

Kapitel 2: Graphen

Algorithmen und Datenstrukturen
WS 2021/22

Prof. Dr. Sándor Fekete

Konzentration



2.1 Historie: Ein Mathematiker geht spazieren



Carl Friedrich Gauß (1777-1855)



Carl Friedrich
Gauss Statue

$$\frac{1 + 2 + 3 + \dots + 100}{100 + 99 + 98 + \dots + 1} = 100 \times 101$$

Also:

$$1+2+3+\dots+100 = 5050$$

Gaußsche Summenformel

Die **Gaußsche Summenformel** (nicht zu verwechseln mit einer **Gaußschen Summe**), auch **kleiner Gauß** genannt, ist eine **Formel** für die **Summe** der ersten n aufeinanderfolgenden **natürlichen Zahlen**:

$$1 + 2 + 3 + 4 + \dots + n = \sum_{k=1}^n k = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$$

2.1 Historie: Ein Mathematiker geht spazieren

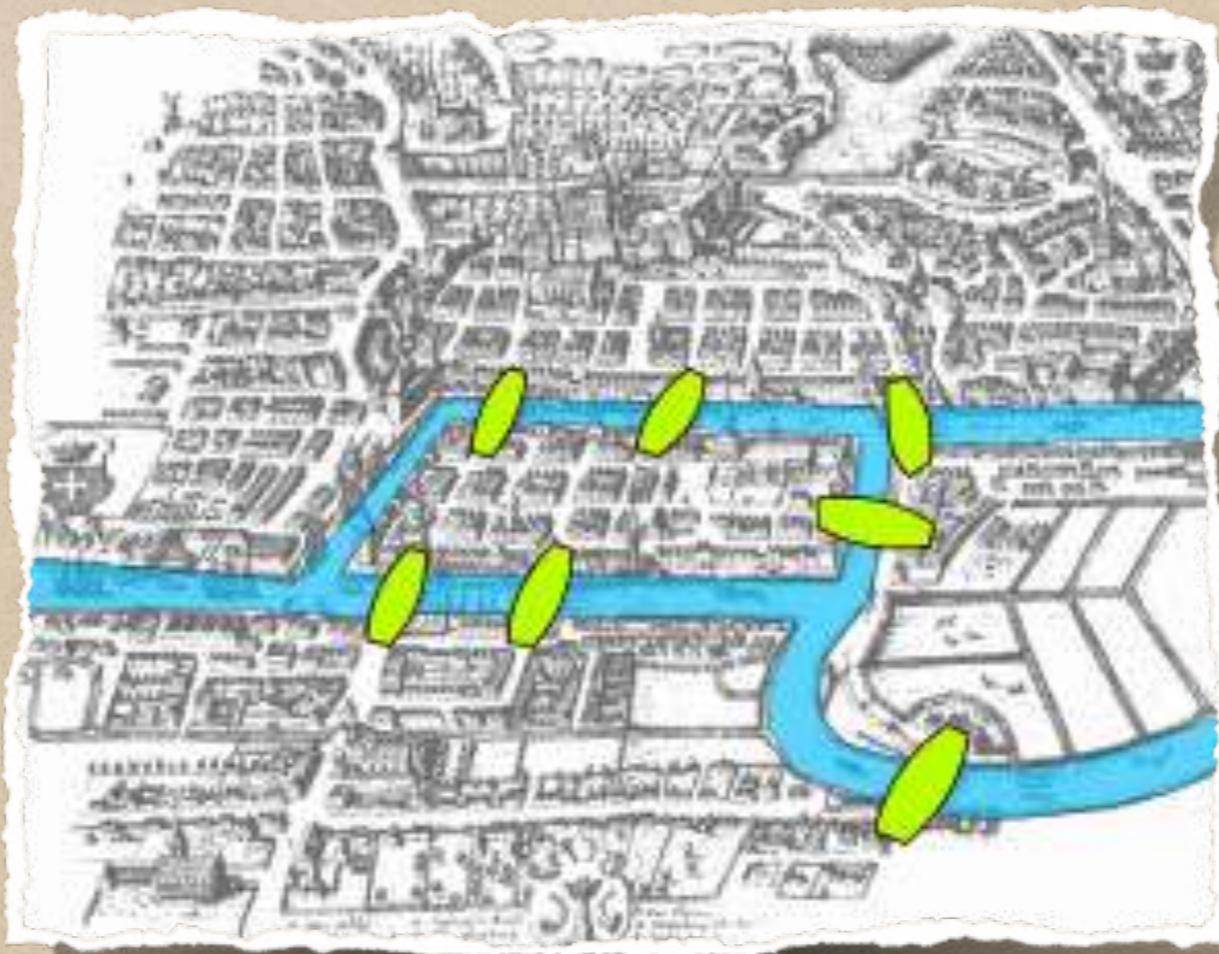


Leonhard Euler (1707-1783)

2.1 Historie: Ein Mathematiker geht spazieren

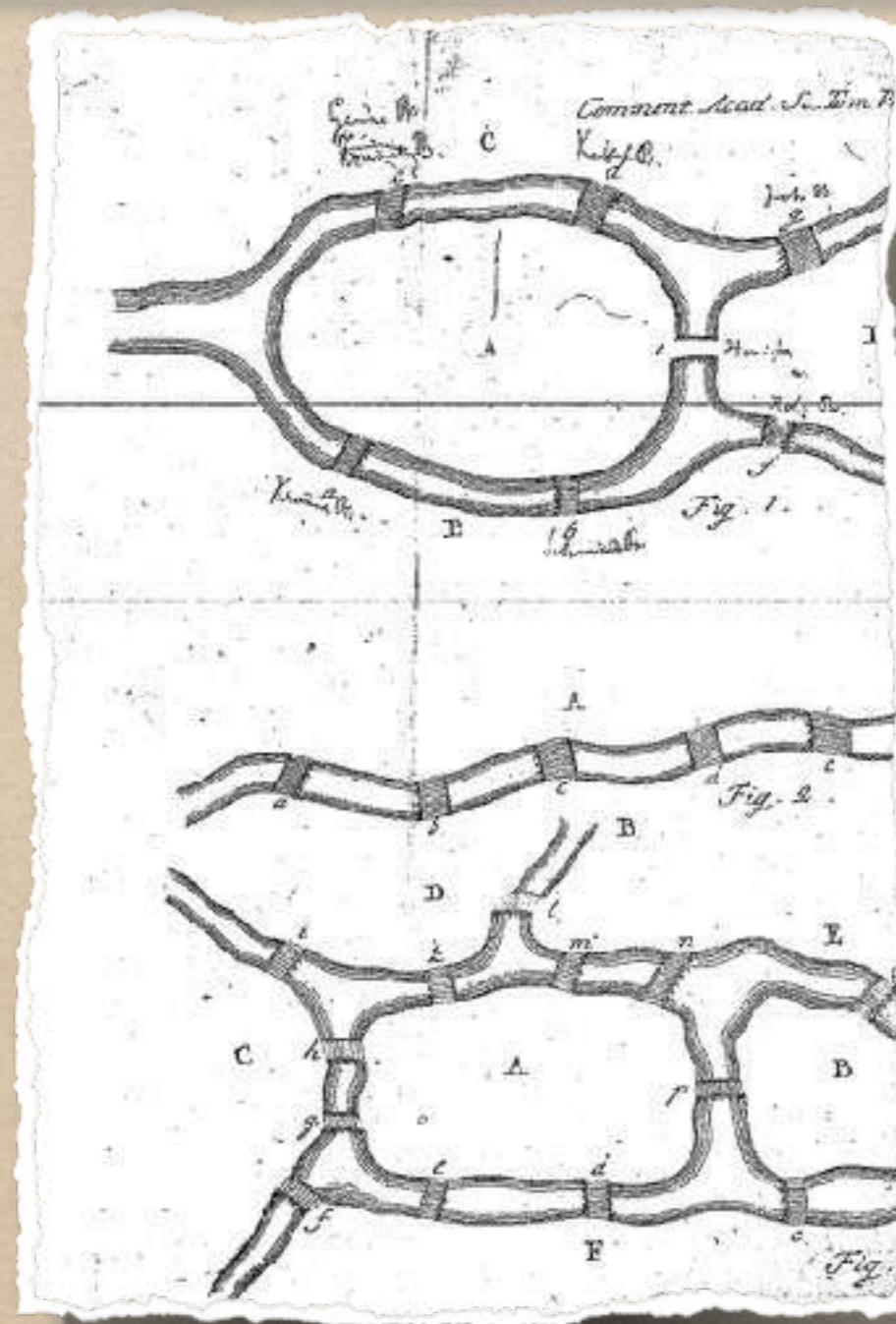


2.1 Historie: Ein Mathematiker geht spazieren



Königsberg und seine 7 Brücken

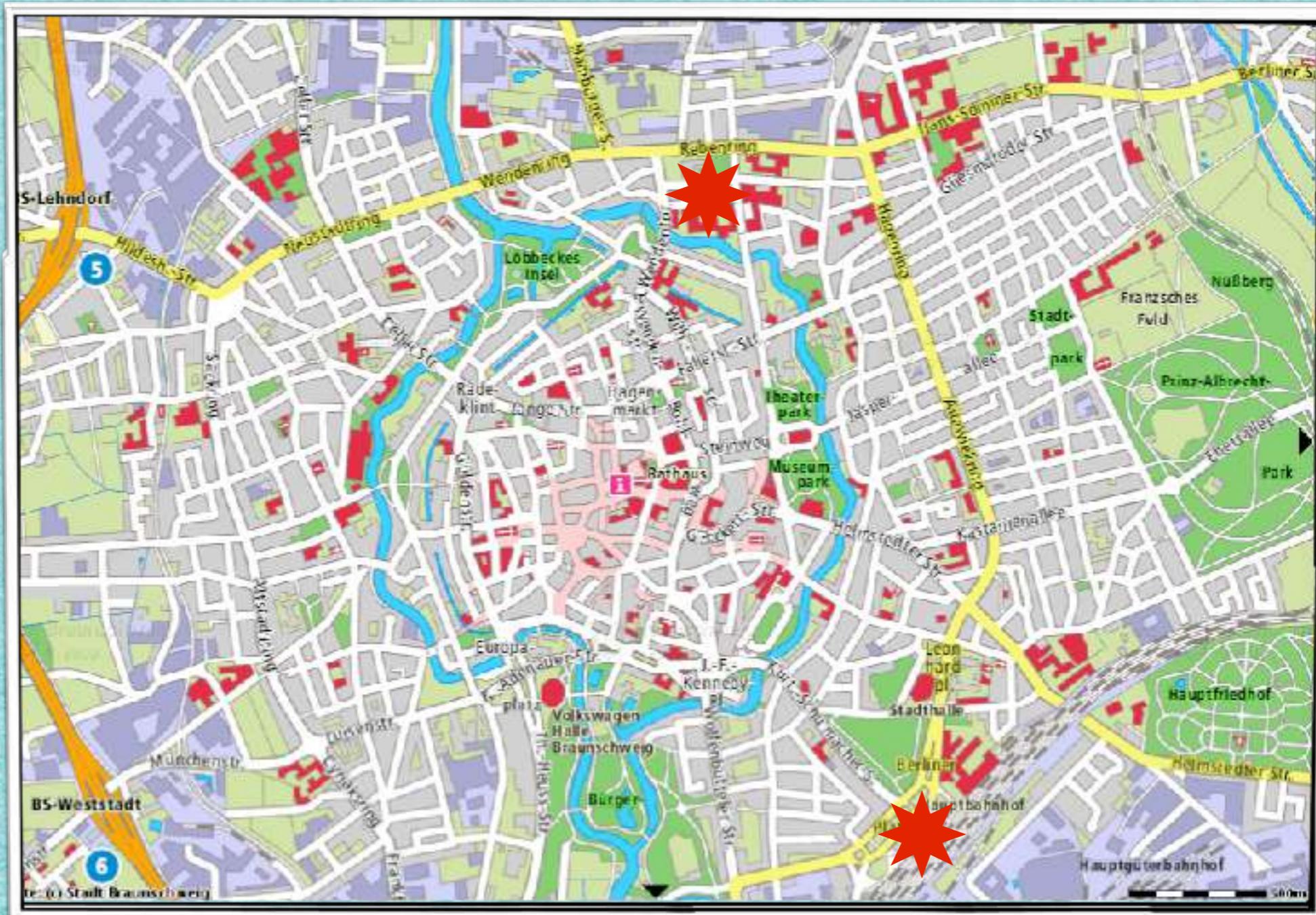
2.1 Historie: Ein Mathematiker geht spazieren



Wie kommt man zur TU?



