

Kapitel 2: Graphen

*Algorithmen und Datenstrukturen
WS 2016/17*

Prof. Dr. Sándor Fekete

Konzentration



2.1 Historie: Ein Mathematiker geht spazieren



Carl Friedrich Gauß (1777-1855)

2.1 Historie: Ein Mathematiker geht spazieren

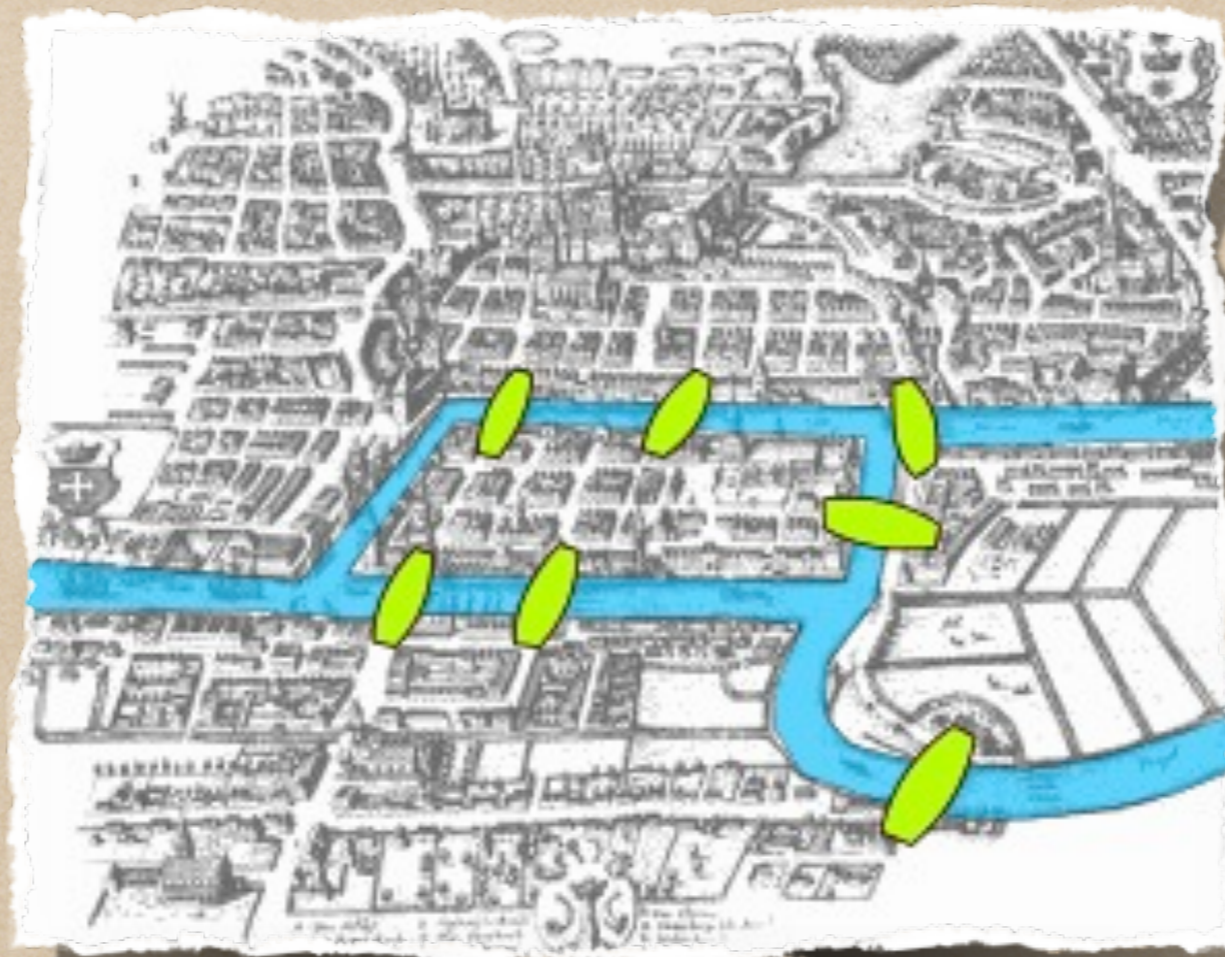


Leonhard Euler (1707-1783)

