

Kapitel 2: Graphen

Algorithmen und Datenstrukturen WS 2022/23

Prof. Dr. Sándor Fekete

Konzentration







Carl Friedrich Gauß (1777-1855)



Carl Friedrich Gauß (1777-1855)

1 + 2 + 3 + ... + 100 100 + 99 + 98 + ... + 1

101 + 101 + 101 + ... + 101

 $= 100 \times 101$

Also:

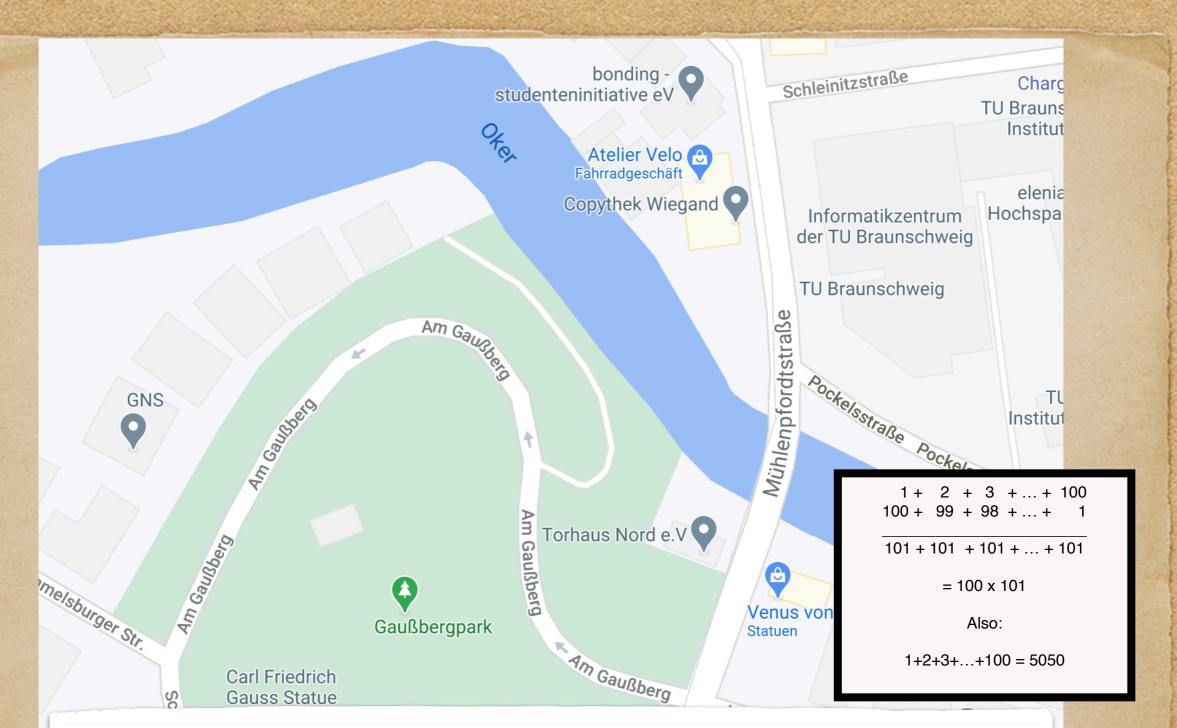
1+2+3+...+100 = 5050



Gaußsche Summenformel

Die Gaußsche Summenformel (nicht zu verwechseln mit einer Gaußschen Summe), auch kleiner Gauß genannt, ist eine Formel für die Summe der ersten n aufeinanderfolgenden natürlichen Zahlen:

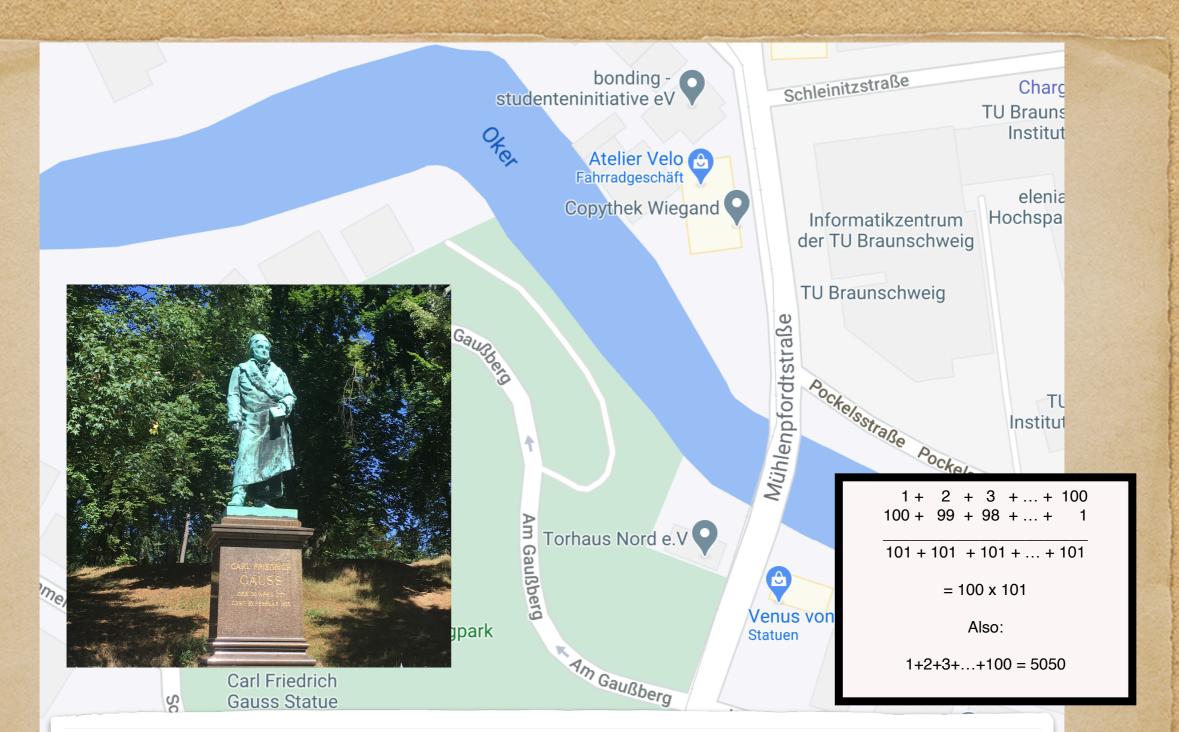
$$1+2+3+4+\cdots+n=\sum_{k=1}^n k=rac{n(n+1)}{2}=rac{n^2+n}{2}$$



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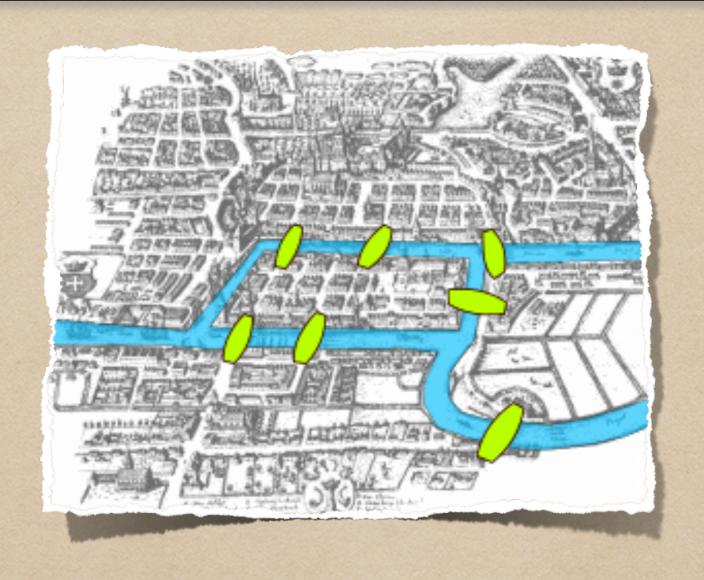


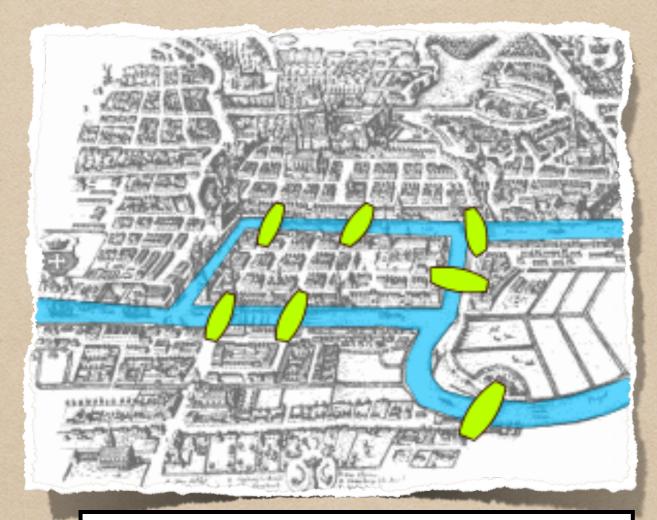




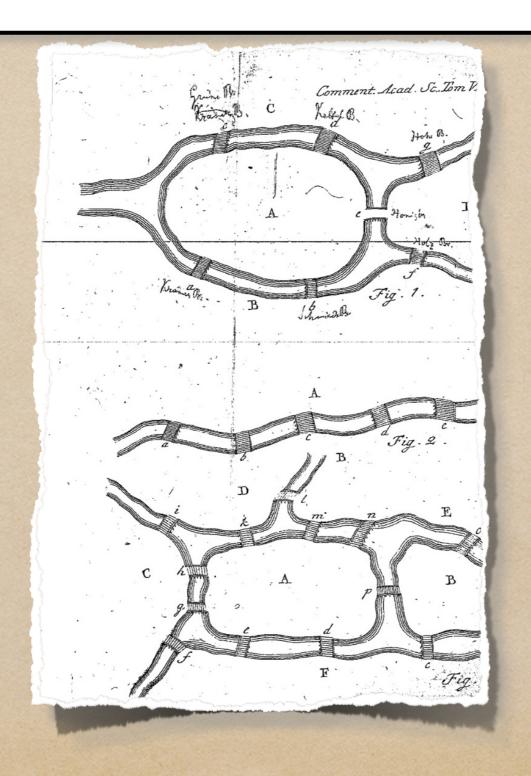
Leonhard Euler (1707-1783)