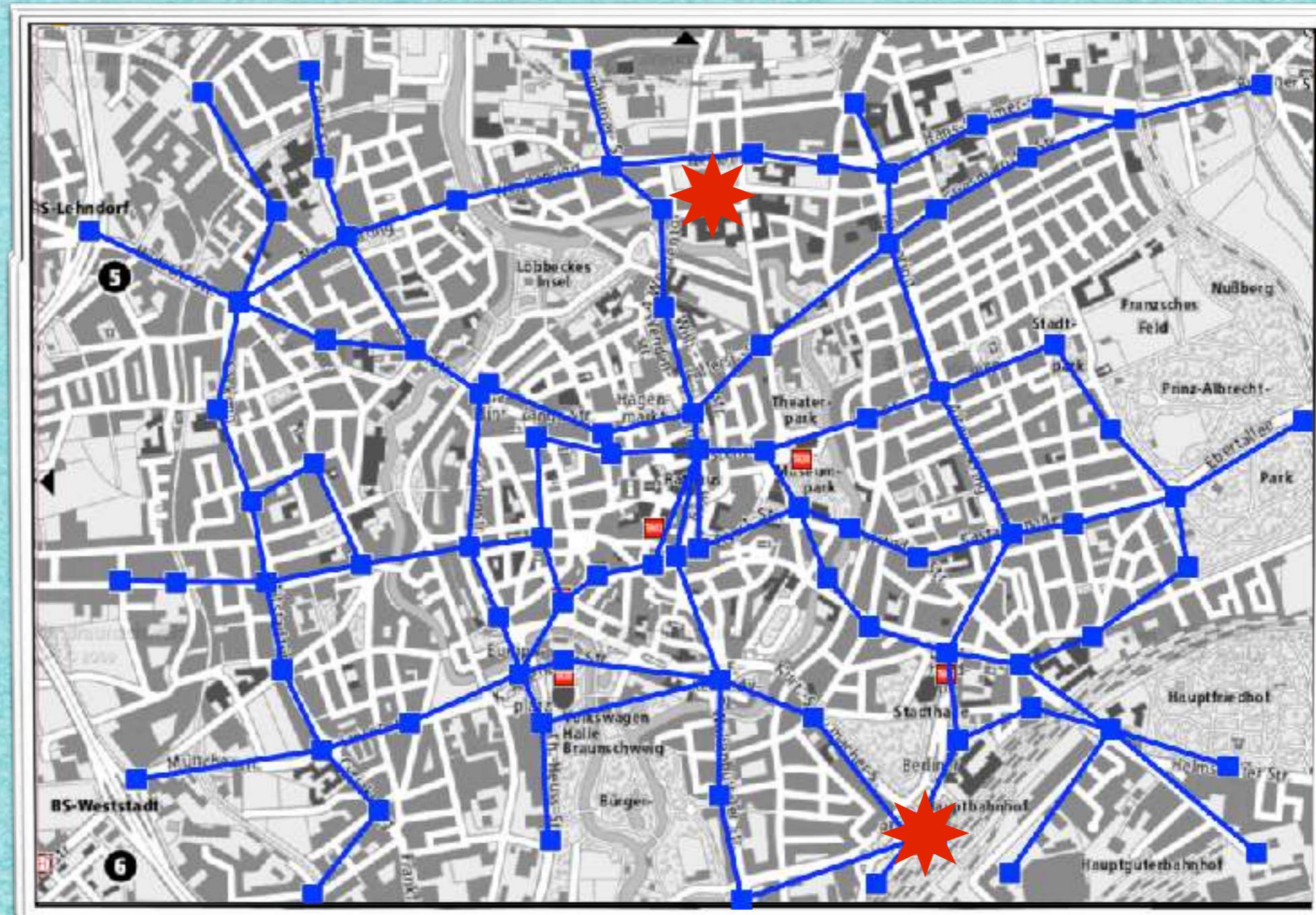
Wie kommt man zur TU?



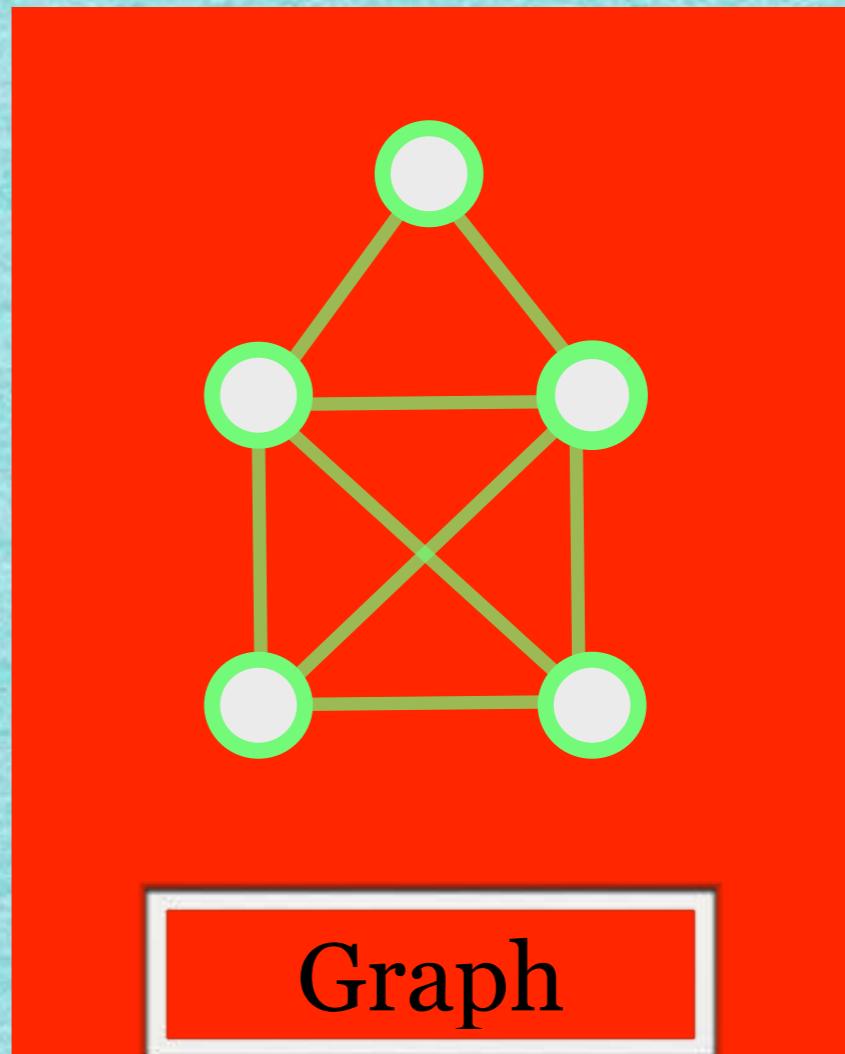
Wie kommt man zur TU?



Wie kommt man zur TU?



Gestatten, Graph!



Graph: Ein Gebilde aus Knoten (Haltestellen)
und Kanten (Verbindungen)