2.1 Historie: Ein Mathematiker geht spazieren

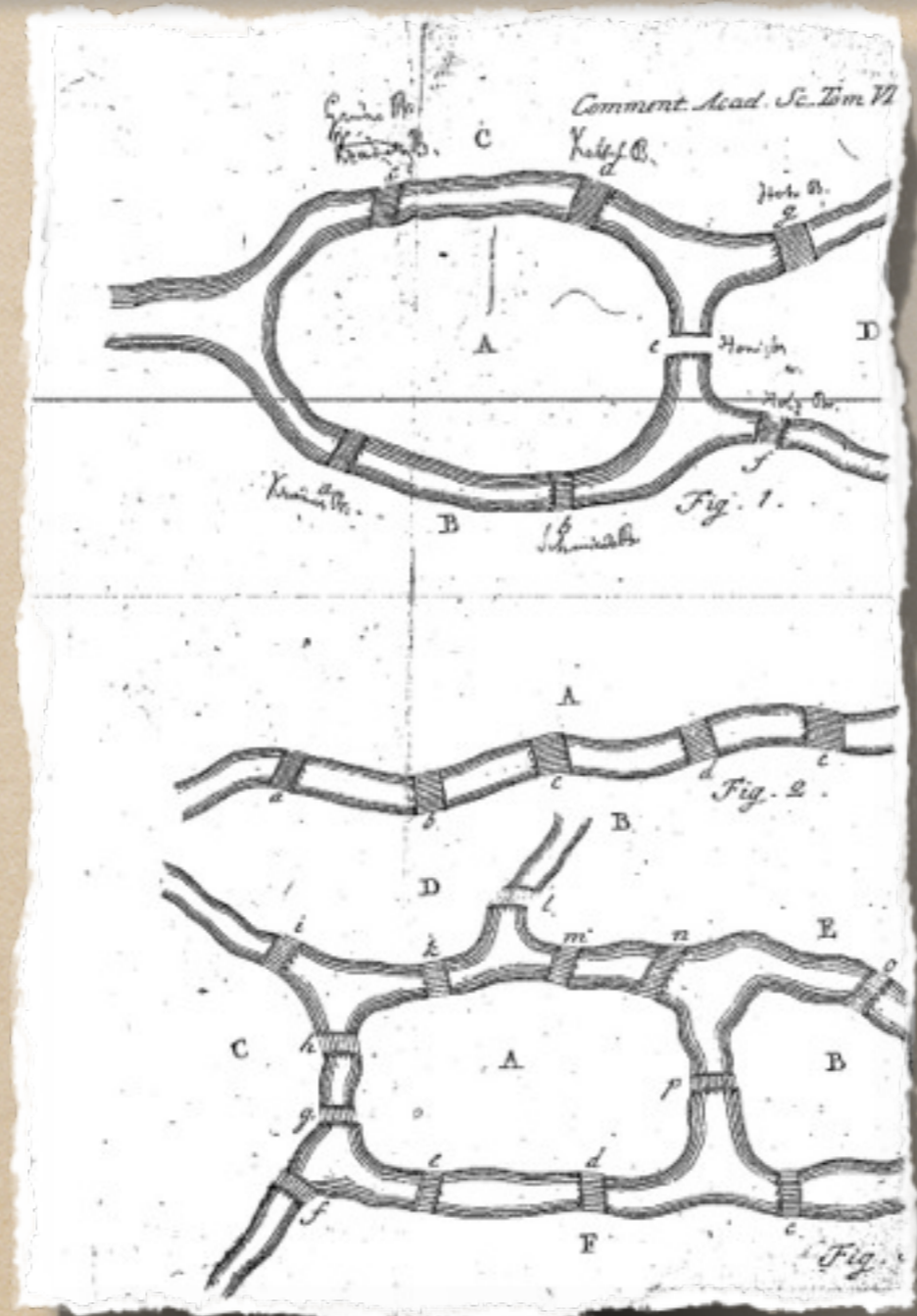


2.1 Historie: Ein Mathematiker geht spazieren



Königsberg und seine 7 Brücken