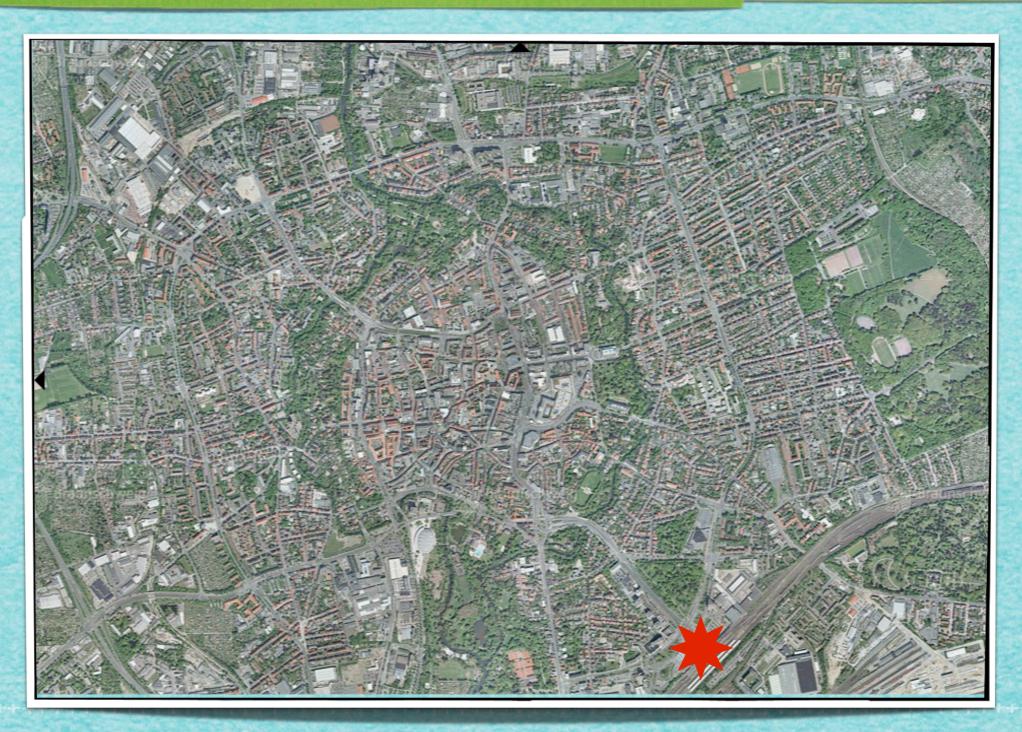




Königsberg und seine 7 Brücken



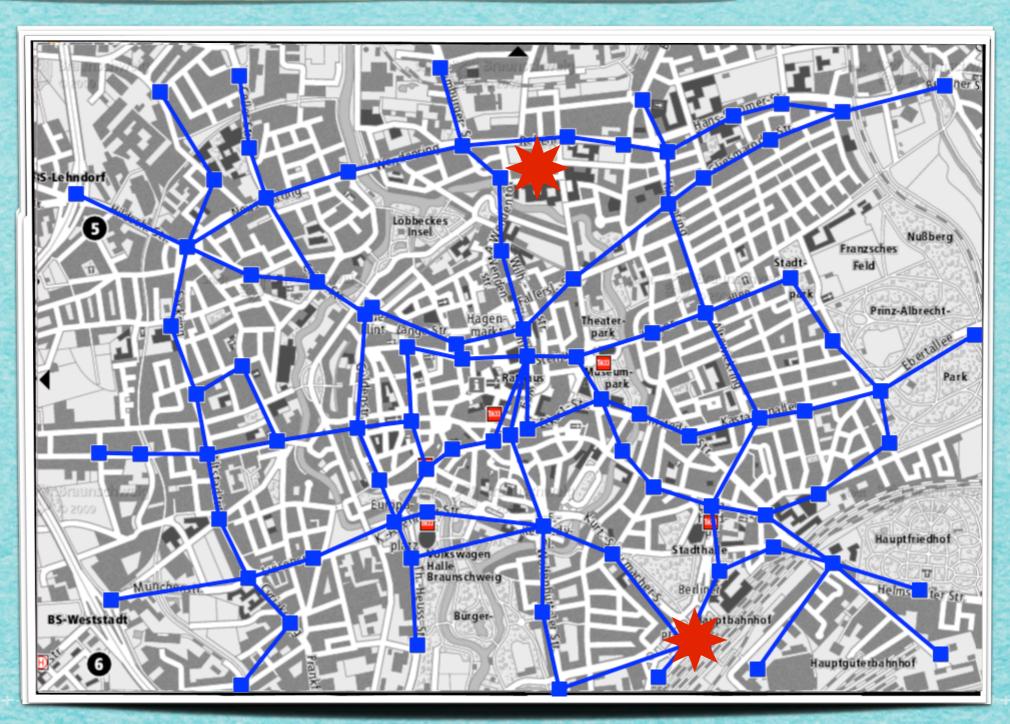


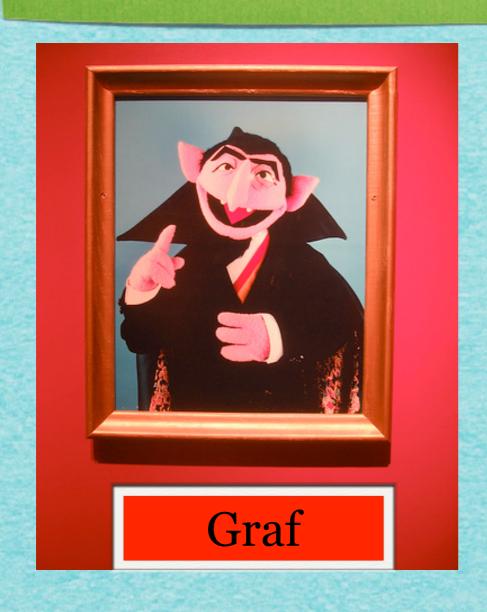


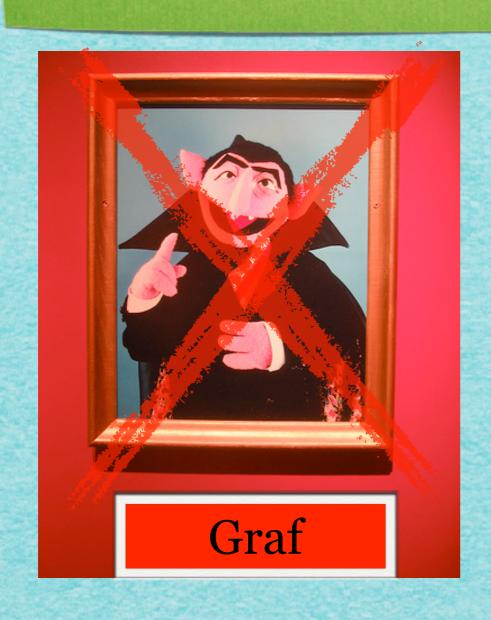




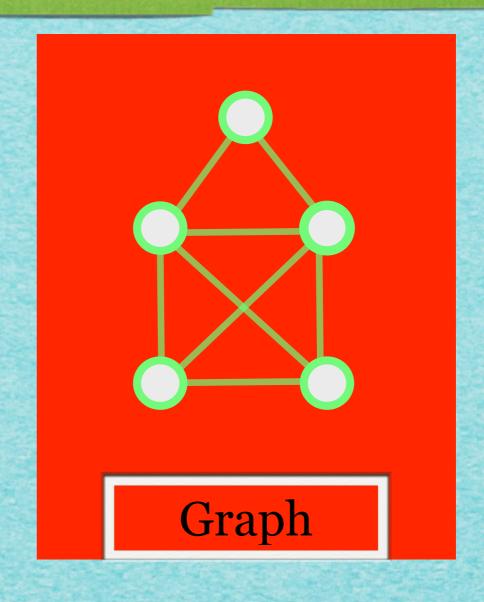


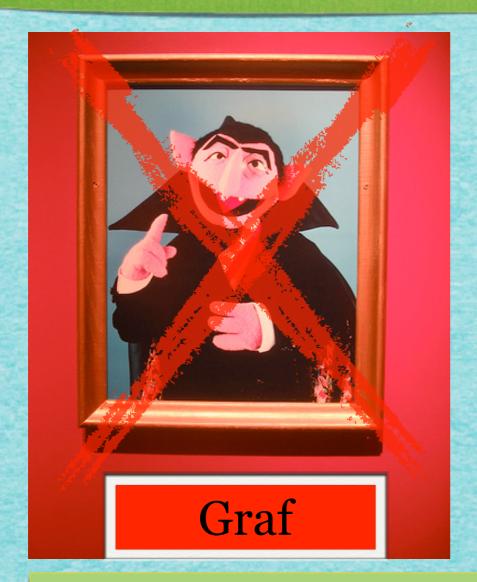


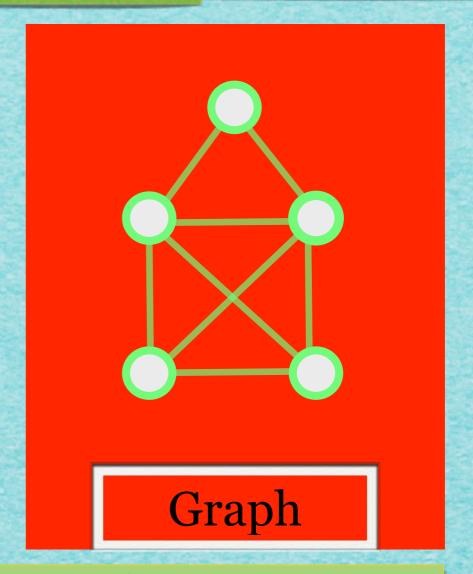




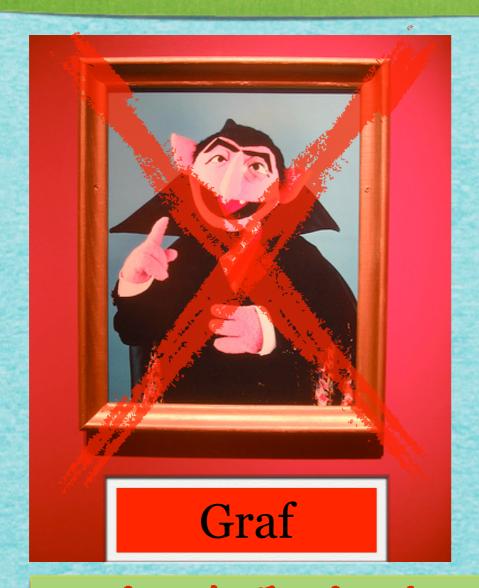


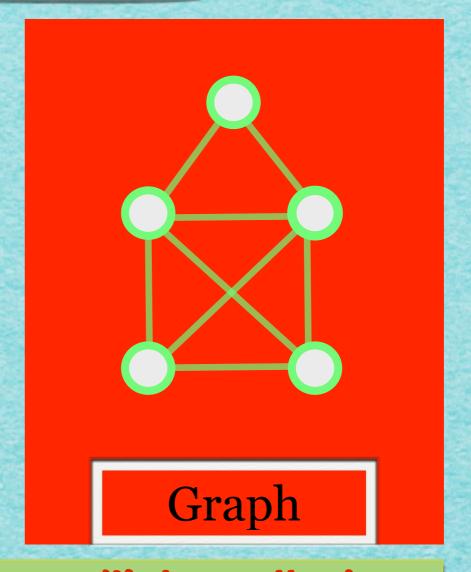




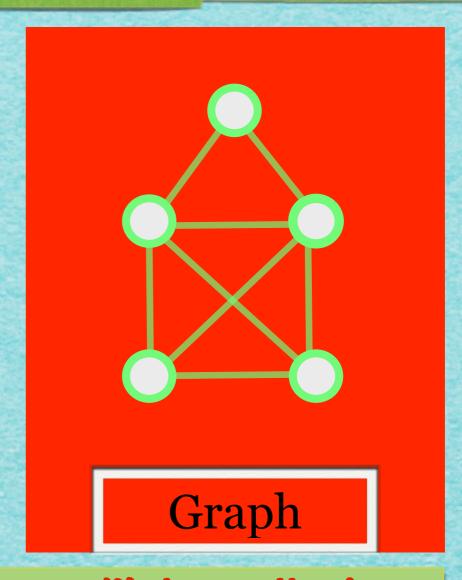


Graph: Ein Gebilde aus Knoten (Haltestellen)