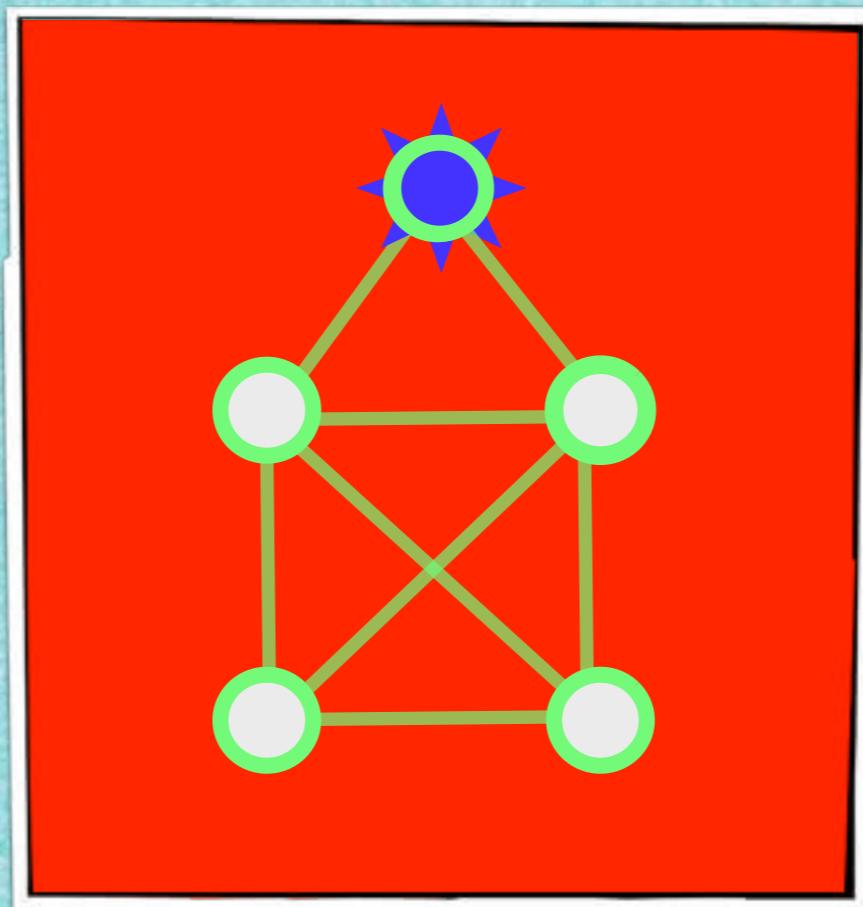
Das Haus des Nikolaus



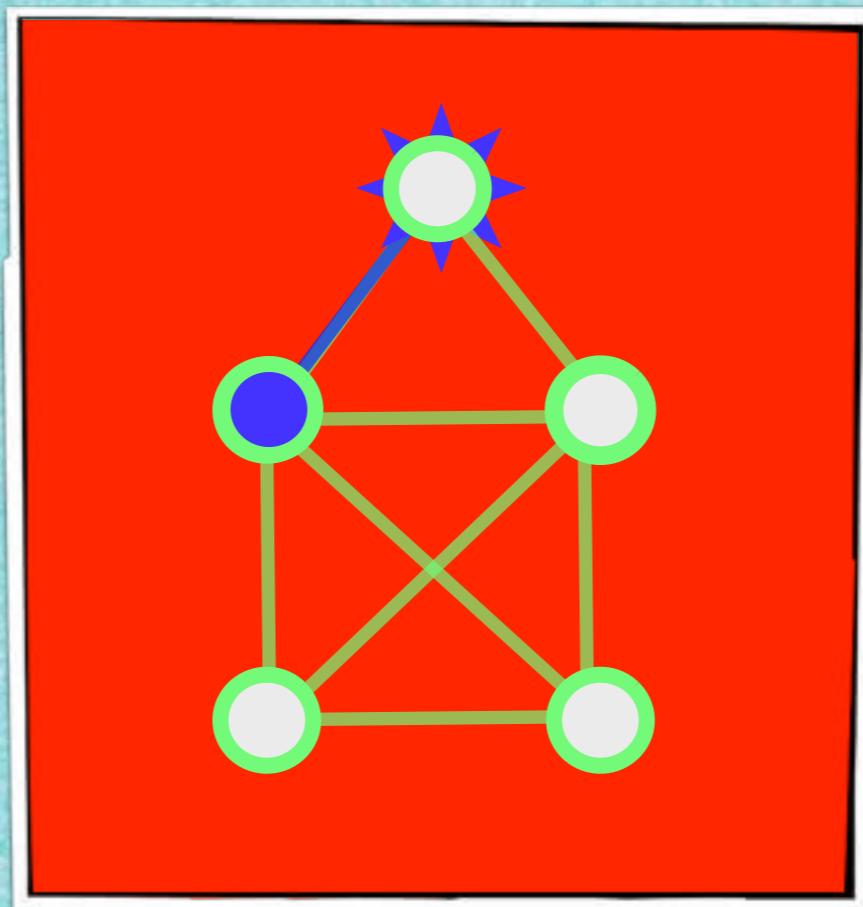
Das Haus des Nikolaus



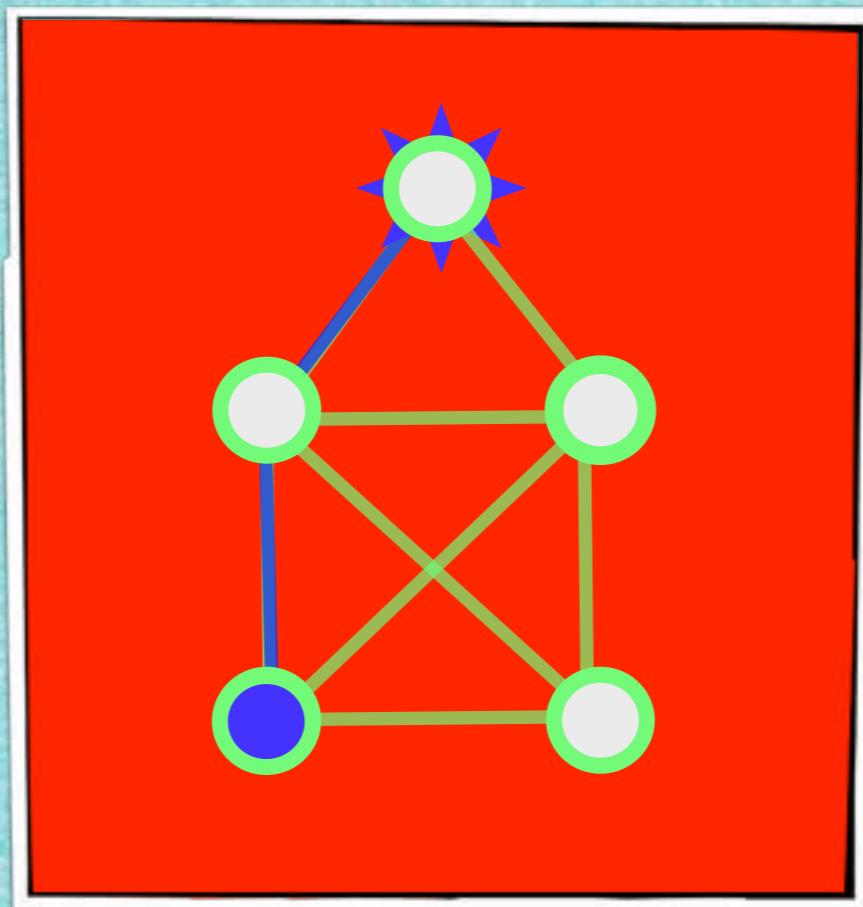
Das Haus des Nikolaus



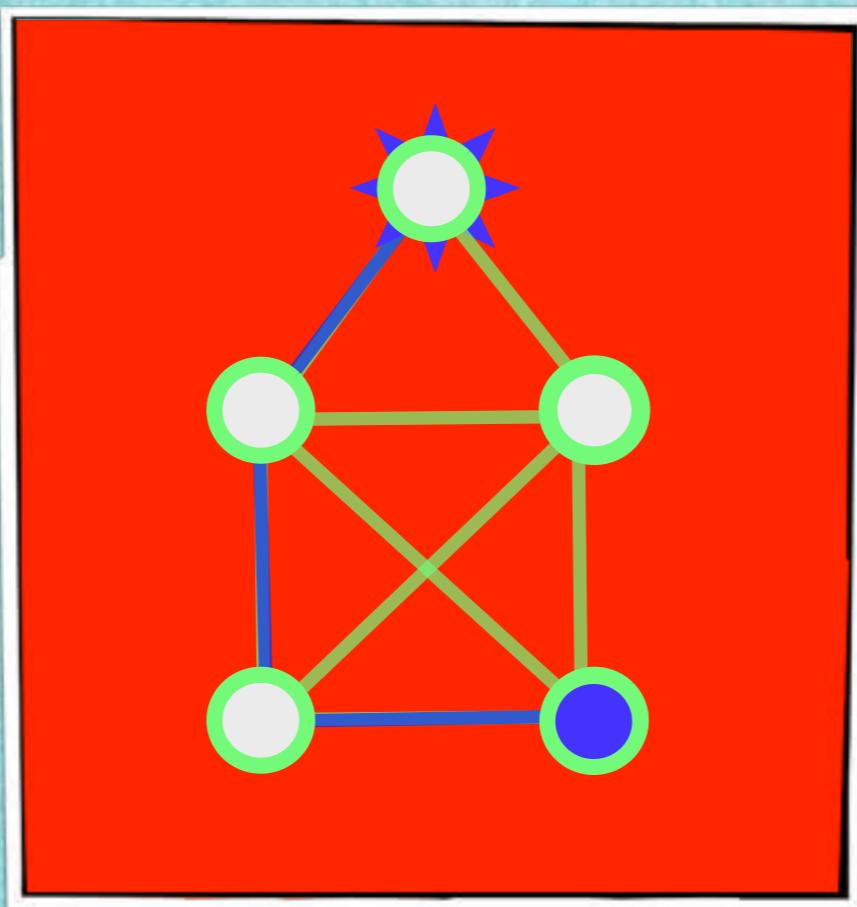
Das Haus des Nikolaus



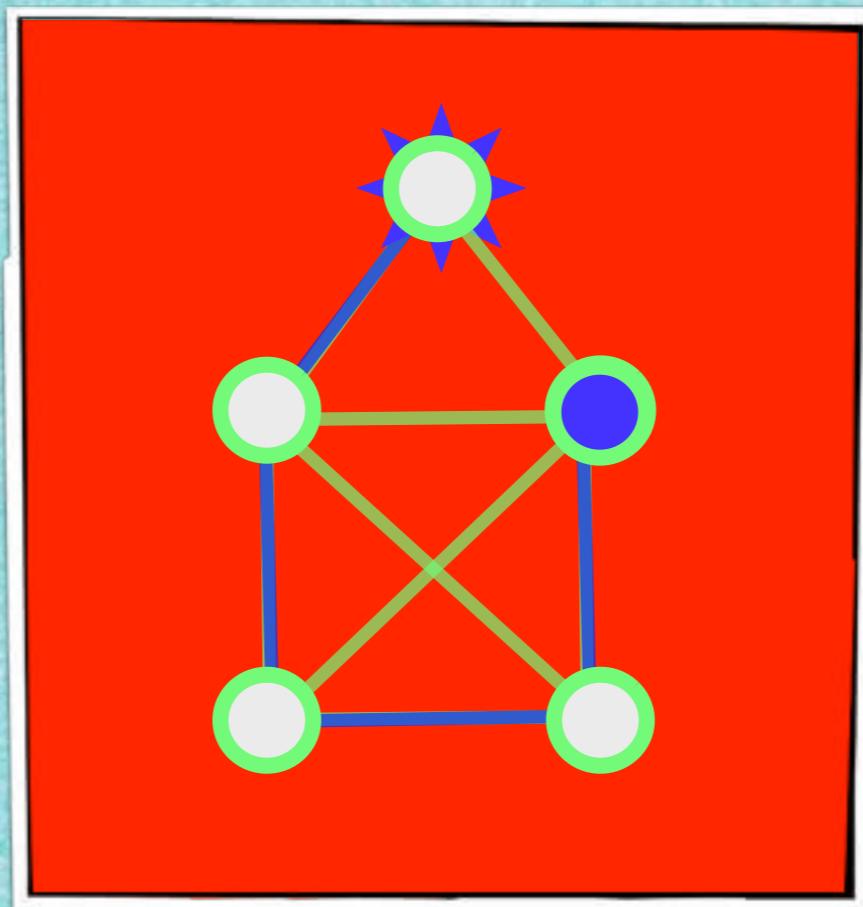
Das Haus des Nikolaus



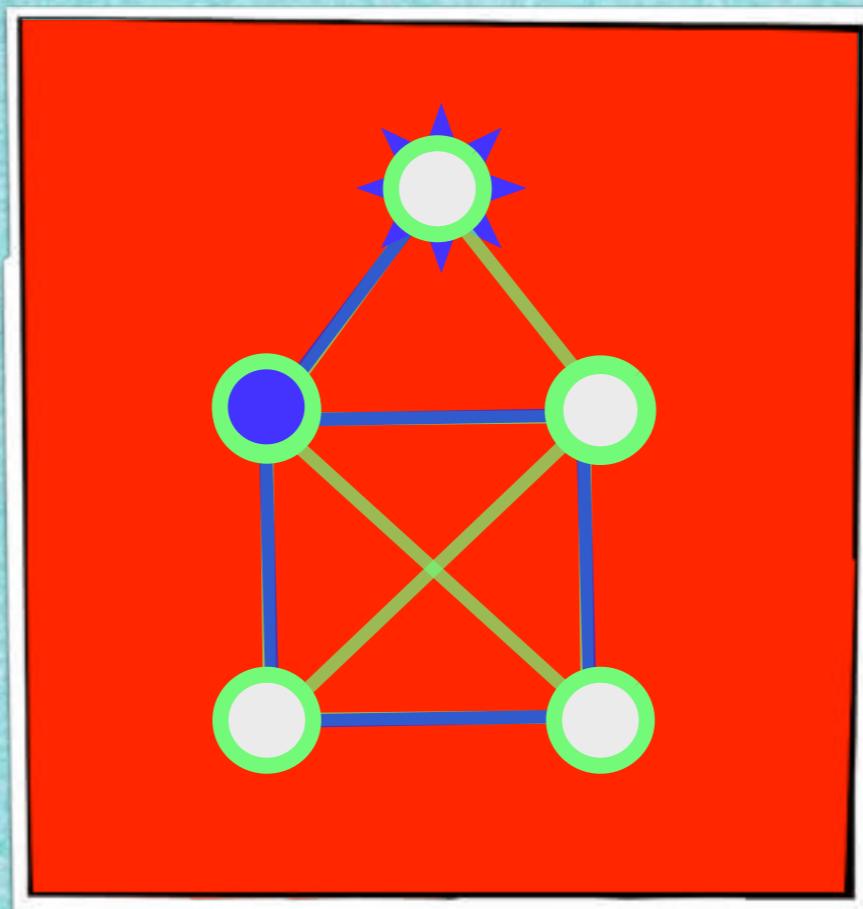
Das Haus des Nikolaus



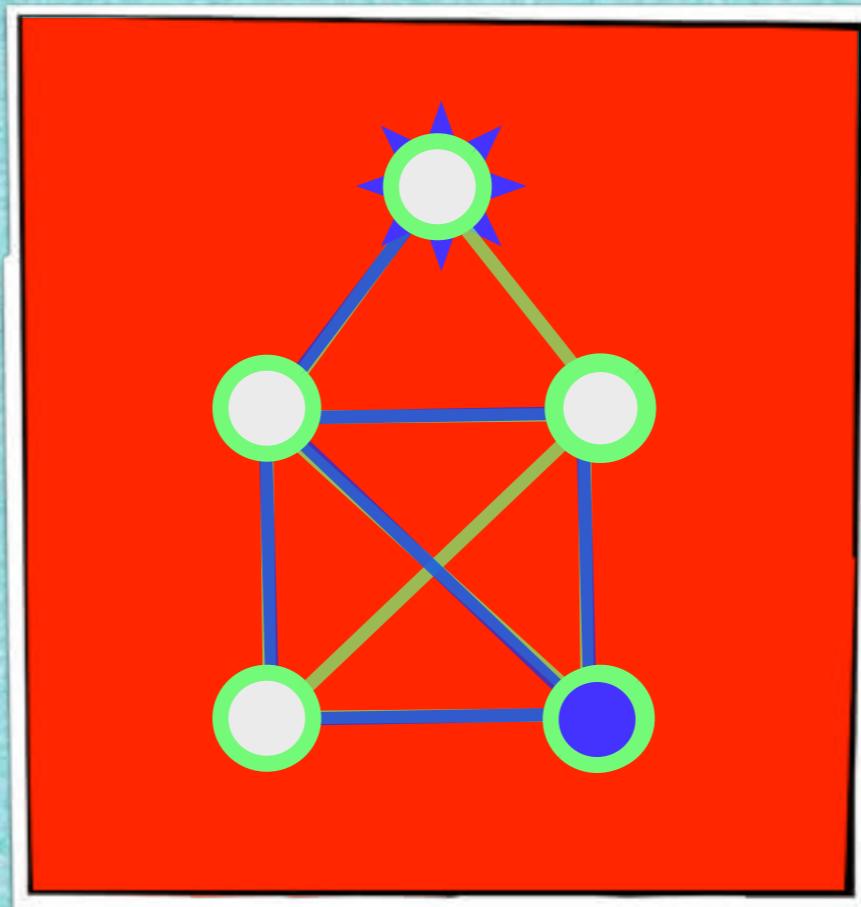
Das Haus des Nikolaus



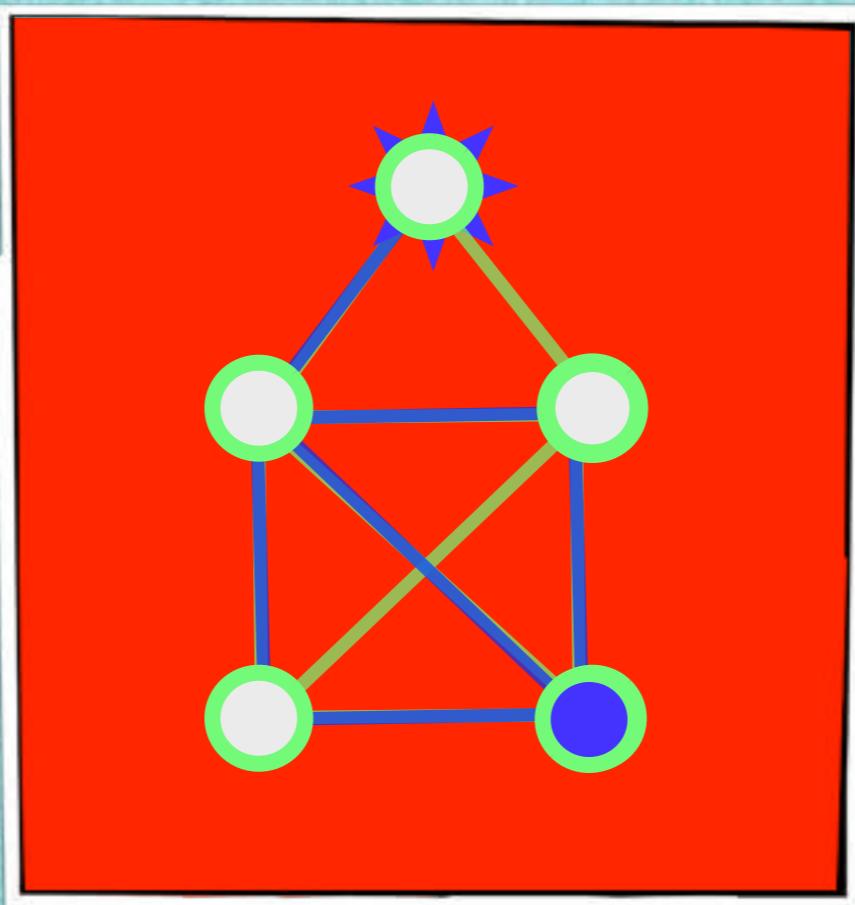
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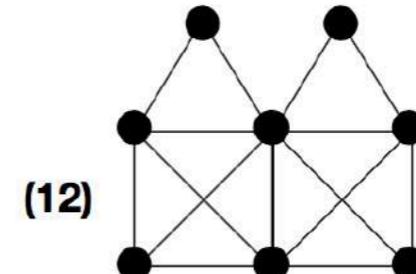
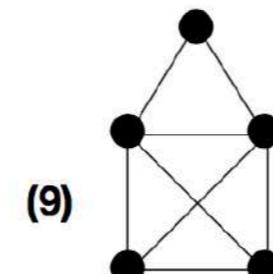
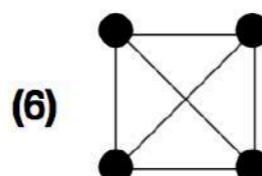
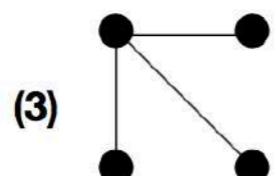
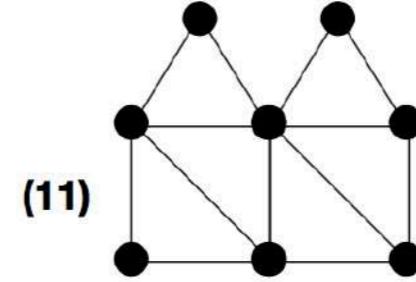
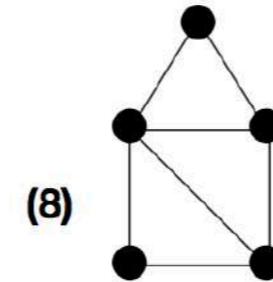
