Computational Geometry **Winter term 2023**
TU Braunschweig, Algorithms Division

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Question Sheet 2

Submit in your solutions, in a single properly (digitally) formatted PDF file, to the folder at `https://nextcloud.ibr.cs.tu-bs.de/s/4pbJromgibzKsZQ` by February 1, 2024. **You may submit this homework sheet in pairs. To do so, please clearly state the full name, field of study, and matriculation number of both partners at the top of the first page.** Name your submission as follows: `[your_full_name(s)].pdf`.

We consider these problems in the two-dimensional variant. Please explain your answers.

Question 1 (Voronoi diagram): **(1+1+2 points)**

Given n points $S \subset \mathbb{R}^2$ in general position (no three points are collinear and no four points lie on a common circle). Let $k \in \mathbb{N}$ be arbitrary but fixed.

a) For $k \geq 1$, what does the kth order Voronoi diagram represent?

b) Consider a region of the kth order Voronoi diagram. Argue into how many regions it will be split in the $(k+1)$th order Voronoi diagram.

c) Argue why for $n \geq 3$, the $(n-1)$th order Voronoi diagram forms a tree.

Question 2 (Polygons and triangulations): **(1+2 points)**

Given a simple polygon P with n vertices in general position (no three vertices are collinear).

a) Argue that every convex polygon permits a triangulation that has a dual graph with maximal vertex degree 2.

b) How can you decide in $\mathcal{O}(n \log n)$, if a given point p is inside of the polygon P?

Question 3 (Point triangulation): **(2+1 points)**

Given n points $S \subset \mathbb{R}^2$ in general position (no three points are collinear and no four points lie on a common circle).

a) Briefly, argue why the dual graph of a point sets Voronoi diagram is a Delauney Triangulation.

b) What does solving instances of the NP-hard problems Minimum-weight Triangulation and Max-Min Triangulation reveal?

Question 4 (Miscellaneous): **(2 points)**

Explain the concept of sweep-line algorithms for geometric problems in your own words. What are its components and requirements? Name examples from the lecture!