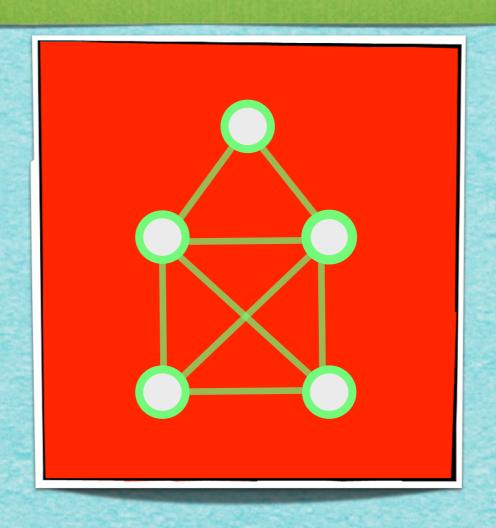




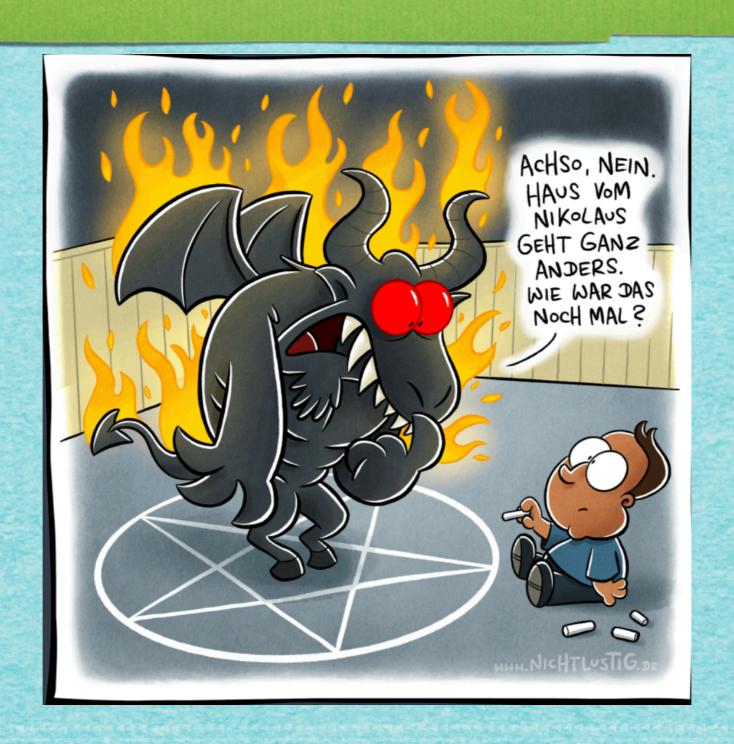
Graph: Ein Gebilde aus Knoten (Haltestellen) und Kanten (Verbindungen)

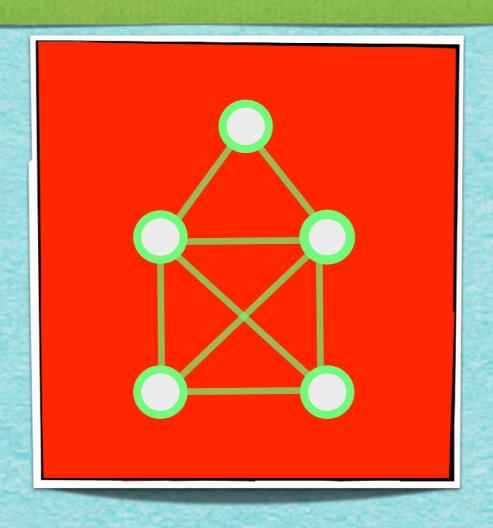


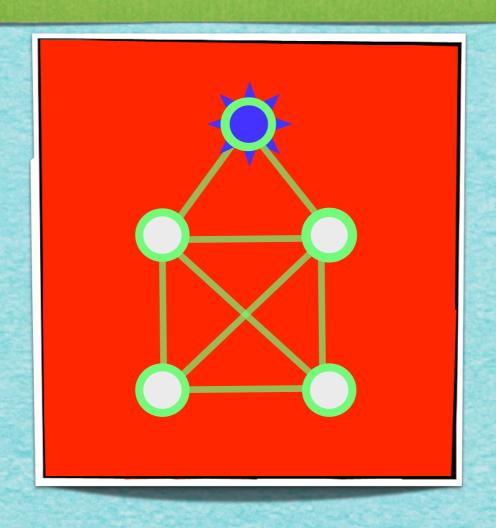
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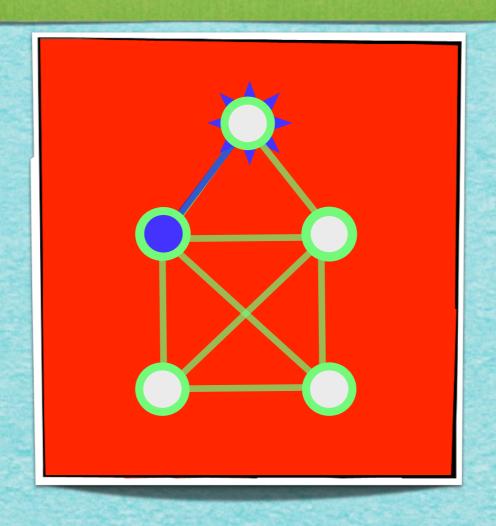


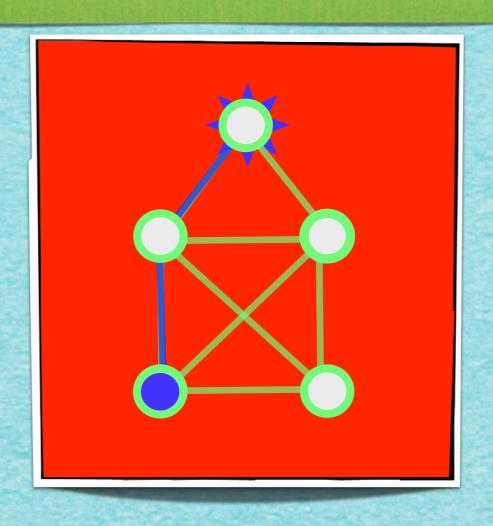


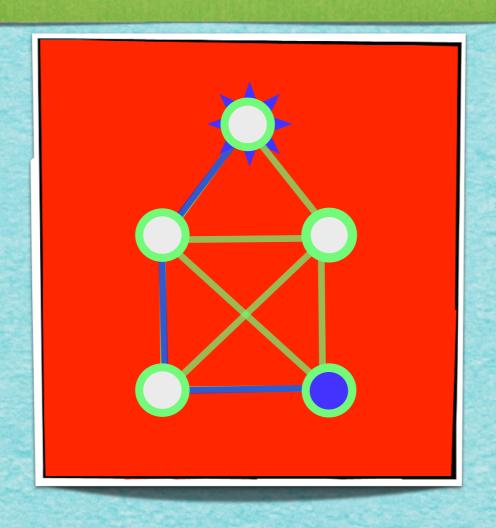


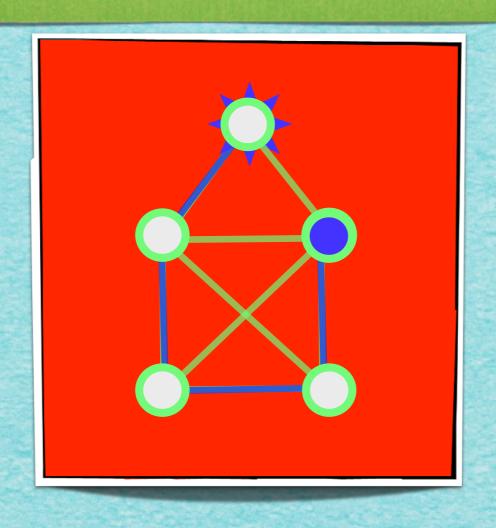


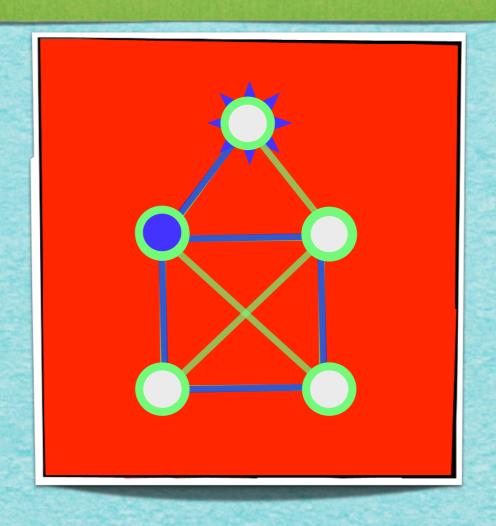


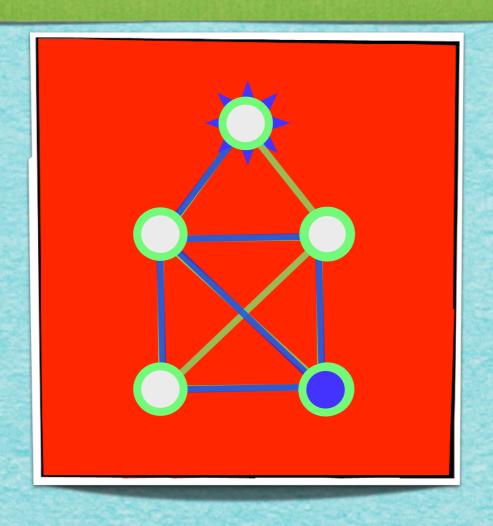


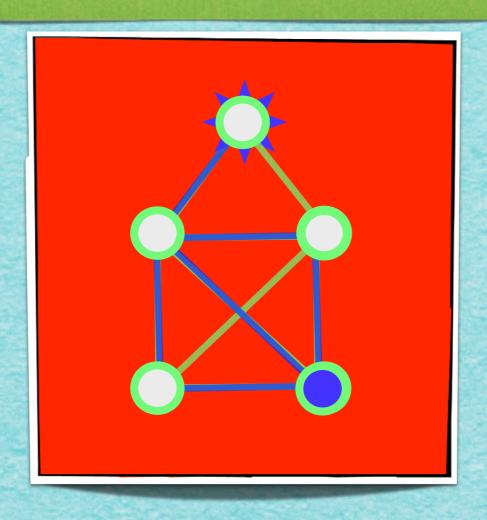




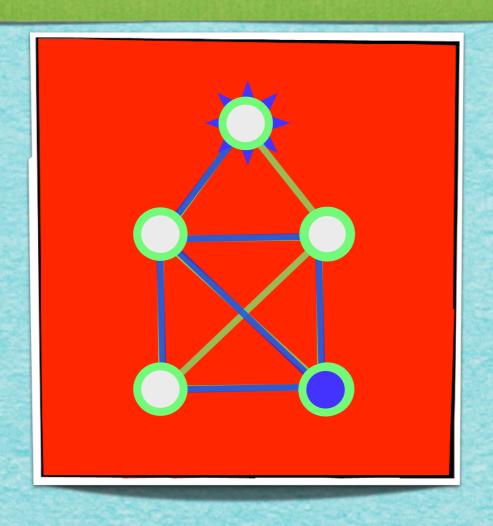






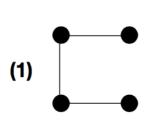


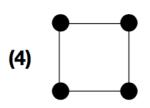
Klappt so nicht...

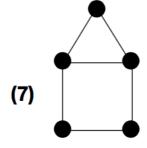


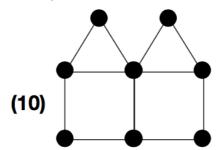
Herausforderung!

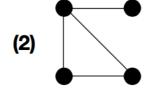
Welche Graphen kannst Du in einem Zug nachzeichnen, ohne den Stift abzusetzen? (Wenn ja, wo kann man anfangen oder aufhören?)

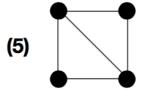


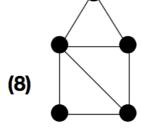


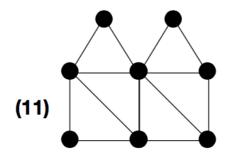


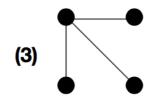


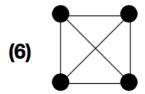


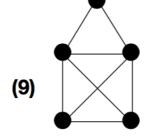


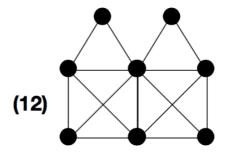












Herausforderung!

App Store Vorschau



One touch Drawing 4+

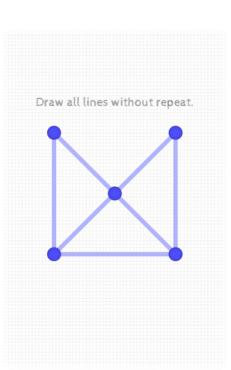
Ecapyc Inc.