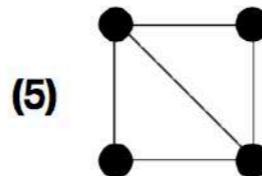
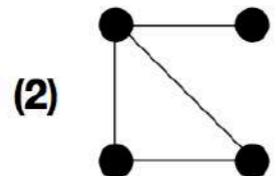
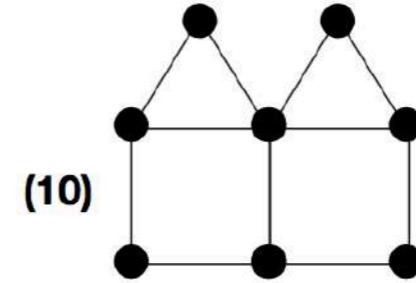
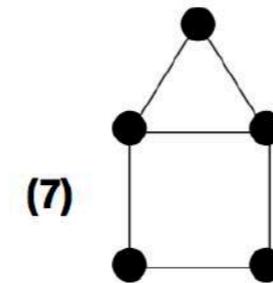
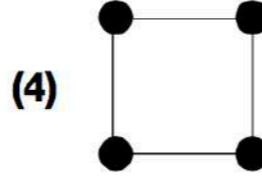
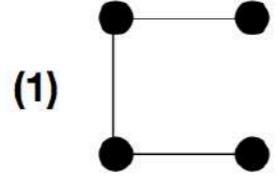
Das Haus des Nikolaus



Klappt so nicht...

Herausforderung !

Welche Graphen kannst Du in einem Zug nachzeichnen, ohne den Stift abzusetzen?
(Wenn ja, wo kann man anfangen oder aufhören?)



Herausforderung !

App Store Vorschau



One touch Drawing 4+

Ecapyc Inc.

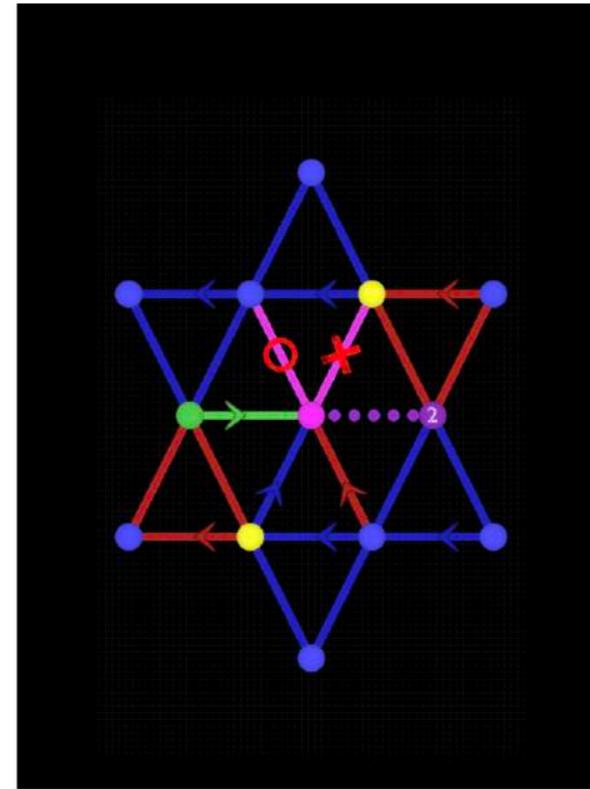
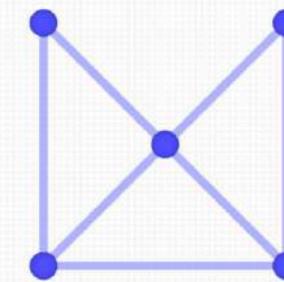
★★★★★ 4,5 • 50 Bewertungen

Gratis · In-App-Käufe möglich

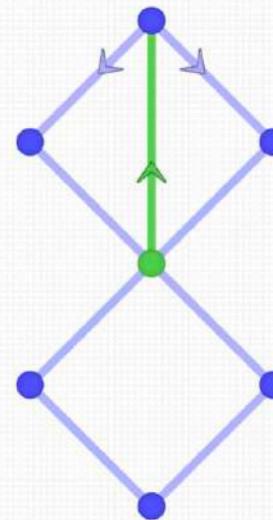
Anzeigen in: Mac App Store ↗

Screenshots iPad iPhone

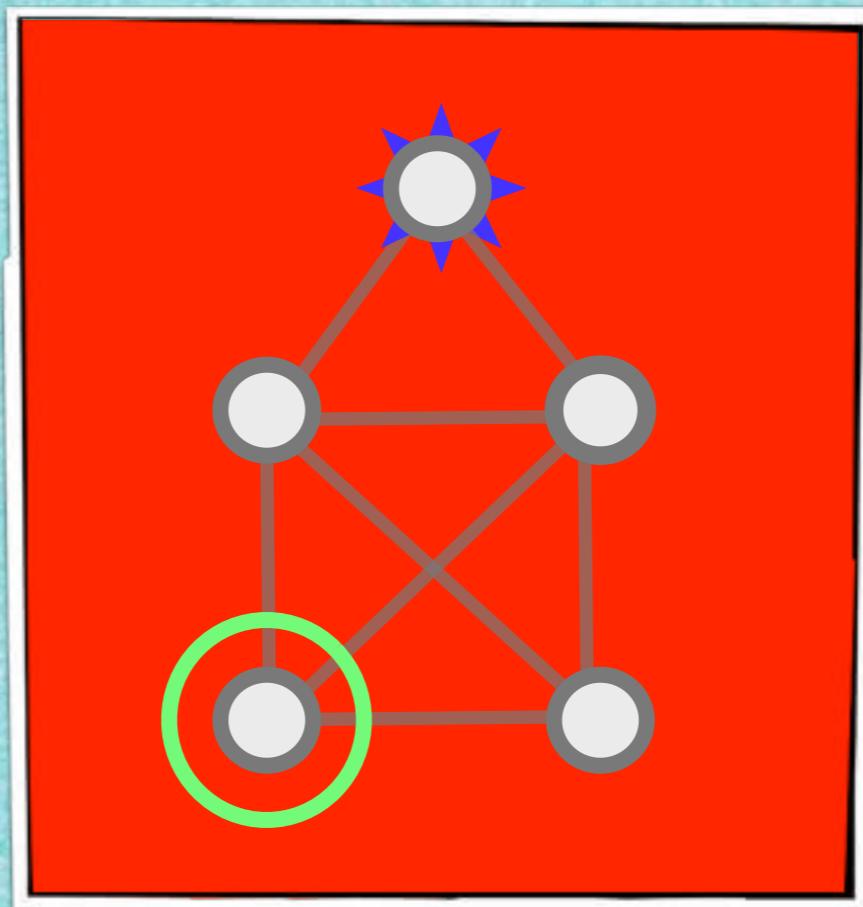
Draw all lines without repeat.



