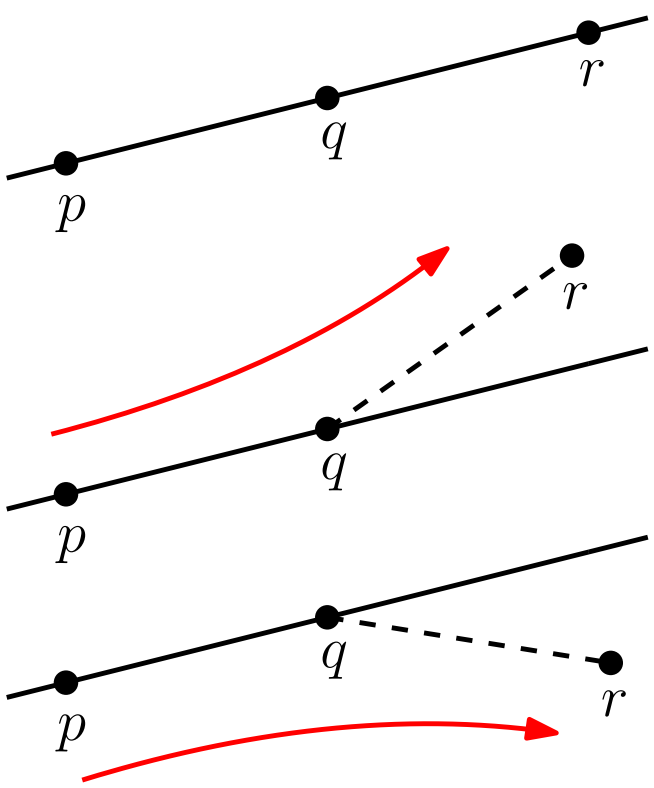
- $A(\triangle(p,q,r)) \left\{ \begin{array}{c} > 0 \\ = 0 \\ < 0 \end{array} \right\} \Leftrightarrow p,q,r \left\{ \begin{array}{c} \text{Left turn} \\ \text{collinear} \\ \text{Right turn} \end{array} \right\}$

- Three predicates:

$$\begin{aligned} \mathit{collinear} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &(p,q,r) \mapsto (A(\triangle(p,q,r)) = 0) \end{aligned}$$

$$\begin{aligned} \mathit{leftTurn} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &(p,q,r) \mapsto (A(\triangle(p,q,r)) > 0) \end{aligned}$$

$$\begin{aligned} \mathit{rightTurn} &: \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{B} \\ &(p,q,r) \mapsto (A(\triangle(p,q,r)) < 0) \end{aligned}$$