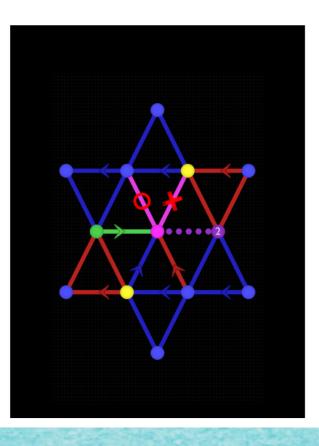
★★★★ 4,5 • 50 Bewertungen

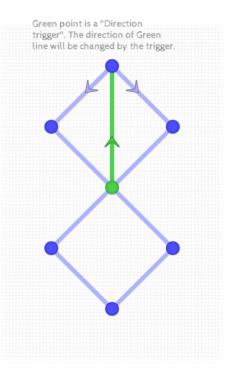
Gratis · In-App-Käufe möglich

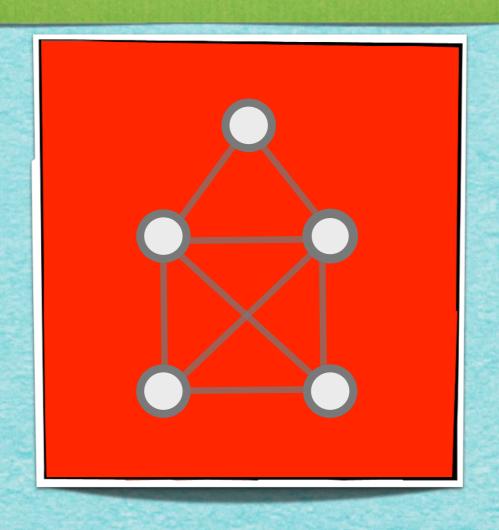
Anzeigen in: Mac App Store ↗

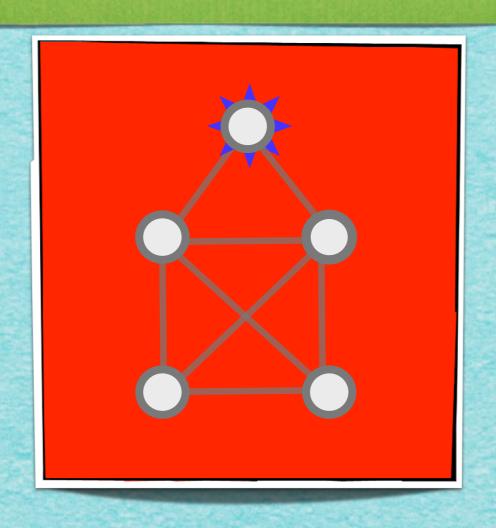
Screenshots iPad

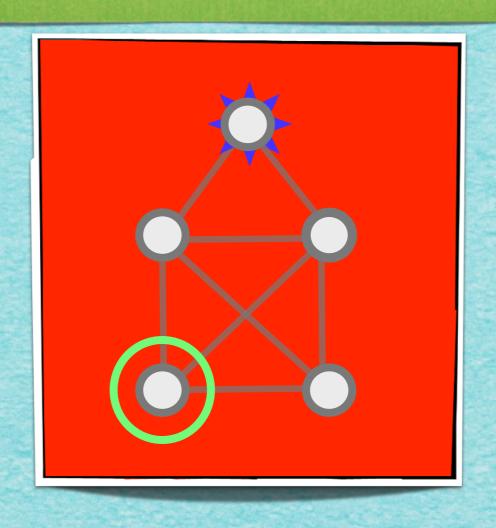


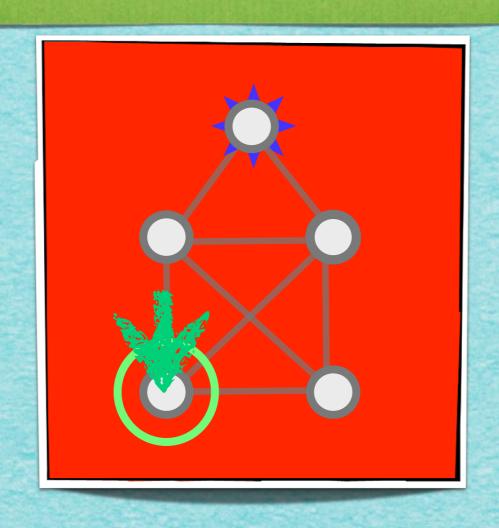


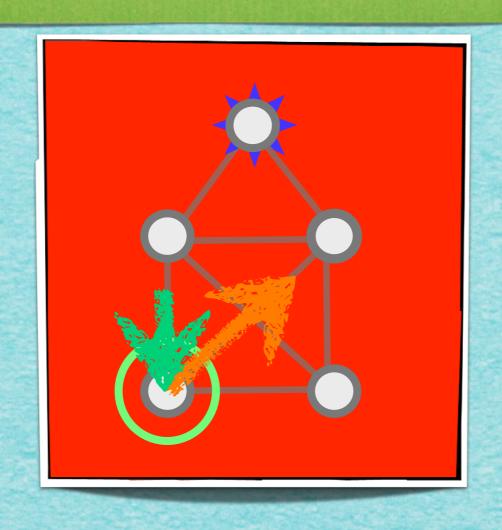


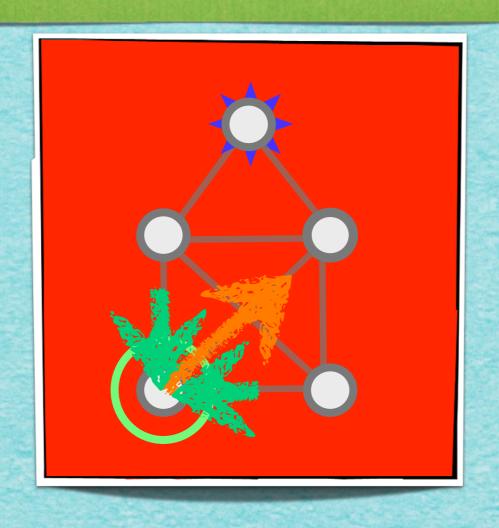


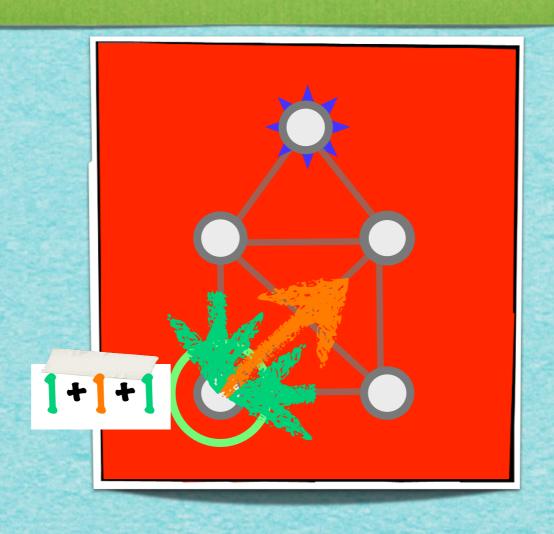


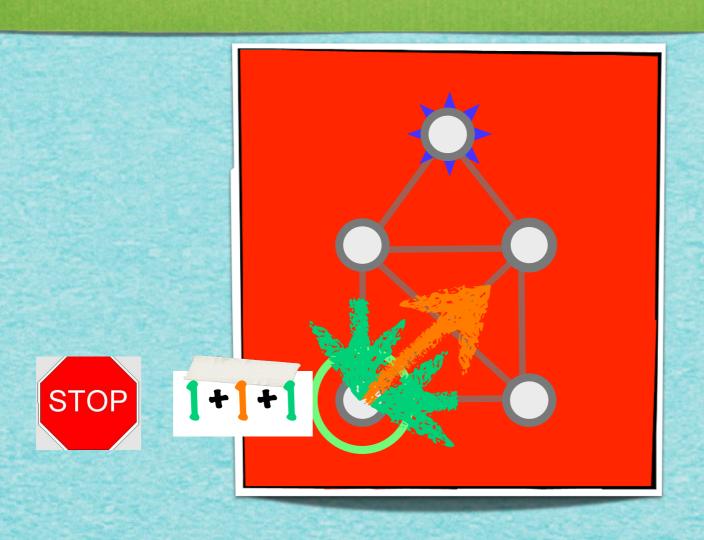


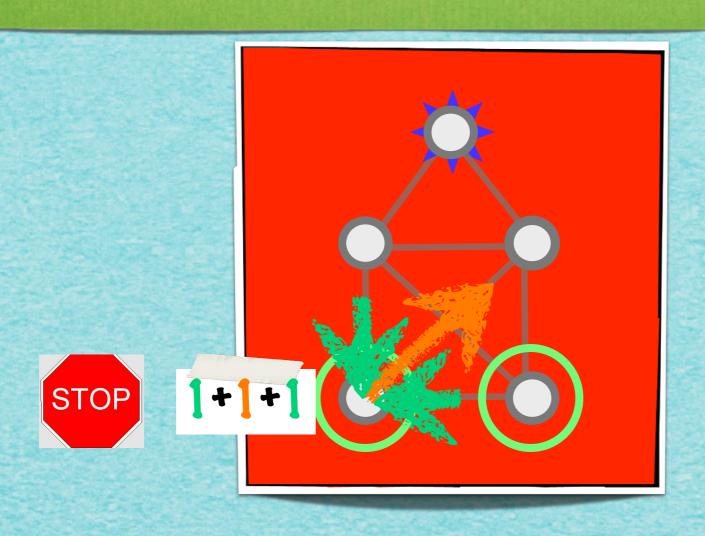


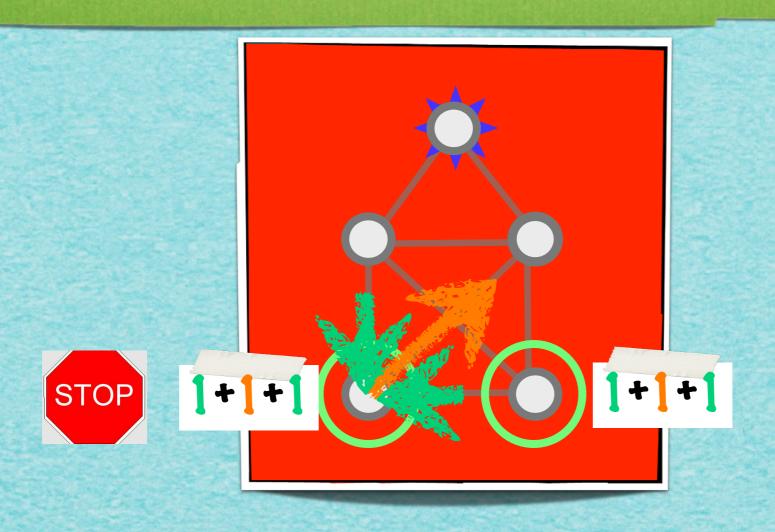