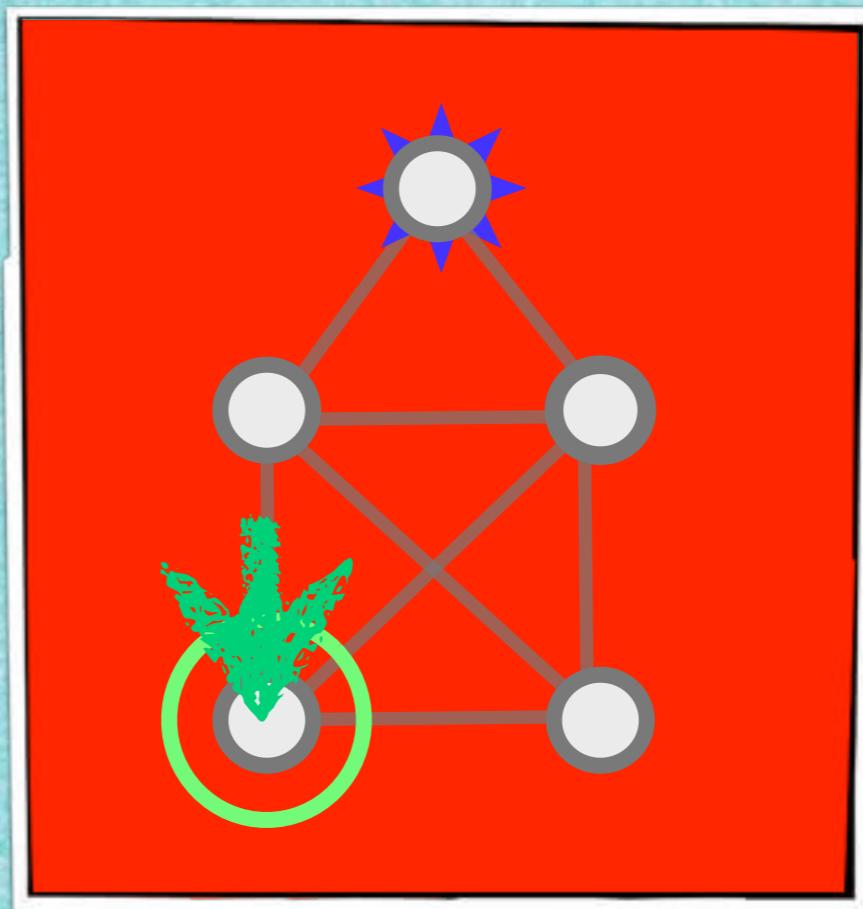
Green point is a "Direction trigger". The direction of Green line will be changed by the trigger.