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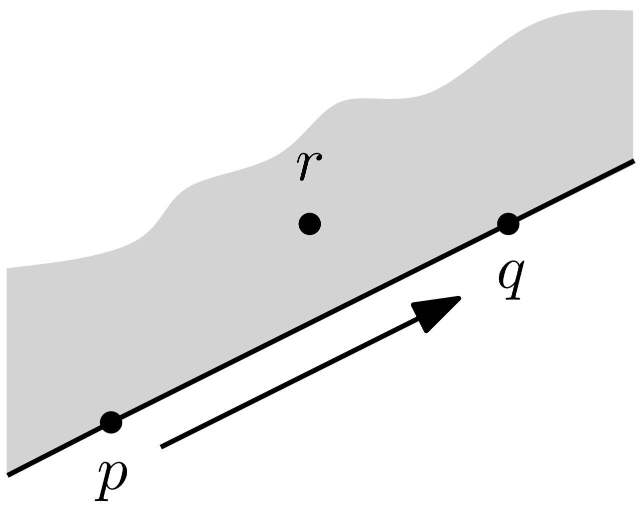
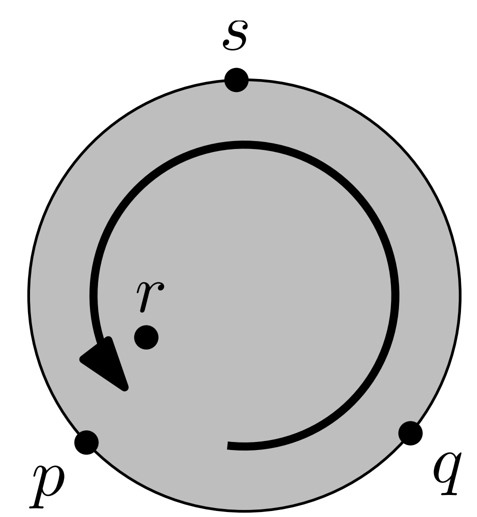
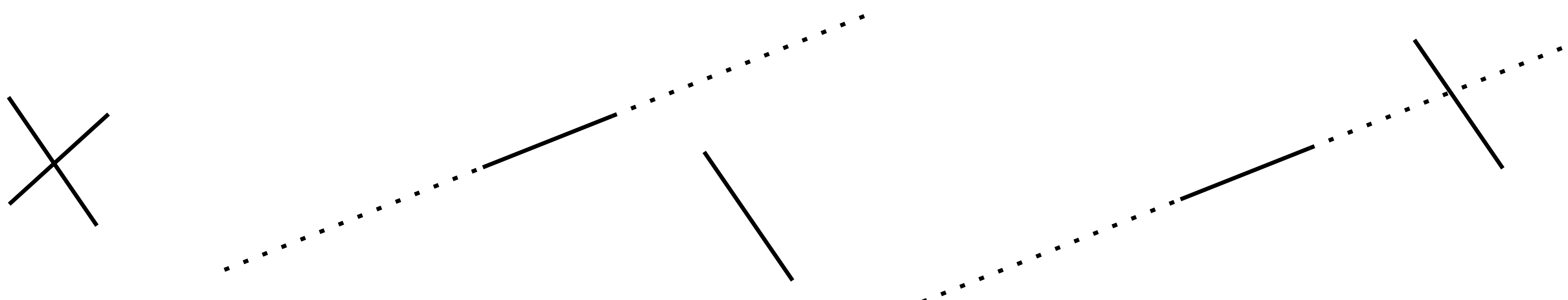


- Does point $r \in \mathbb{R}^2$ lie on (CCW) oriented circle?
  (induced by $s, q, p \in \mathbb{R}^2$)



- Intersection test (given pairs of segments, etc…)



Abbreviations:

- # := cardinality (e.g., #vertices of a polygon)
- w.l.o.g.: Without loss of generality

Notation:

- Unless stated otherwise
- $\overline{ab}$:= segment between a, b
- $|A|$:= cardinality (but also, e.g., $|P|$ = #vertices of polygon P)
- ∂A := Boundary of set A
- A° := Interior of set A

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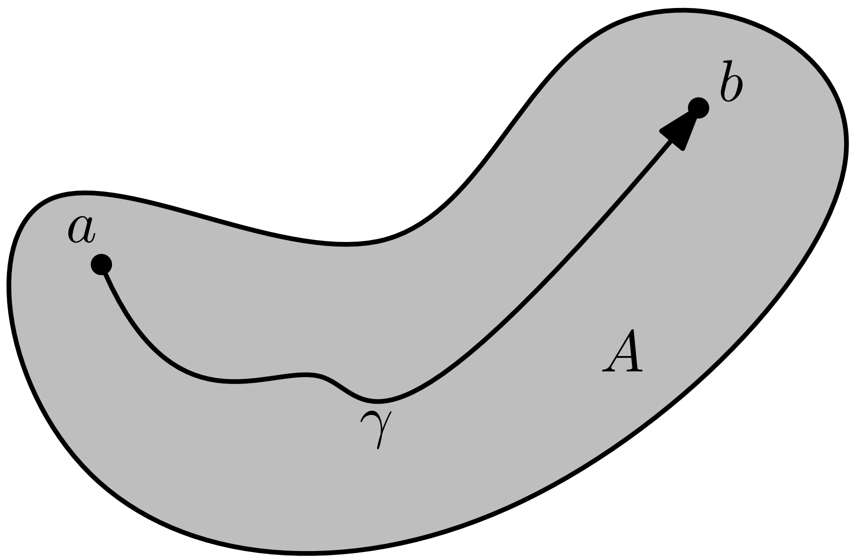
Path connectivity:

- $A \subset \mathbb{R}^d$ (path-) connected

  $:\Leftrightarrow$

  $\forall a, b \in A : \exists$ continuous curve $\gamma : [0,1] \to A$

  with $\gamma(0) = a, \gamma(1) = b$



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