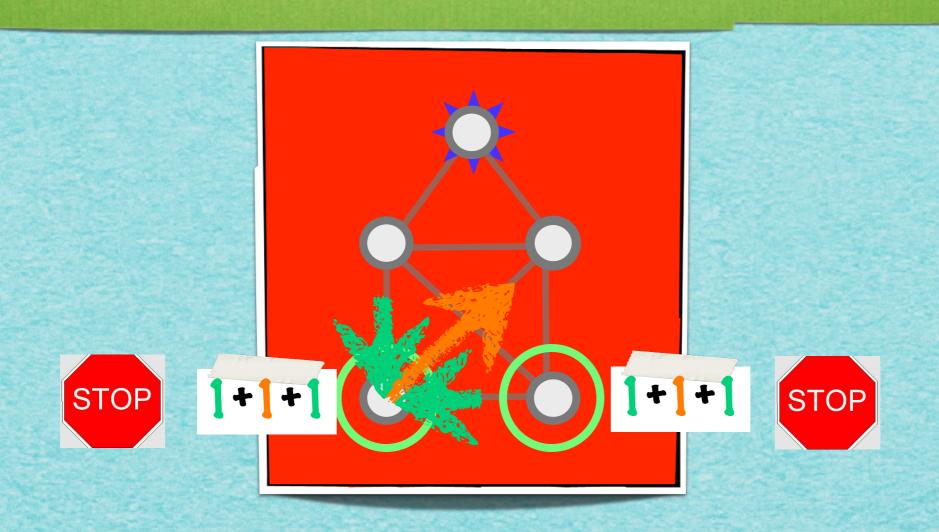


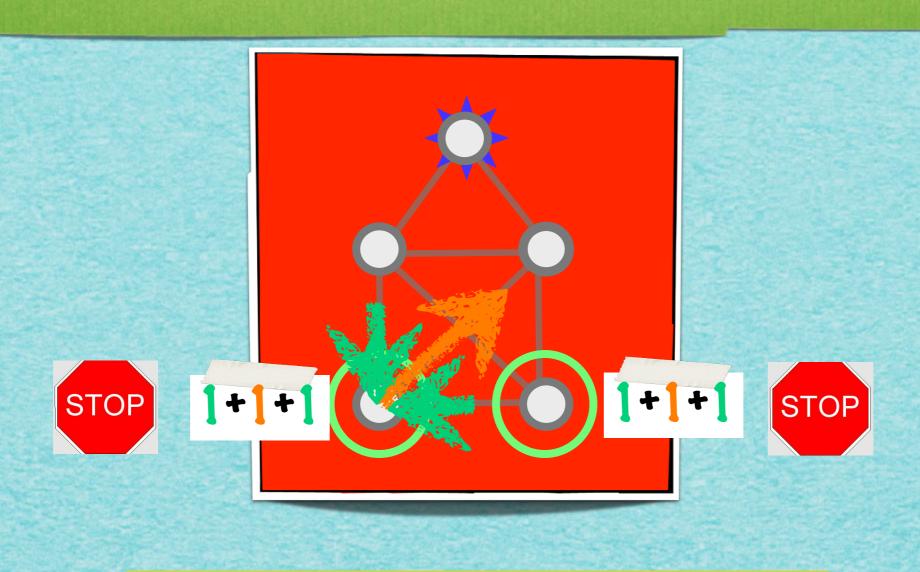




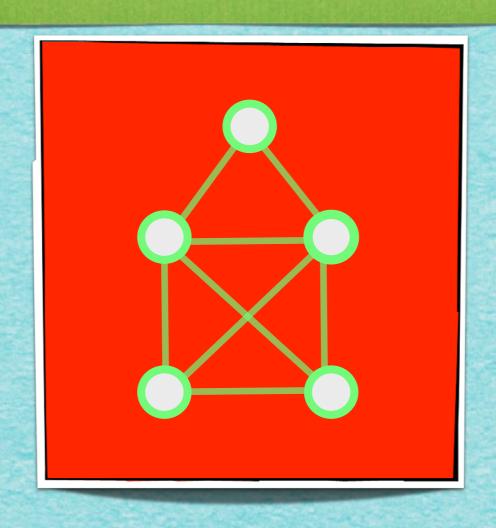


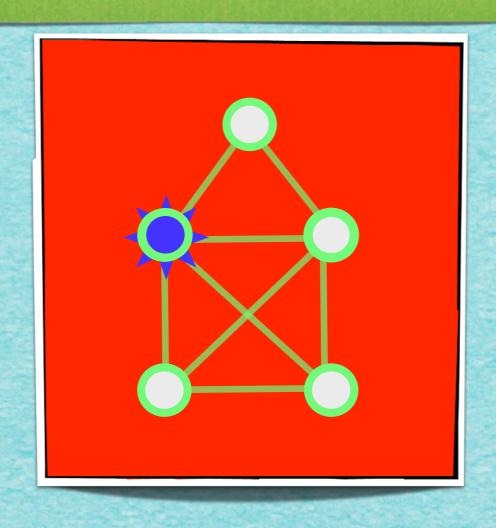


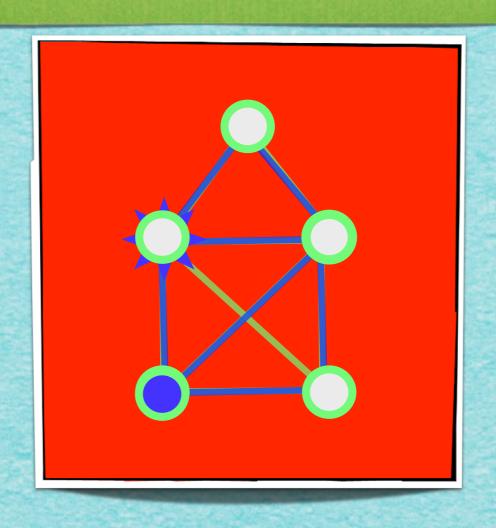


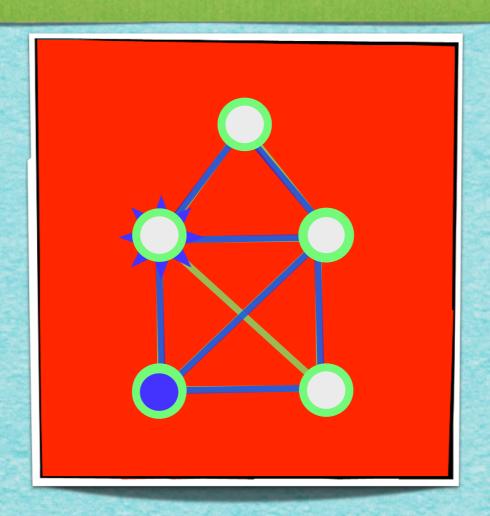


Hmmmmm...

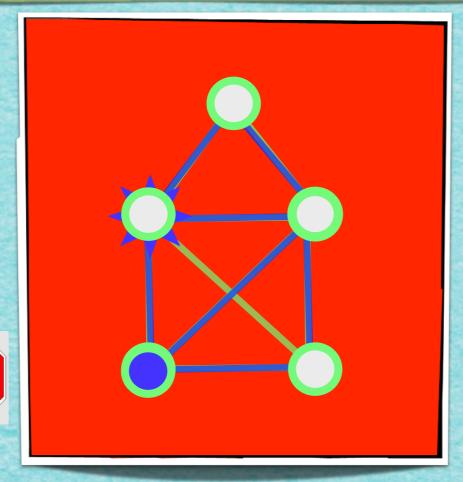






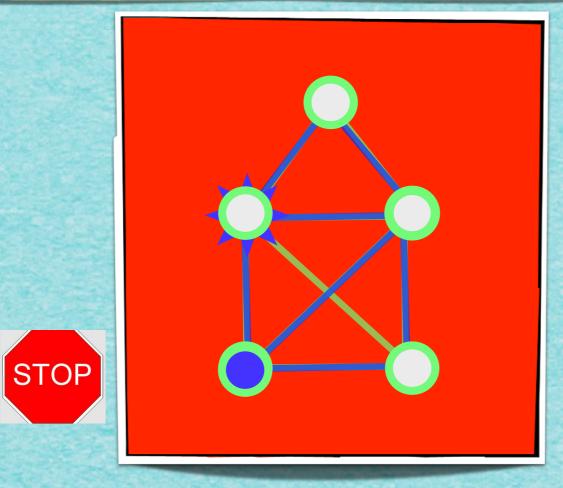


Ooooooooch...



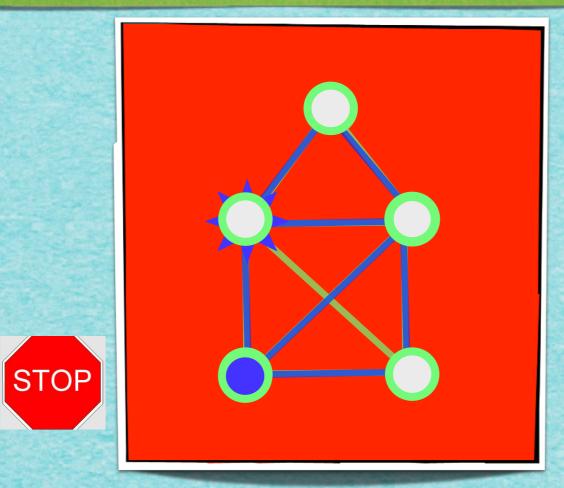
STOP

Ooooooooch...

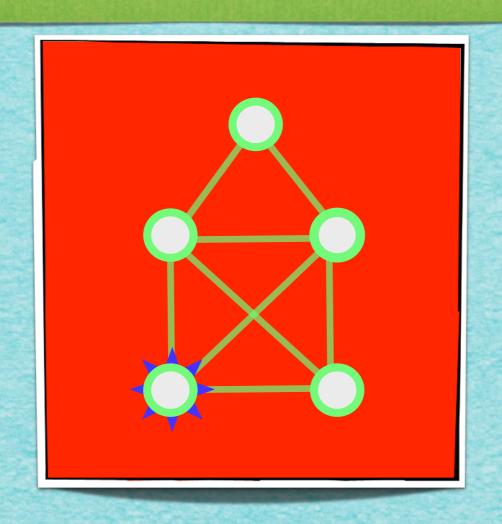


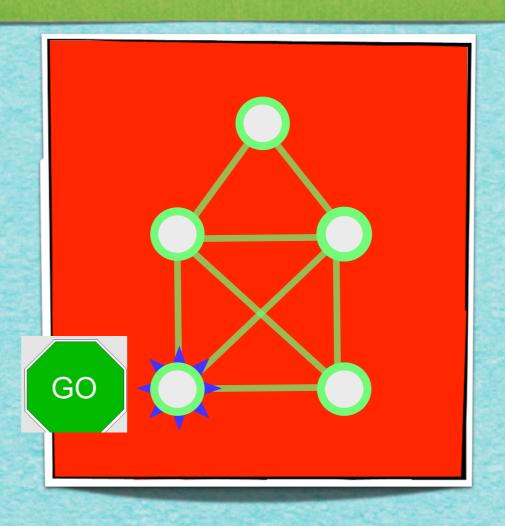


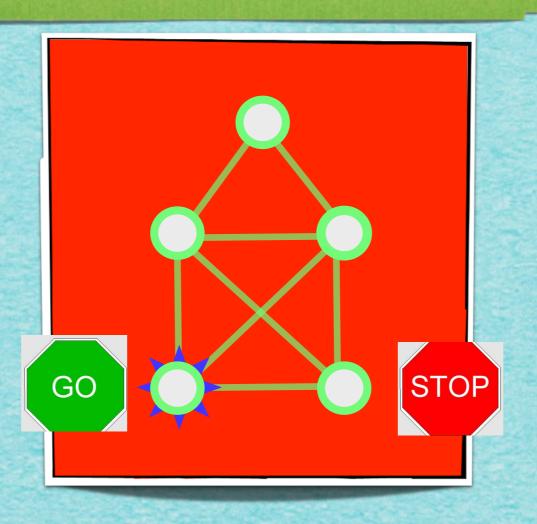
Ooooooooch...