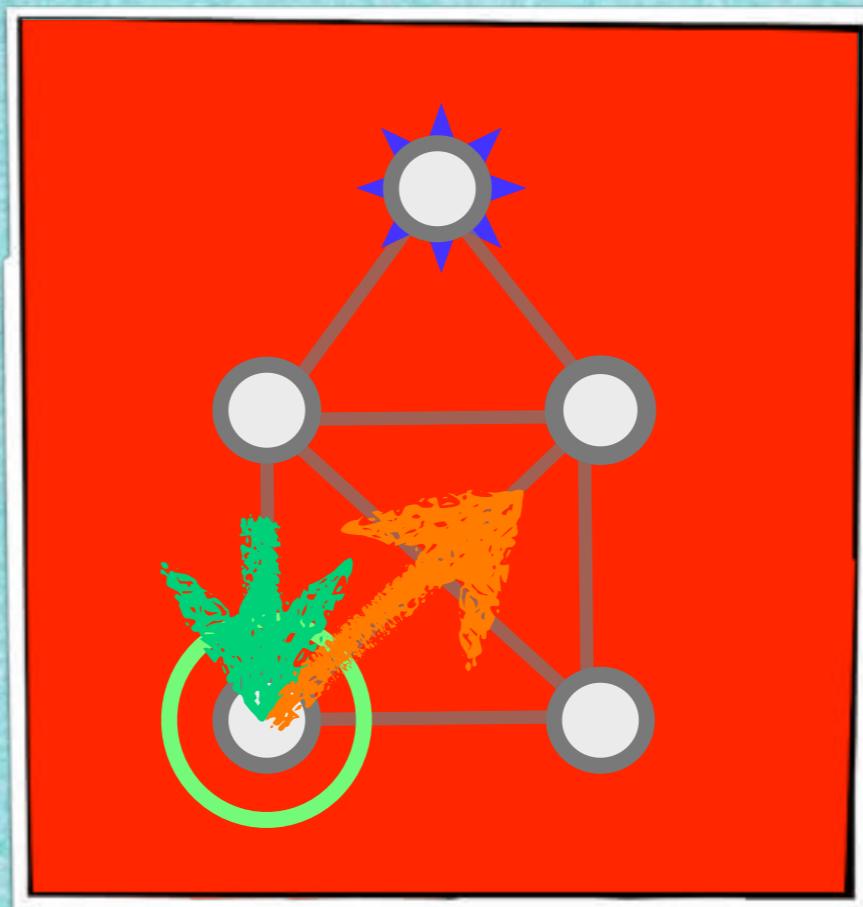
Das Haus des Nikolaus



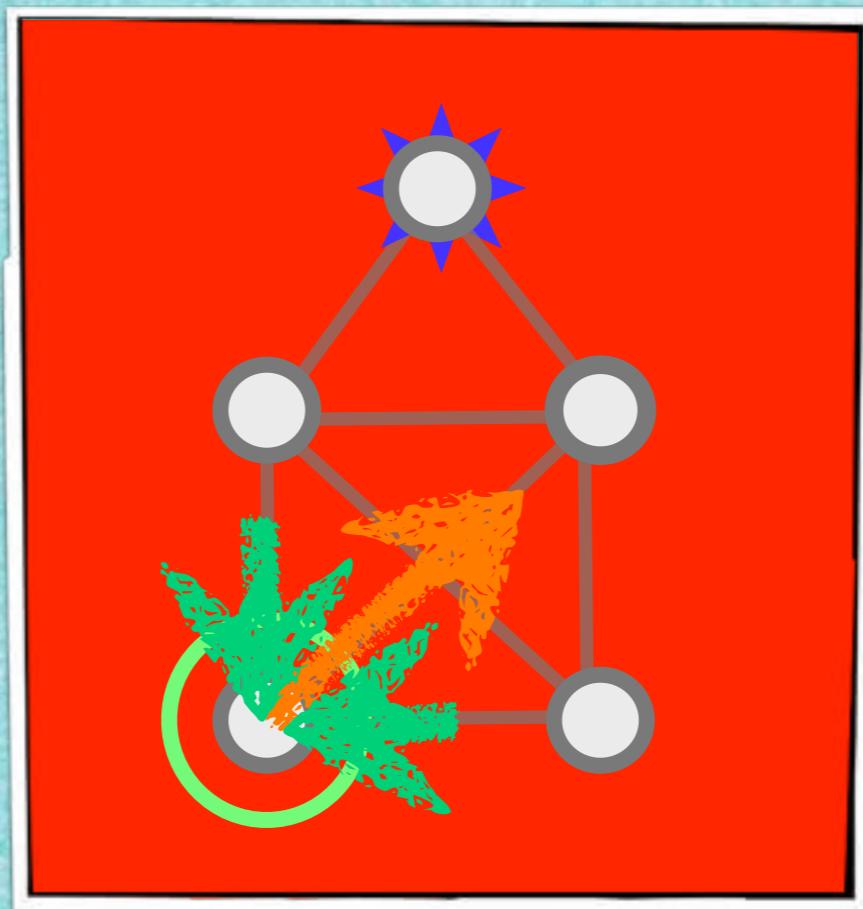
Das Haus des Nikolaus



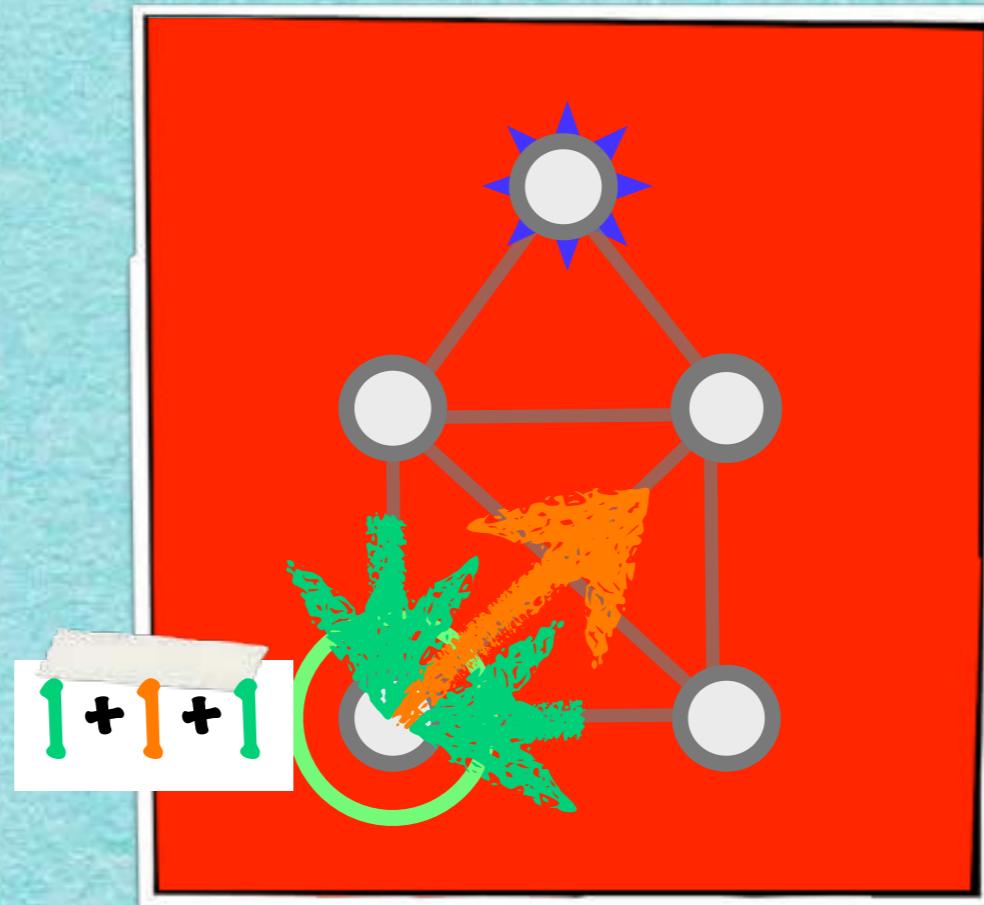
Das Haus des Nikolaus



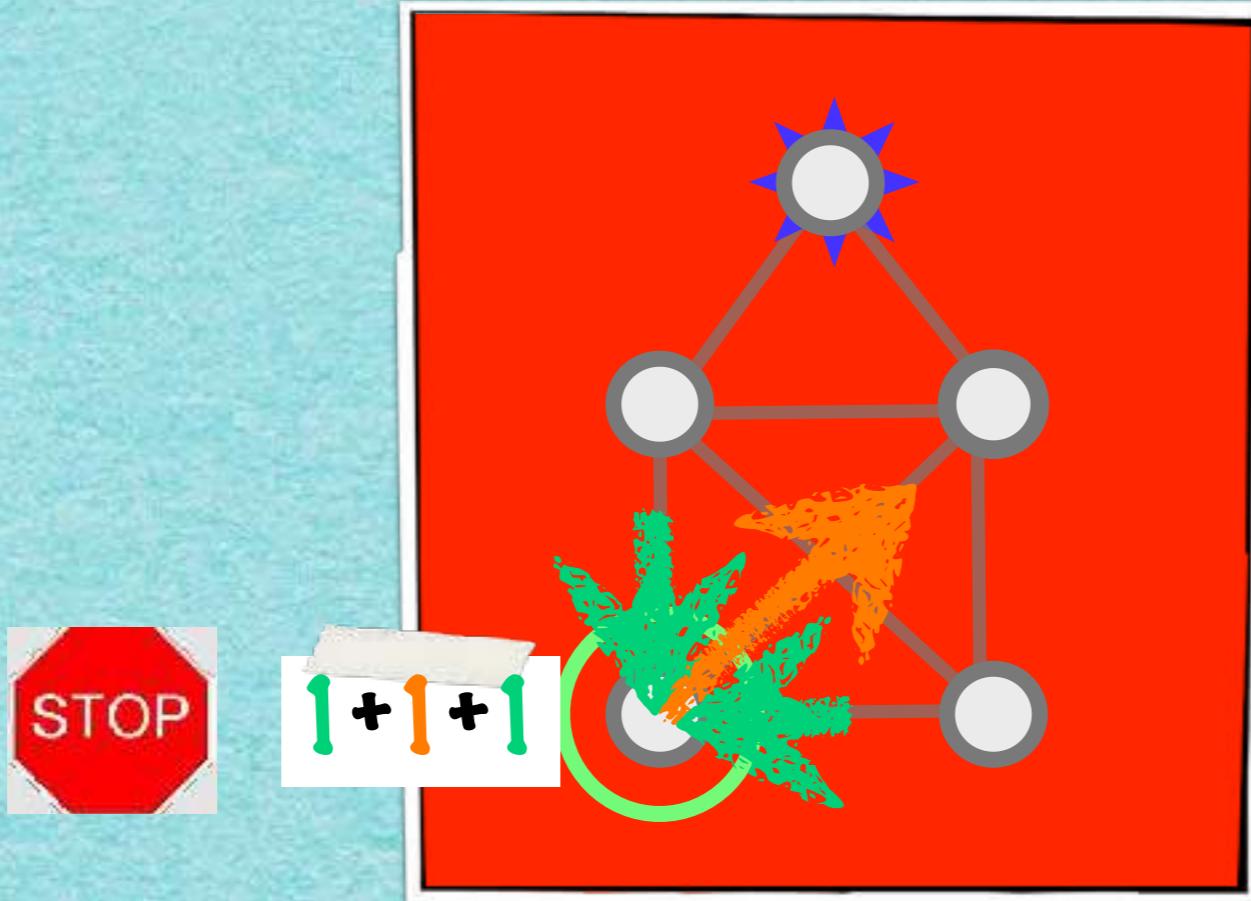
Das Haus des Nikolaus



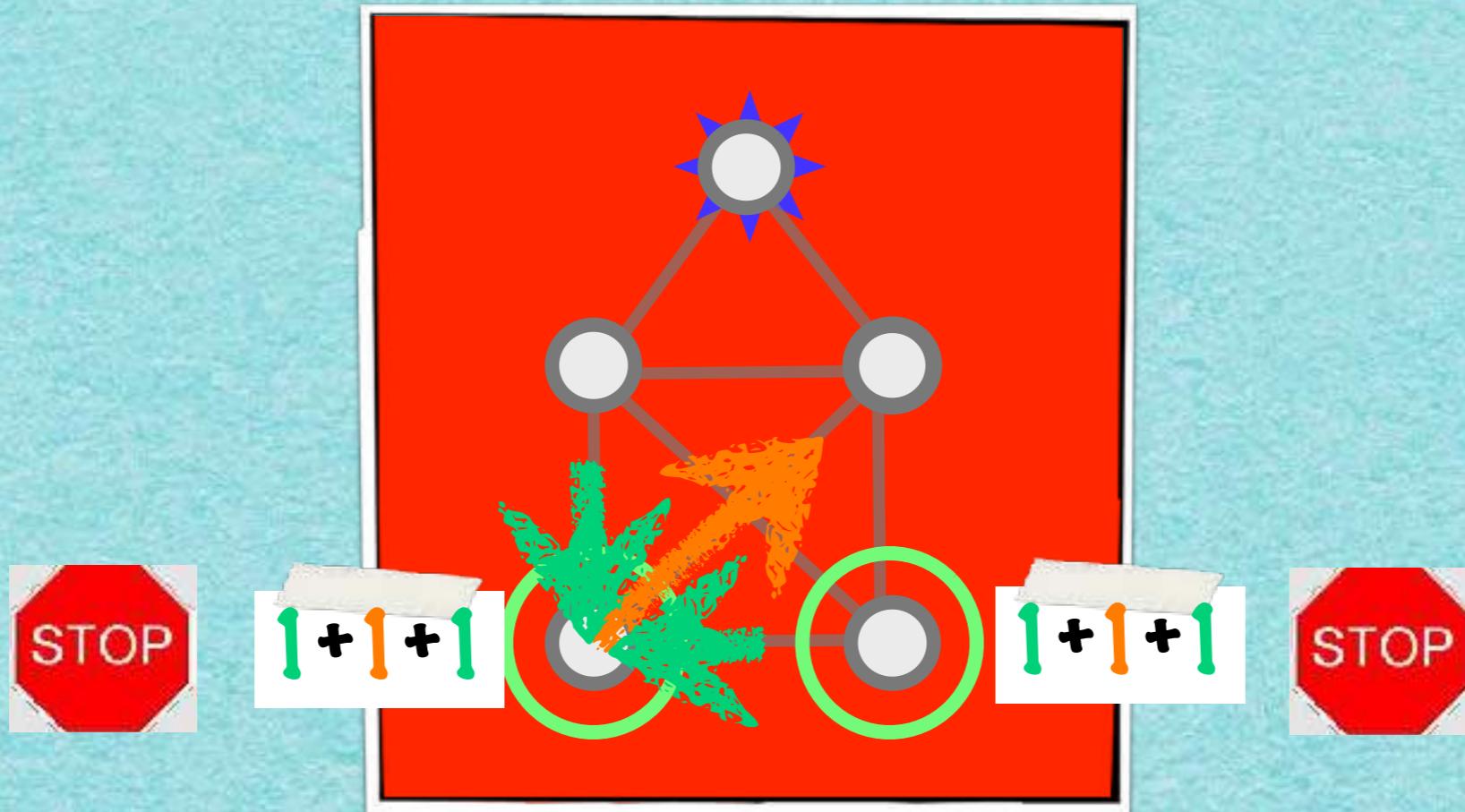
Das Haus des Nikolaus



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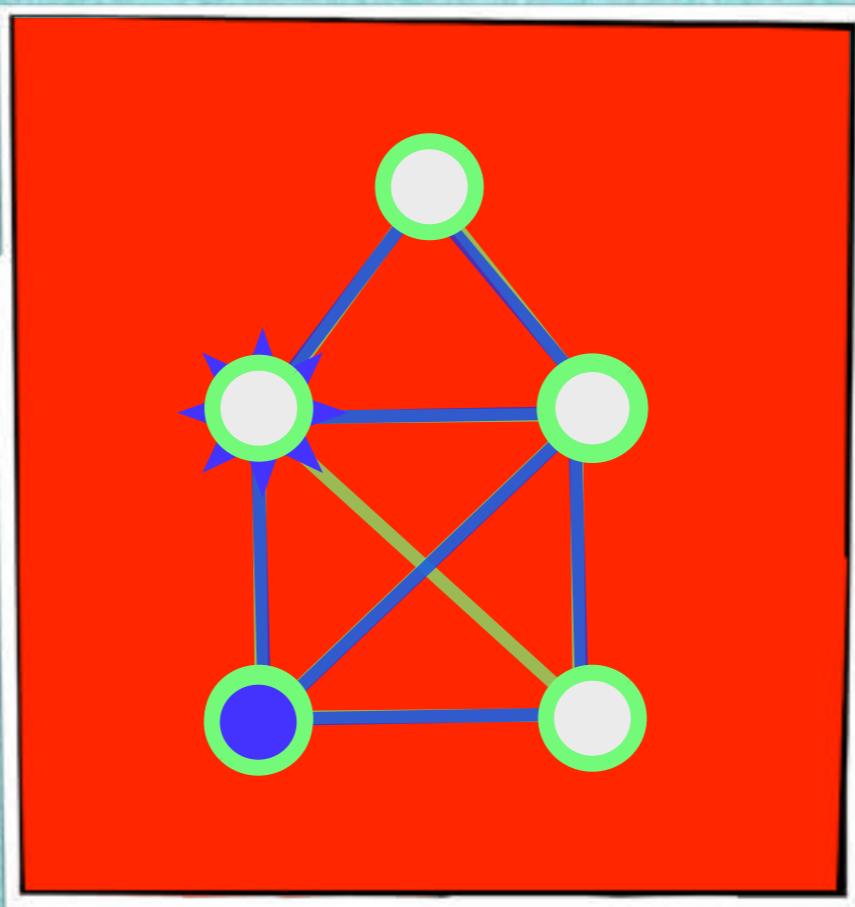


Das Haus des Nikolaus



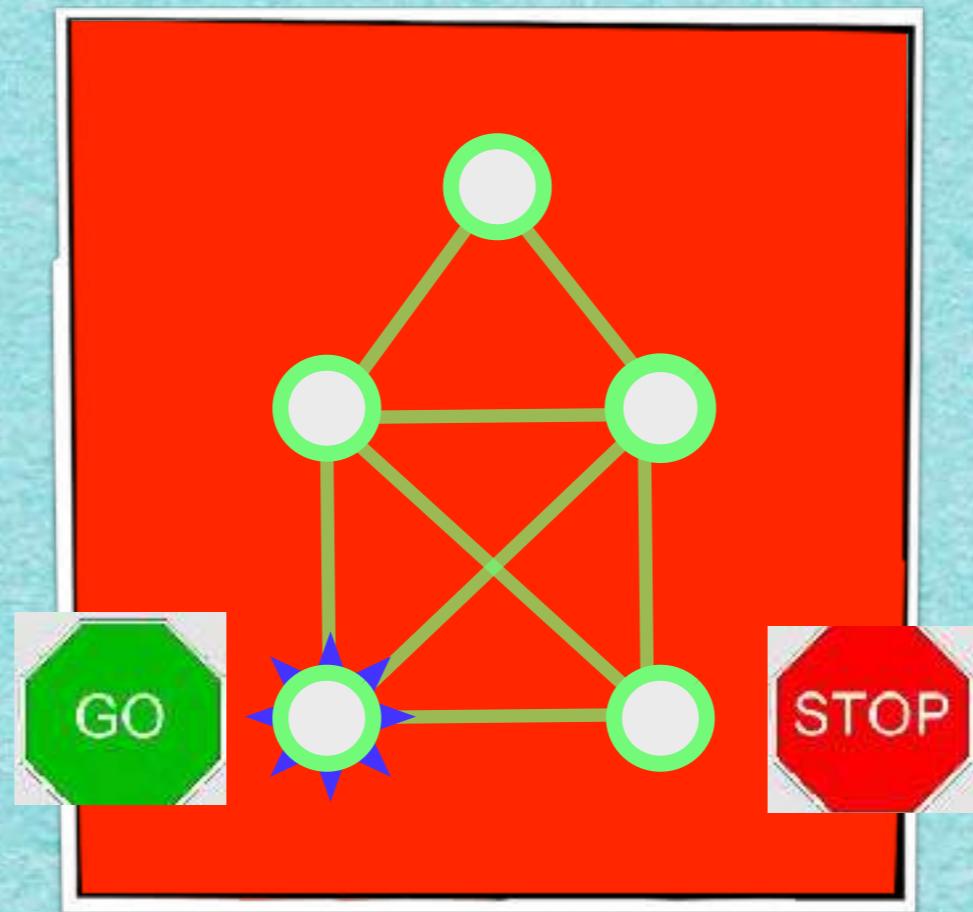
Hmmmm...

Das Haus des Nikolaus

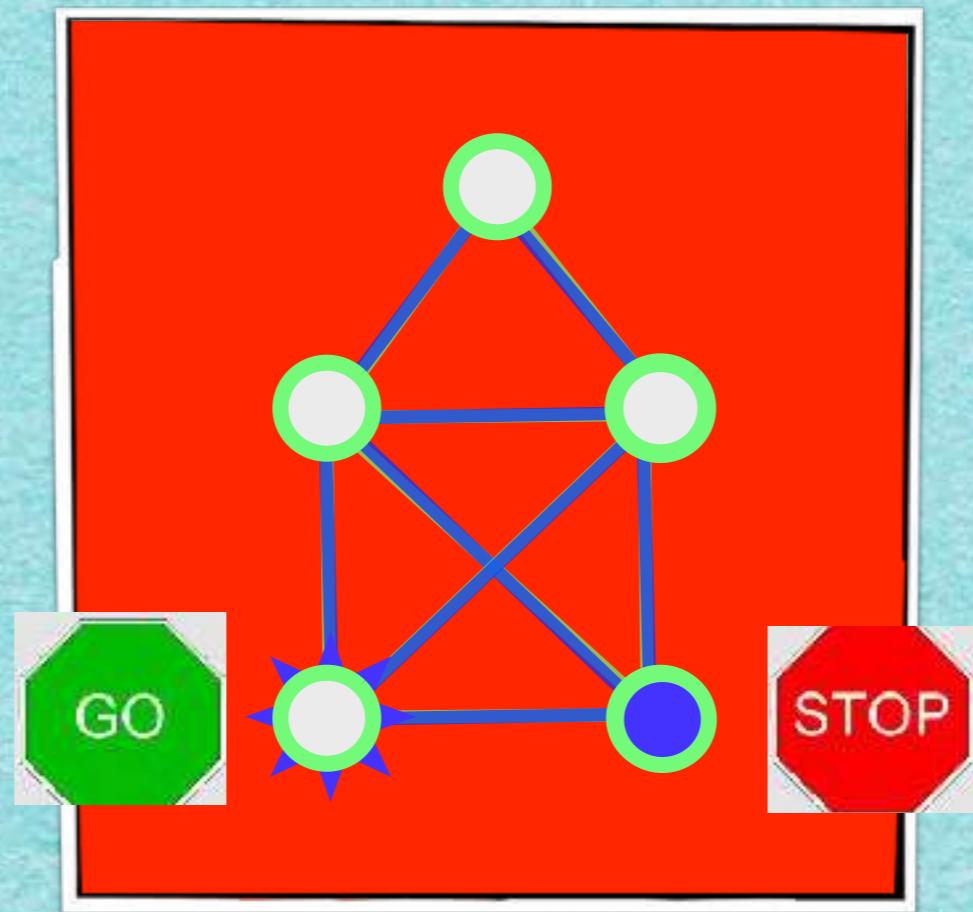


Ohhhhhhhh...

Das Haus des Nikolaus

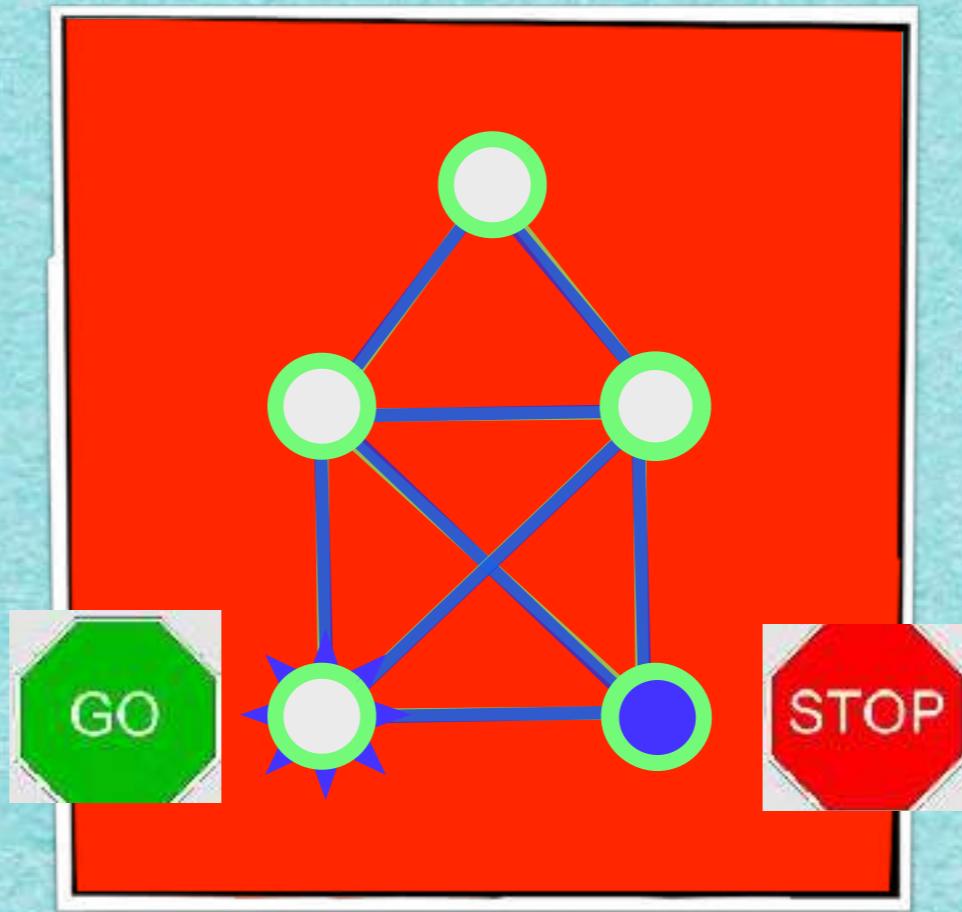


Das Haus des Nikolaus

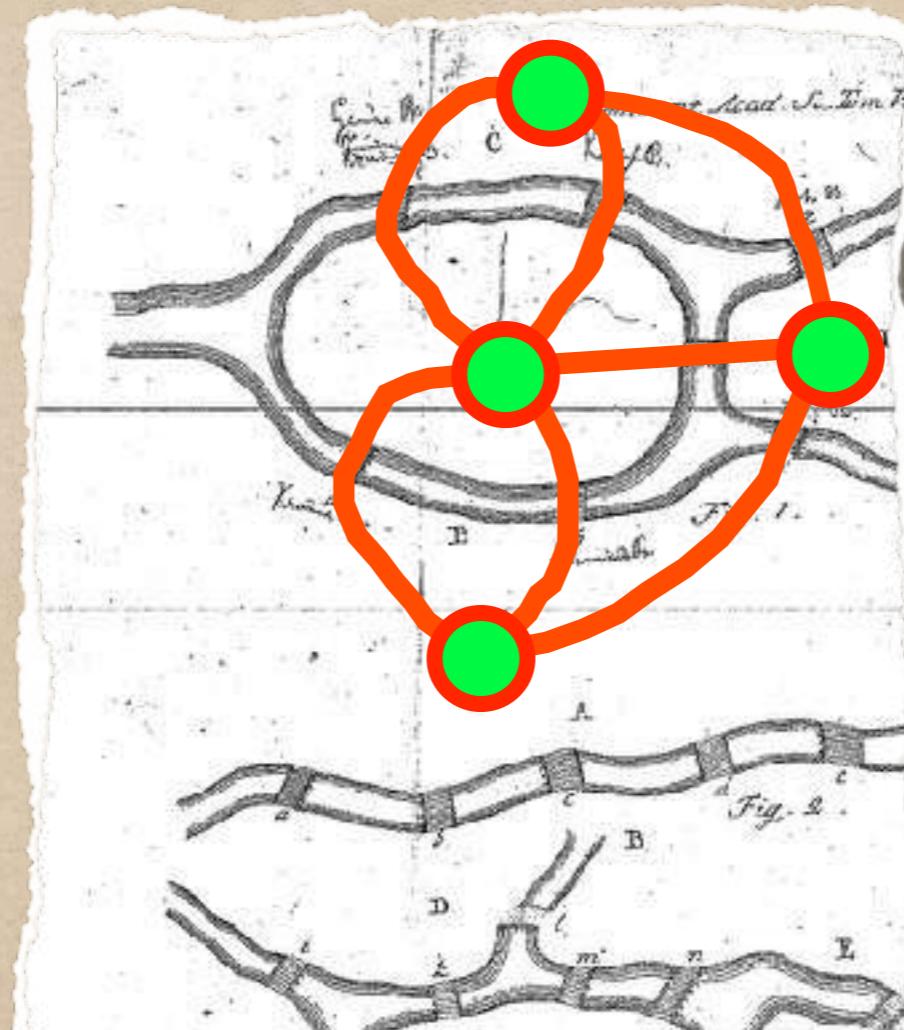


Ahhhhhhh!

Das Haus des Nikolaus



Wichtig: An einem der Knoten mit drei Kanten anfangen, weil man sonst irgendwann dort nicht mehr weg kommt!



- Alle Knoten sind ungerade?!
- Man müsste an allen anfangen oder aufhören!
- Das geht nicht an einem Stück!

Euler: (1) Das gilt für jede beliebige Instanz: Mit mehr als zwei ungeraden Knoten gibt es keinen solchen Weg.

(2) Man kann auch charakterisieren, unter welchen Bedingungen es einen Weg tatsächlich gibt.

108 SOLVITIO PROBLEMATIS
SOLVITIO PROBLEMATIS
AD
GEOMETRIAM SITVS
PERTINENTIS
AVCTORIS
Leob. Euler.

§. 1.