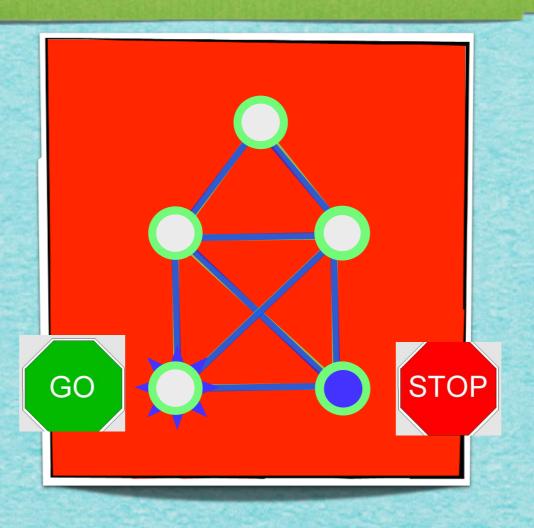


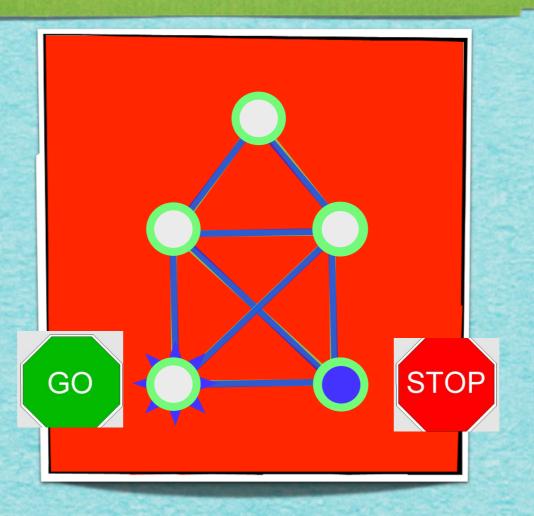




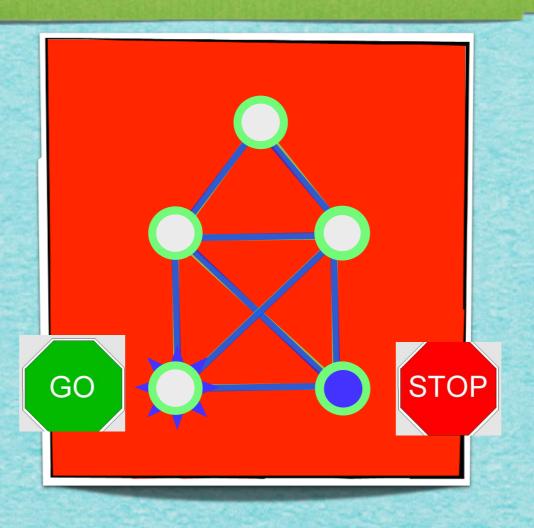


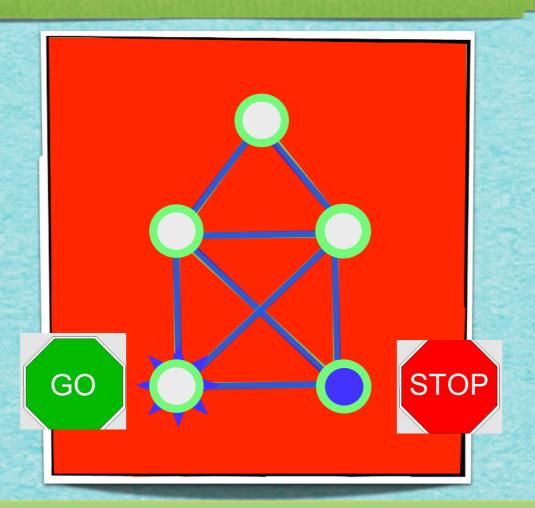




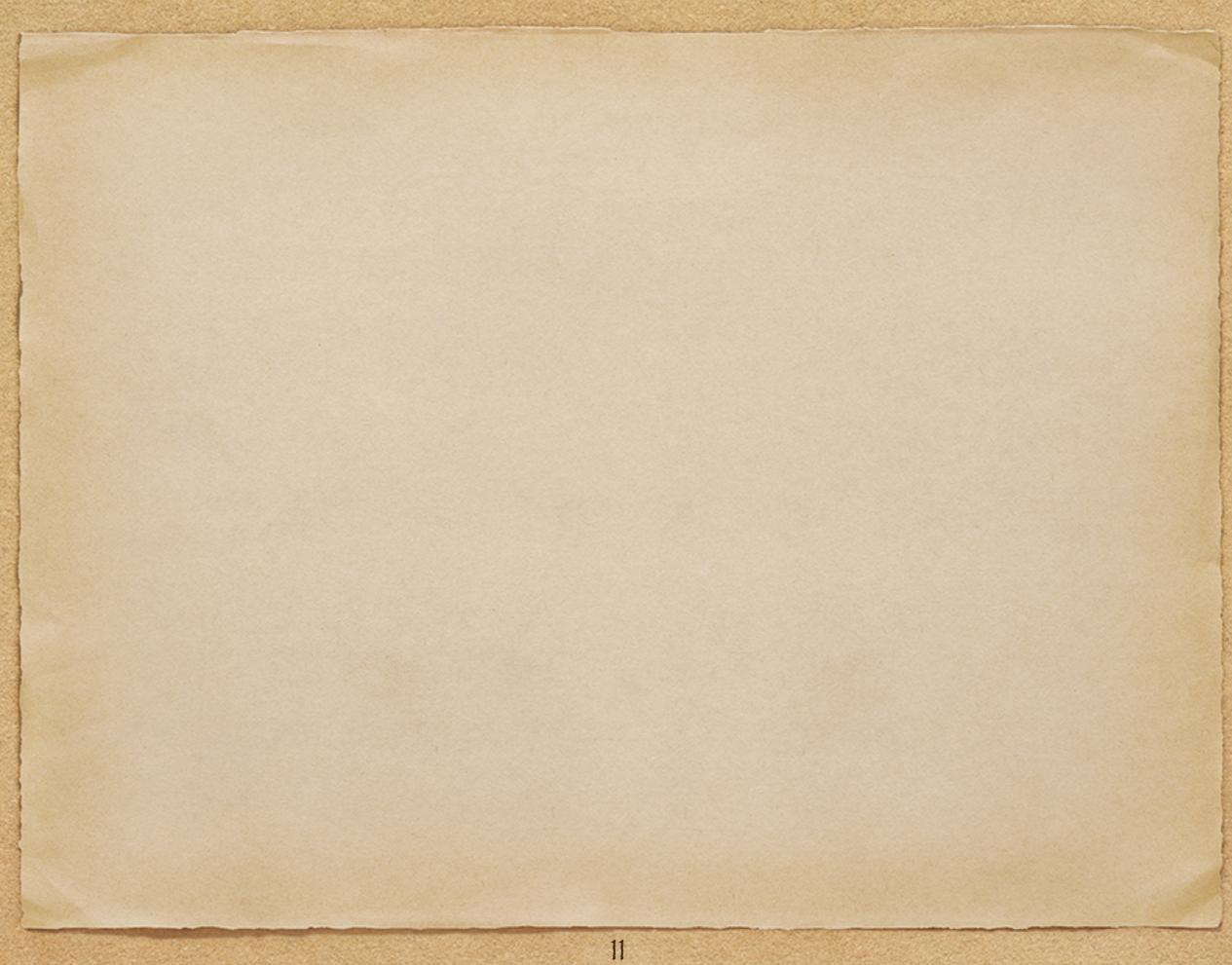


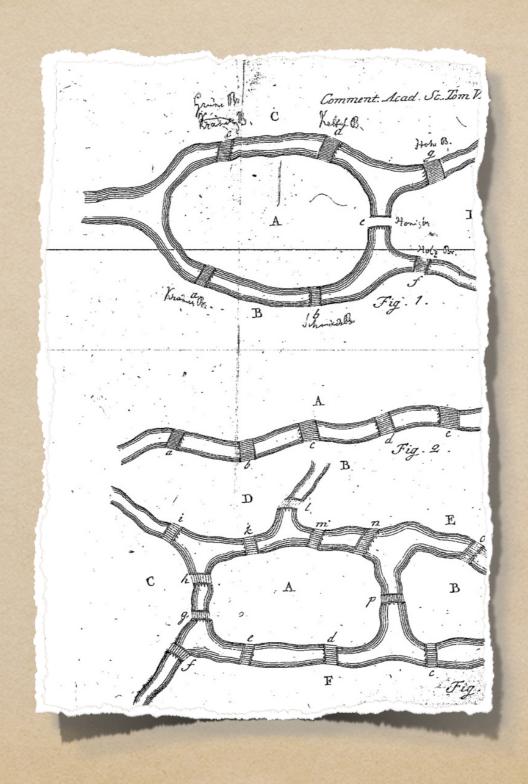
Ahhhhhhhh!

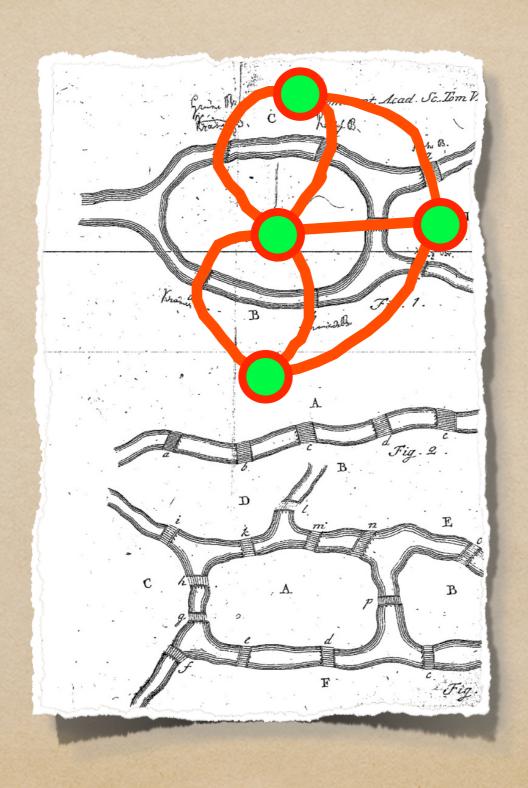


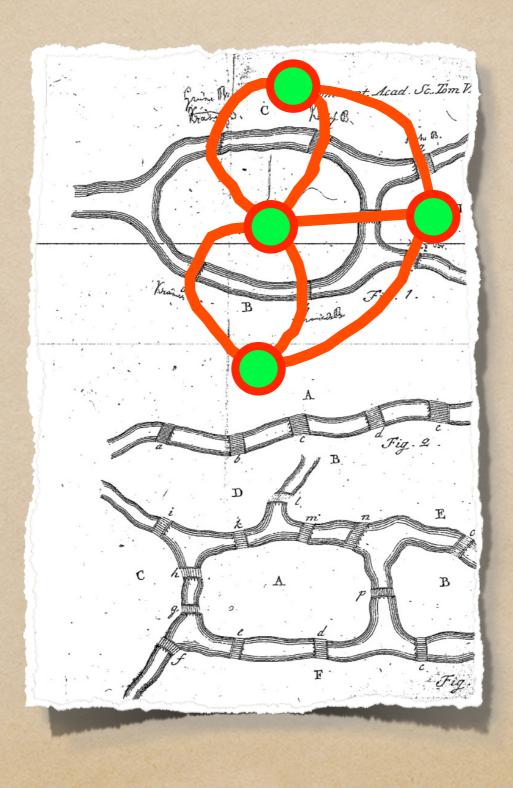


Wichtig: An einem der Knoten mit drei Kanten anfangen, weil man sonst irgendwann dort nicht mehr weg kommt!