Praeterea illam Geometriae partem, quae circa quadrilateris versatur, et omni tempore summo studio illi excusa, alterius partis etiamnum admodum ignotus primus mentitionem fecit Leibnizius, quam Geometriam sive vocavit. Ista pars ab ipso in libro suo determinando, siueque proprietatibus eiusdem occupata esse statuitur; in quo negotio neque ad quantitates respondendam, neque calculo quantitatum verendum sit. Cuicunque autem problema ad hanc sive Geometriam pertinet, et quali methodo in us resoluenda uti oporteat, non fatus est definitum. Quomodo enim cum super problematis eiusdem mentio esset facta, quod quidem ad generarium perire videbatur, ut ita esset comparandum, ut neque determinacionem quantitatum requiriatur, neque laboriosum calculi quantitatum opus admittatur, id ad geometriam sive resolvendam haud dubitamus: praeterit quod in eius solutione sive sive in conclusionem veniat, calculus vero nullus propositus sit via. Methodum ergo meum quasi ad hanc generis proble-

- Alle Knoten sind ungerade?!
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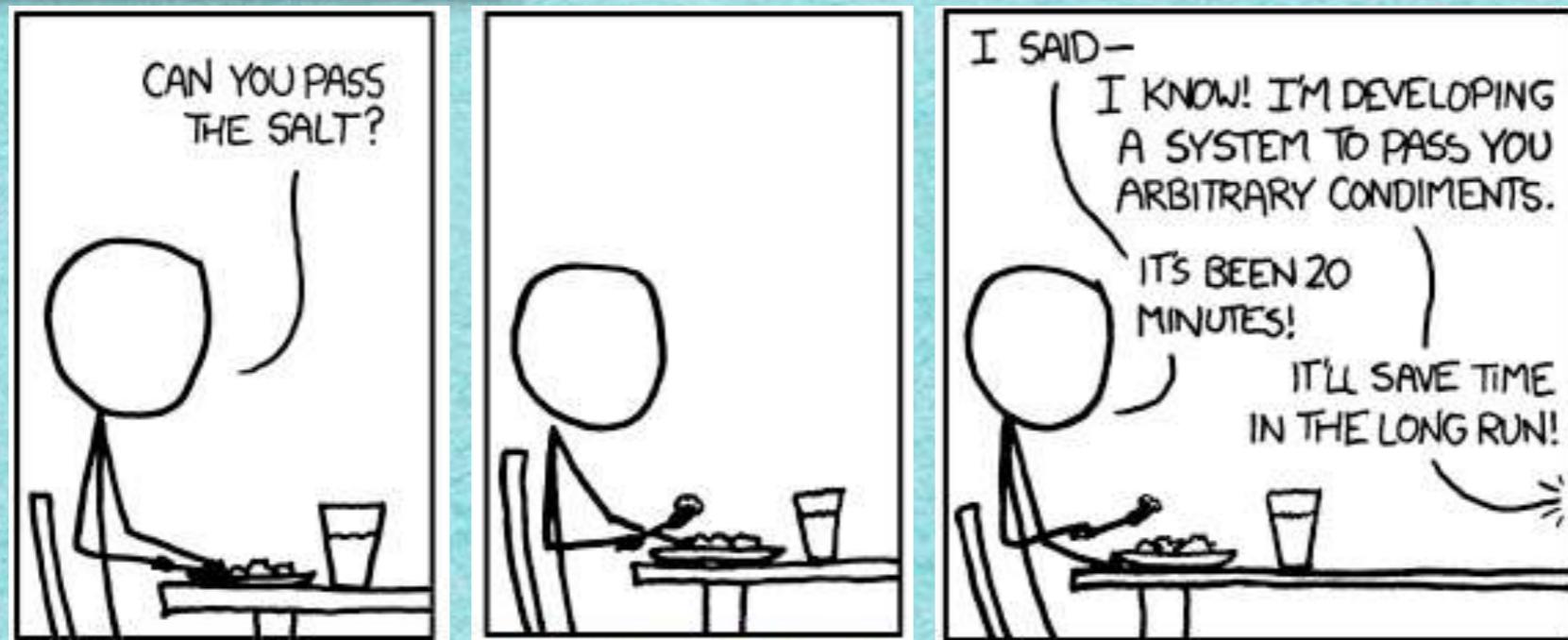
2.1 Historie

Euler hat:

- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet



2.1 Historie

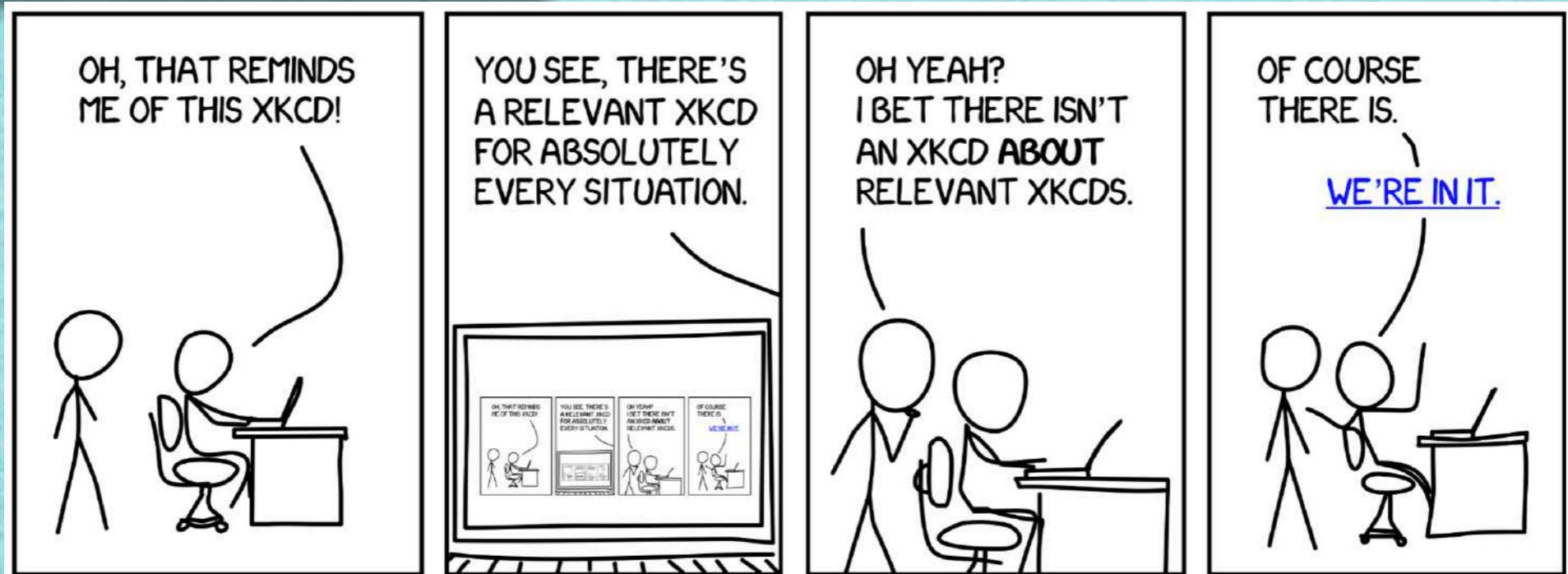


Euler hat:

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- ein Gebiet begründet



2.1 Historie



Euler hat:

- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet

2.1 Historie

Leonhard Euler:

1707 Geboren in Basel
1720 Studienbeginn in Basel
1723 Magister
1727 Berufung an Petersburger Akademie
1731 Professur für Physik

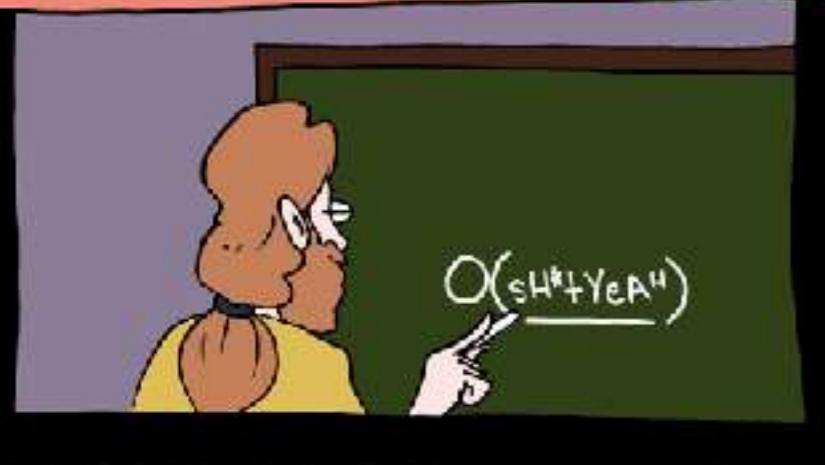


Erik Demaine:

1981 Geboren in Halifax
1993 Studienbeginn in Halifax
1995 Bachelor
1996 Master
2001 Ph.D.
2001 Assistenzprofessor am MIT
2005 Full Professor am MIT



DR. DEMAINE CREATED AN ALGORITHM THAT SOLVED ALL MATHEMATICAL THEOREMS.



SOON AFTER, ALL PHYSICS QUESTIONS WERE ANSWERED

INSIDE PROTONS ARE QUARKS, INSIDE QUARKS ARE STRINGS, INSIDE STRINGS IS GOD TELLING YOU TO FUCK OFF.



THEN ENGINEERING, CHEMISTRY, BIOLOGY, NEUROSCIENCE, PSYCHIATRY...

P ≠ NP, COLD FUSION WILL NEVER WORK, THE RED QUEEN HYPOTHESIS IS RIGHT, CONSCIOUSNESS IS AN ILLUSION, AND YOUR MOTHER NEVER LOVED YOU.



HAVING COMPLETED SCIENCE, HE MOVED ON TO PHILOSOPHICAL AND LITERARY QUESTIONS.

IS ALL THAT WE SEE OR SEEM BUT A DREAM, WITHIN A DREAM?



THEN UNINTERESTING RHETORICAL QUESTIONS

ARE WE GONNA GIVE 110% TONIGHT?!

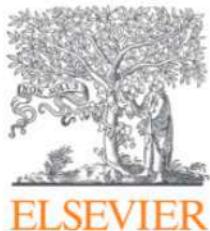
WHO'S A CUTE KITTY? SCRUFFLES IS.



FINALLY, ALL THAT WAS LEFT WAS SENSELESS HALF-CONCEIVED QUESTIONS FROM STONED PHILOSOPHY UNDERGRADS.

DO THINGS, LIKE, MAN, YOU KNOW, WOAH?





New geometric algorithms for fully connected staged self-assembly ☆

Erik D. Demaine ^a✉ Sándor P. Fekete ^b✉, Christian Scheffer ^b✉, Arne Schmidt ^b✉

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<https://doi.org/10.1016/j.tcs.2016.11.020>

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Abstract

We consider *staged self-assembly systems*, in which shapes can be added to bins in several stages. Within these systems, shapes connect to each other, depending on the *glue types* of their faces. Previous work by Demaine et al. showed that a relatively small number of stages suffices to produce arbitrary shapes in this model. However, their constructions were only based on a spanning tree of the graph, so they did not produce full connectivity of the underlying shape. In this paper, we consider the case of shapes with holes; self-assembly of fully connected shapes with a polylogarithmic number of stages was left as a

Volume 93 February 2021 101700



Folding polyominoes with holes into a cube ☆

Oswin Aichholzer ^a✉, Hugo A. Akitaya ^b✉, Kenneth C. Cheung ^c✉, Erik D. Demaine ^d✉, Martin L. Demaine ^d✉, Sándor P. Fekete ^e✉, Linda Kleist ^e✉, Irina Kostitsyna ^f✉, Maarten Löffler ^g✉, Zuzana Masárová ^h✉, Klara Mundilová ^d✉, Christiane Schmidt ⁱ✉

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Abstract

Jetzt wird's genauer!

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