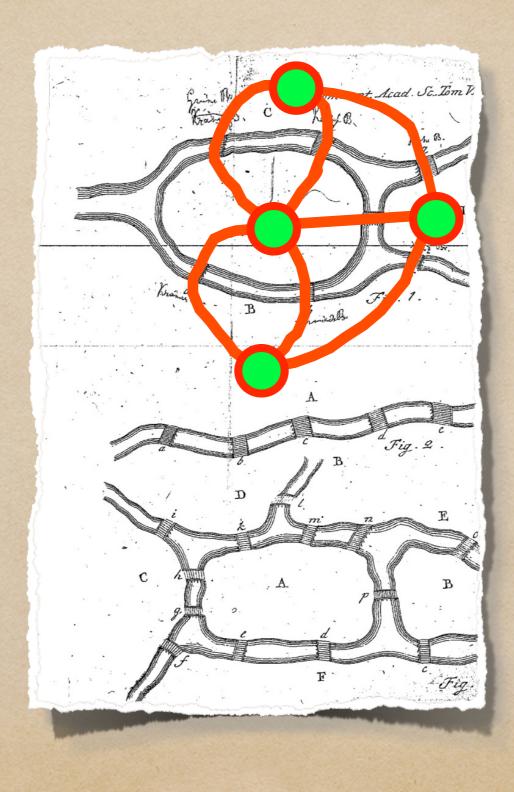




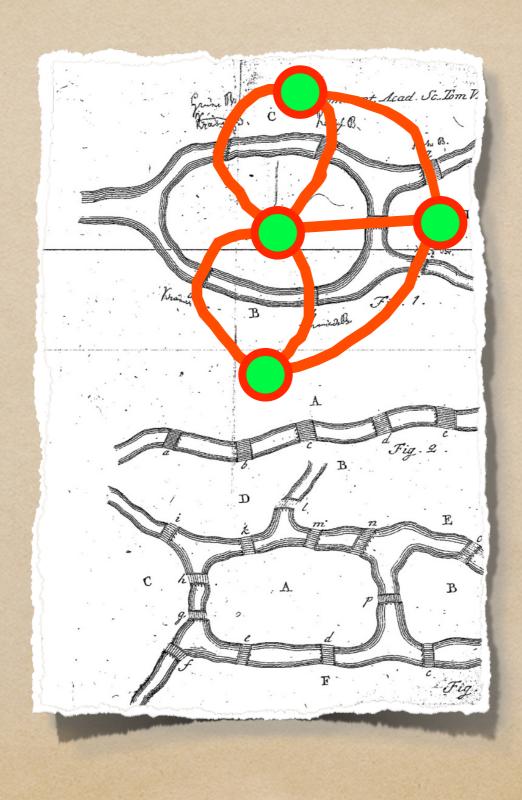




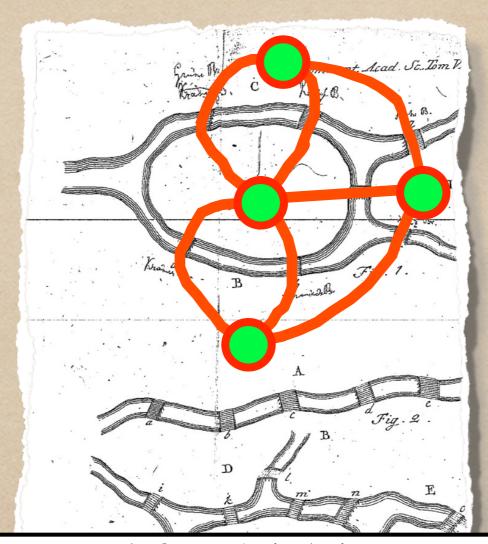
• Alle Knoten sind ungerade?!



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- Das geht nicht an einem Stück!



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Euler: (1) Das gilt für jede beliebige Instanz: Mit mehr als zwei ungeraden Knoten gibt es keinen solchen Weg.

(2) Man kann auch charakterisieren, unter welchen Bedingungen es einen Weg tatsächlich gibt.

SOLVTIO PROBLEMATIS

SOLVTIO PROBLEMATIS

GEOMETRIAM SITVS

AVCTORE Leonb. Eulero.

Tabula VIII. Raeter illam Geometriae partem, quae circa quantitates versatur, et omni tempore summo studio est exculta, alterius partis etiamnum admodum ignotae primus mentionem fecit Leibnitzius, quam Geometriam fitus vocauit. Ista pars ab ipso in solo fitu determinando, fitusque proprietatibus eruendis occupata effe statuitur; in quo negotio neque ad quantitates respiciendum, neque calculo quantitatum vtendum sit. Cuiusmodi autem problemata ad hanc fitus Geometriam pertineant, et quali methodo in iis resoluendis vti oporteat, non fatis est definitum. Quamobrem, cum nuper problematis cuiusdam mentio esset sacta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, vt neque determinationem quantitatum requireret, neque folutionem calculi quantitatum ope admitteret, id ad geometriam fitus referre haud dubitaui; praesertim quod in eius solutione solus situs in considerationem vemat, calculus vero nullius prorfus fit vius. Methodum ergo meam quam ad huius generis proble-

- Alle Knoten sind ungerade?!
- Man müsste an allen anfangen oder aufhören!
- Das geht nicht an einem Stück!

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Euler hat:

· eine Instanz betrachtet



- eine Instanz betrachtet
- ein Problem gelöst



- · eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet



CAN YOU PASS THE SALT?

Euler hat:

- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet

2.1 Historie





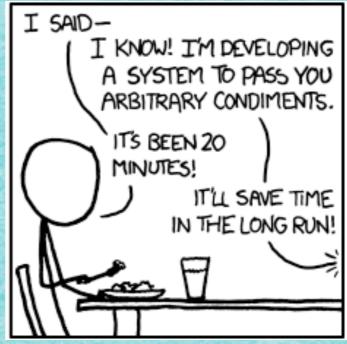


- · eine Instanz betrachtet
- ein Problem gelöst
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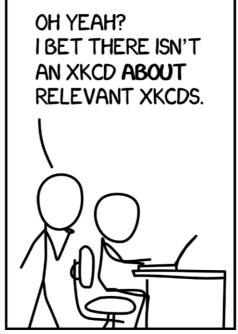


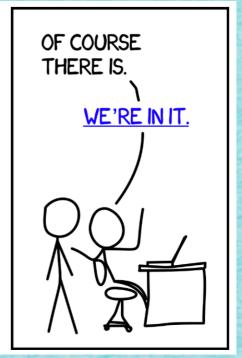
- eine Instanz betrachtet
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- ein Gebiet begründet









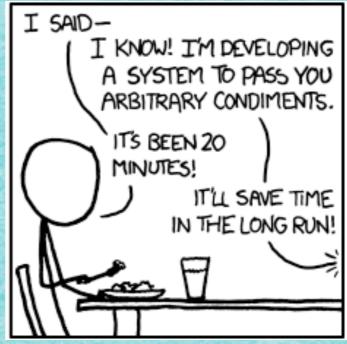


- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet









- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet



Leonhard Euler:



Leonhard Euler:

1707 Geboren in Basel



Leonhard Euler:

1707 Geboren in Basel 1720 Studienbeginn in Basel



Leonhard Euler:

1707 Geboren in Basel 1720 Studienbeginn in Basel 1723 Magister



Leonhard Euler:

1707 Geboren in Basel 1720 Studienbeginn in Basel 1723 Magister 1727 Berufung an Petersburger Akademie



Leonhard Euler:

1707 Geboren in Basel 1720 Studienbeginn in Basel 1723 Magister 1727 Berufung an Petersburger Akademie 1731 Professur für Physik



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Leonhard Euler:

1707 Geboren in Basel

1720 Studienbeginn in Basel 1723 Magister

1727 Berufung an Petersburger

Akademie

1731 Professur für Physik



Erik Demaine:



Leonhard Euler:

1707 Geboren in Basel

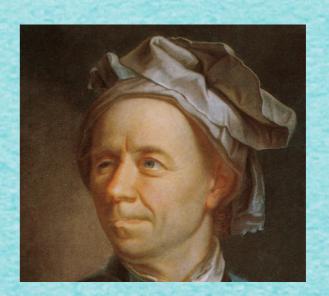
1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

Akademie

1731 Professur für Physik



Erik Demaine:

1981 Geboren in Halifax



Leonhard Euler:

1707 Geboren in Basel

1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

Akademie

1731 Professur für Physik



Erik Demaine:

1981 Geboren in Halifax 1993 Studienbeginn in Halifax



Leonhard Euler:

1707 Geboren in Basel

1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

Akademie

1731 Professur für Physik



Erik Demaine:

1981 Geboren in Halifax

1993 Studienbeginn in Halifax 1995 Bachelor



Leonhard Euler:

1707 Geboren in Basel

1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

Akademie

1731 Professur für Physik



Erik Demaine:

1981 Geboren in Halifax

1993 Studienbeginn in Halifax 1995 Bachelor

1996 Master



Leonhard Euler:

1707 Geboren in Basel

1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

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1995 Bachelor

1996 Master

2001 Ph.D.



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1707 Geboren in Basel

1720 Studienbeginn in Basel

1723 Magister

1727 Berufung an Petersburger

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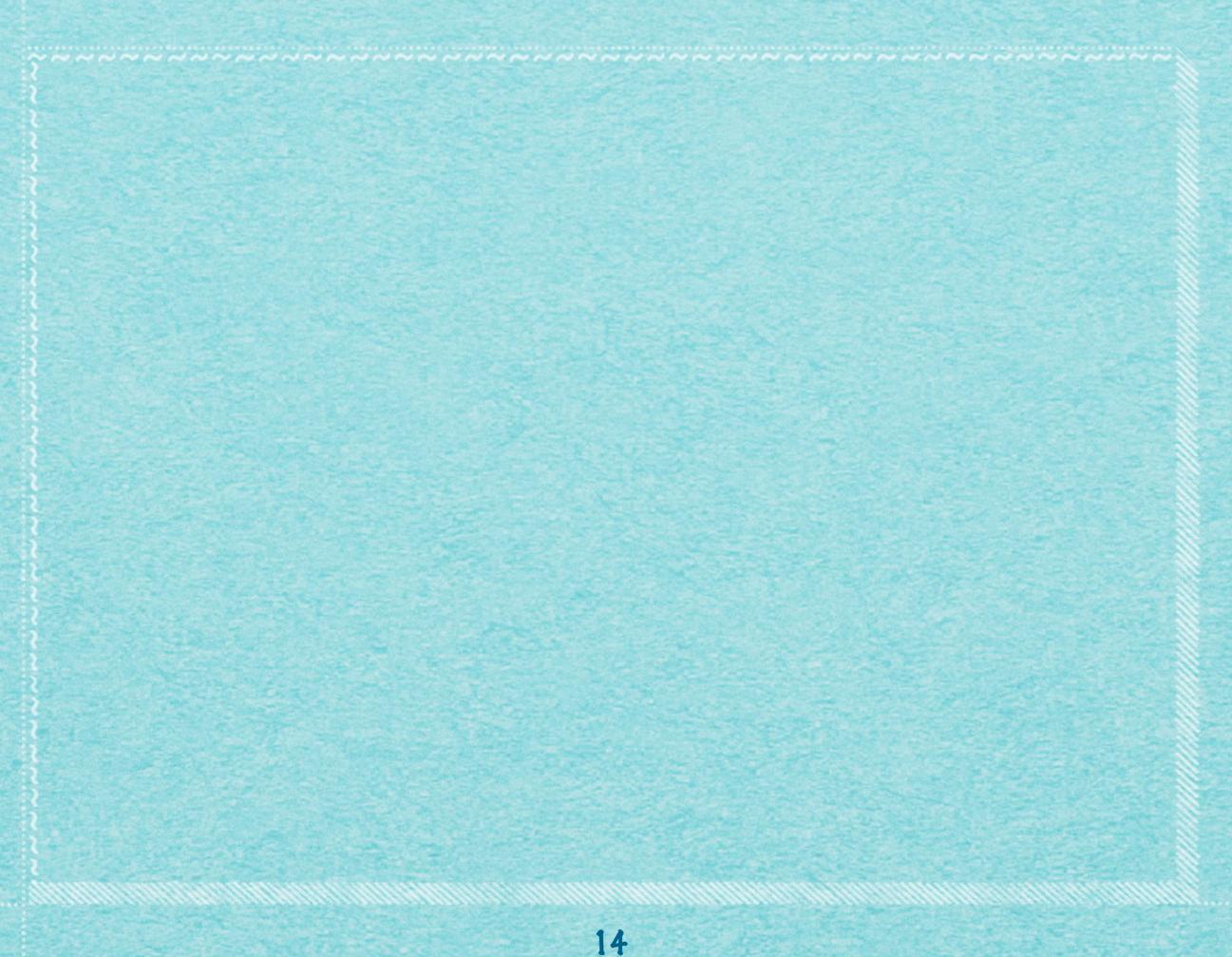
1995 Bachelor

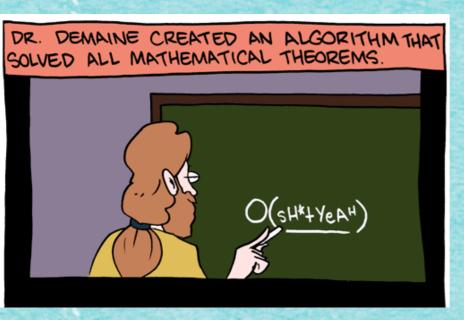
1996 Master

2001 Ph.D.

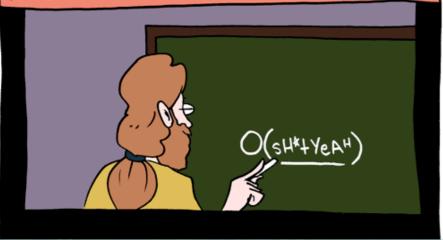
2001 Assistenzprofessor am MIT 2005 Full Professor am MIT

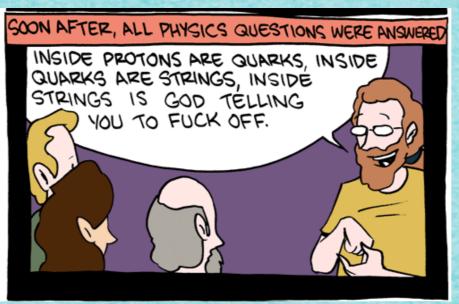






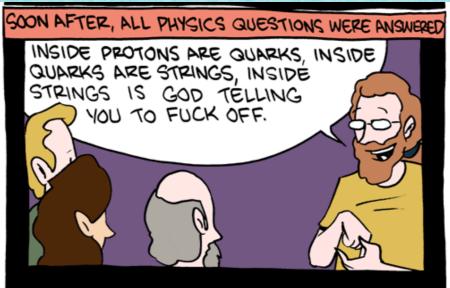
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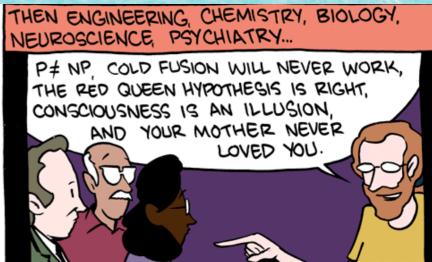


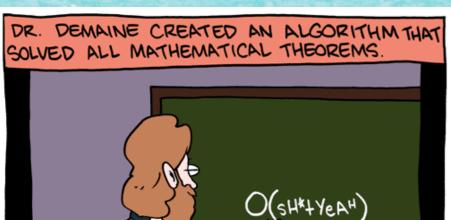


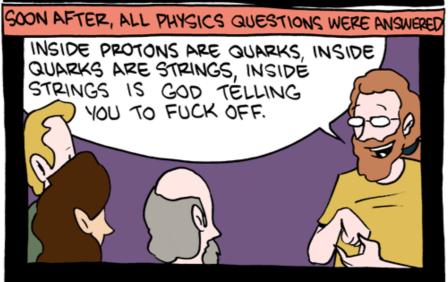
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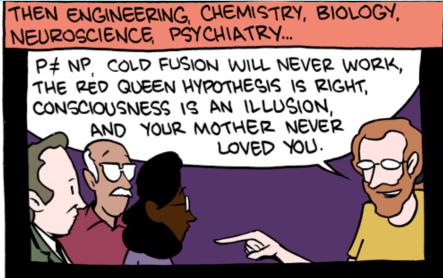


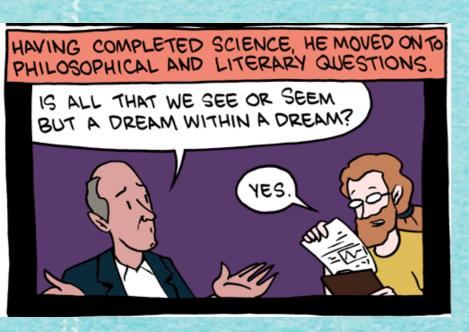






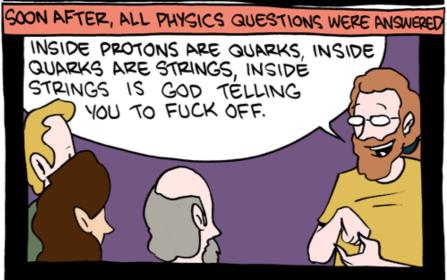


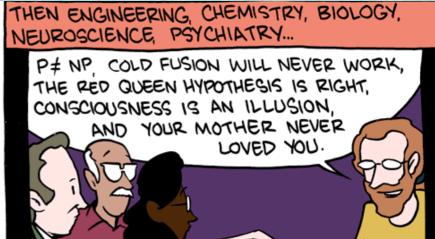


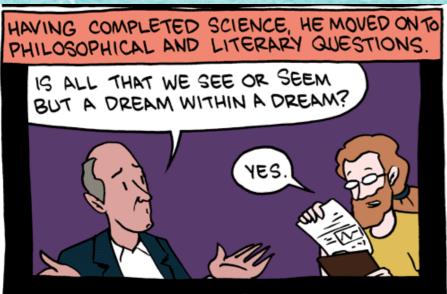


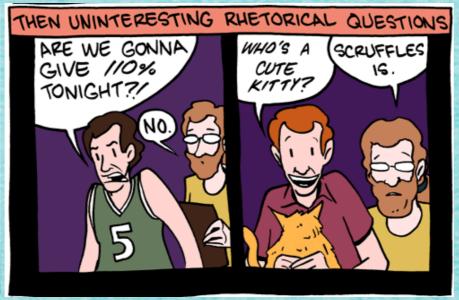


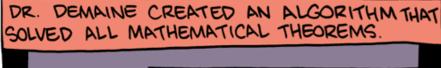


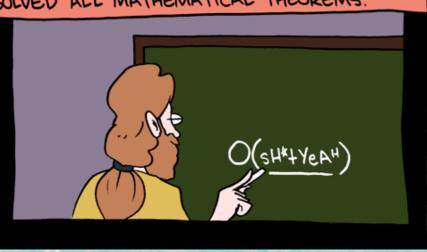


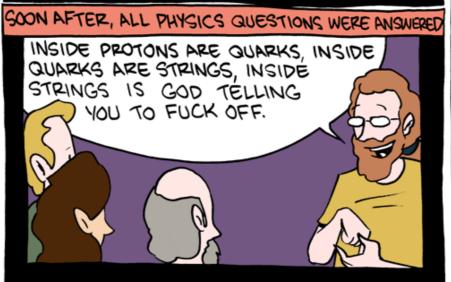


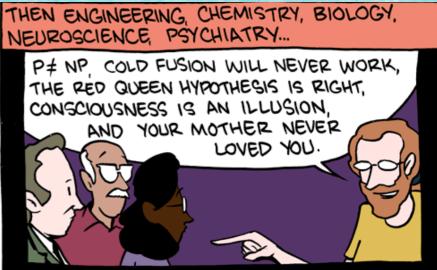


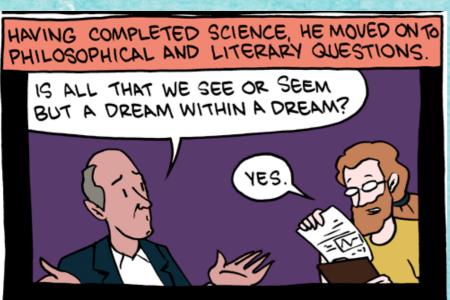






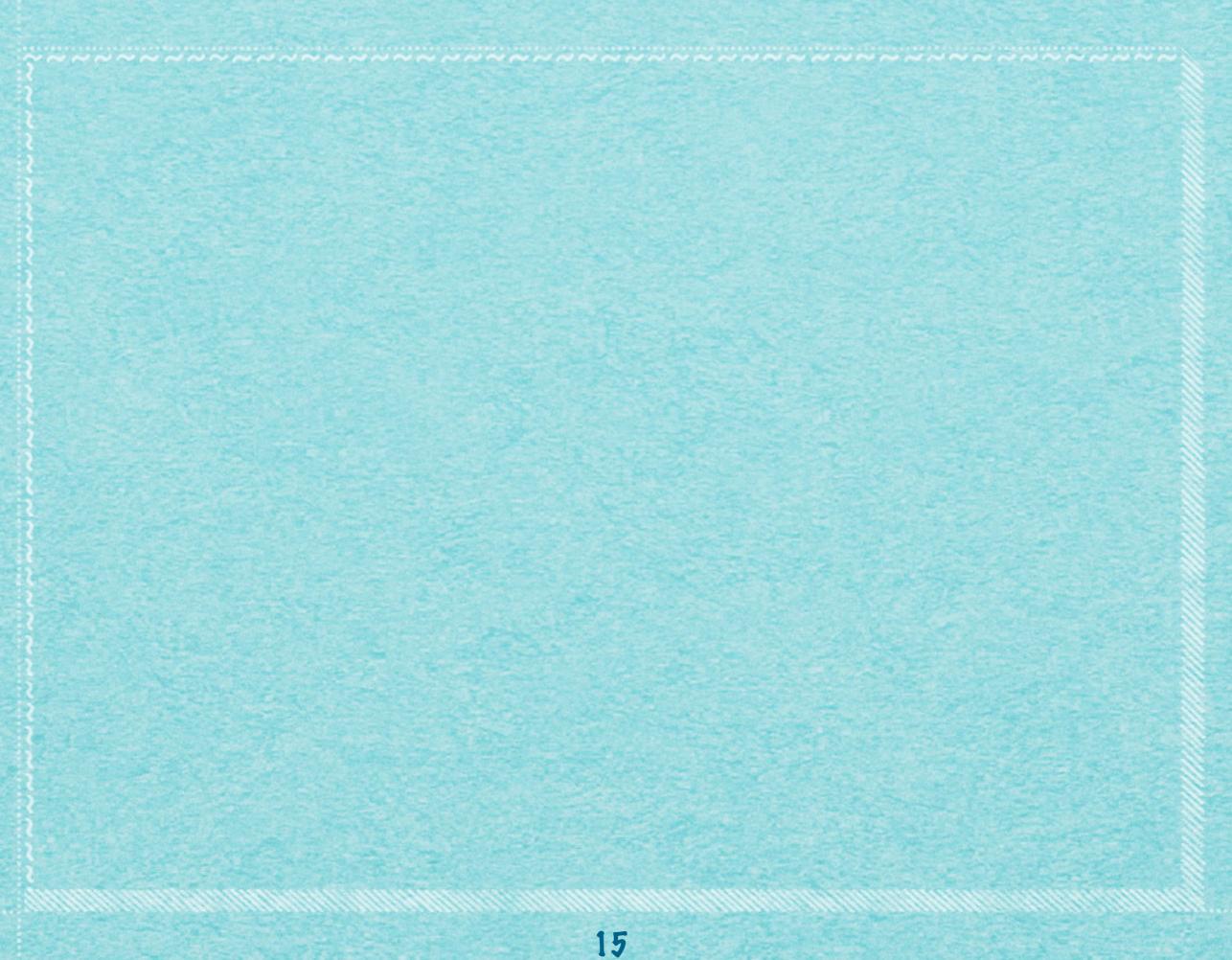














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New geometric algorithms for fully connected staged self-assembly ☆

Erik D. Demaine ^a ▶ Sándor P. Fekete ^b △ ☒, Christian Scheffer ^b ☒, Arne Schmidt ^b ☒

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Abstract

We consider *staged self-assembly systems*, in which square-shaped tiles can be added to bins in several stages. Within these bins, the tiles may connect to each other, depending on the *glue types* of their edges. Previous work by Demaine et al. showed that a relatively small number of tile types suffices to produce arbitrary shapes in this model. However, these constructions were only based on a spanning tree of the geometric shape, so they did not produce full connectivity of the underlying grid graph in the case of shapes with holes; self-assembly of fully connected assemblies with a polylogarithmic number of stages was left as a major open problem.



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Volume 93, February 2021, 101700



Folding polyominoes with holes into a cube ☆

Oswin Aichholzer ^a ⋈, Hugo A. Akitaya ^b ⋈, Kenneth C. Cheung ^c⋈, Erik D. Demaine ^d ⋈, Martin L. Demaine ^d ⋈, Sándor P. Fekete ^e ⋈, Linda Kleist ^e ⋈, Irina Kostitsyna ^f ⋈, Maarten Löffler ^g ⋈, Zuzana Masárová ^h ⋈, Klara Mundilova ^d ⋈, Christiane Schmidt ⁱ ⋈

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