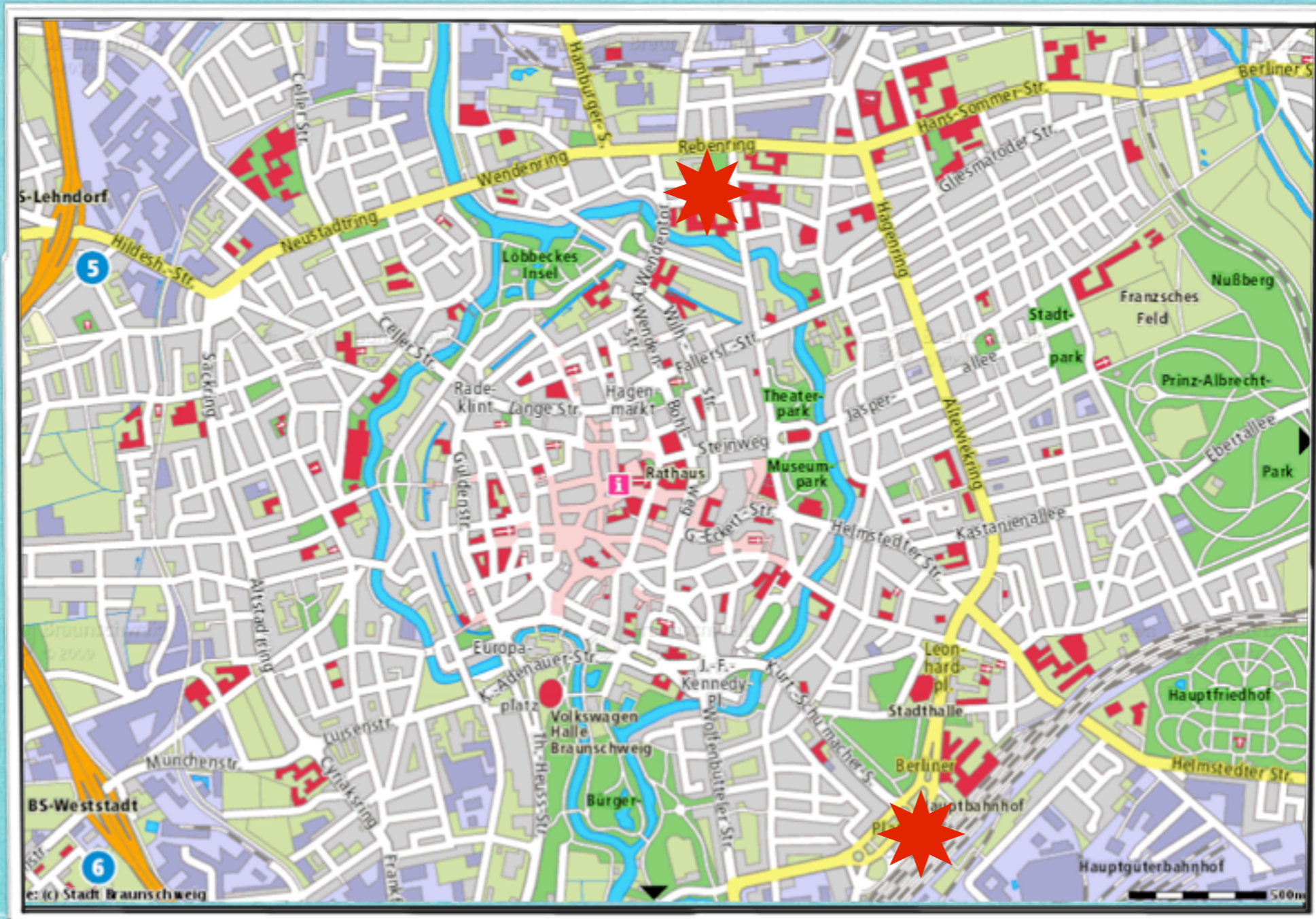
2.1 Historie: Ein Mathematiker geht spazieren



Wie kommt man zur TU?



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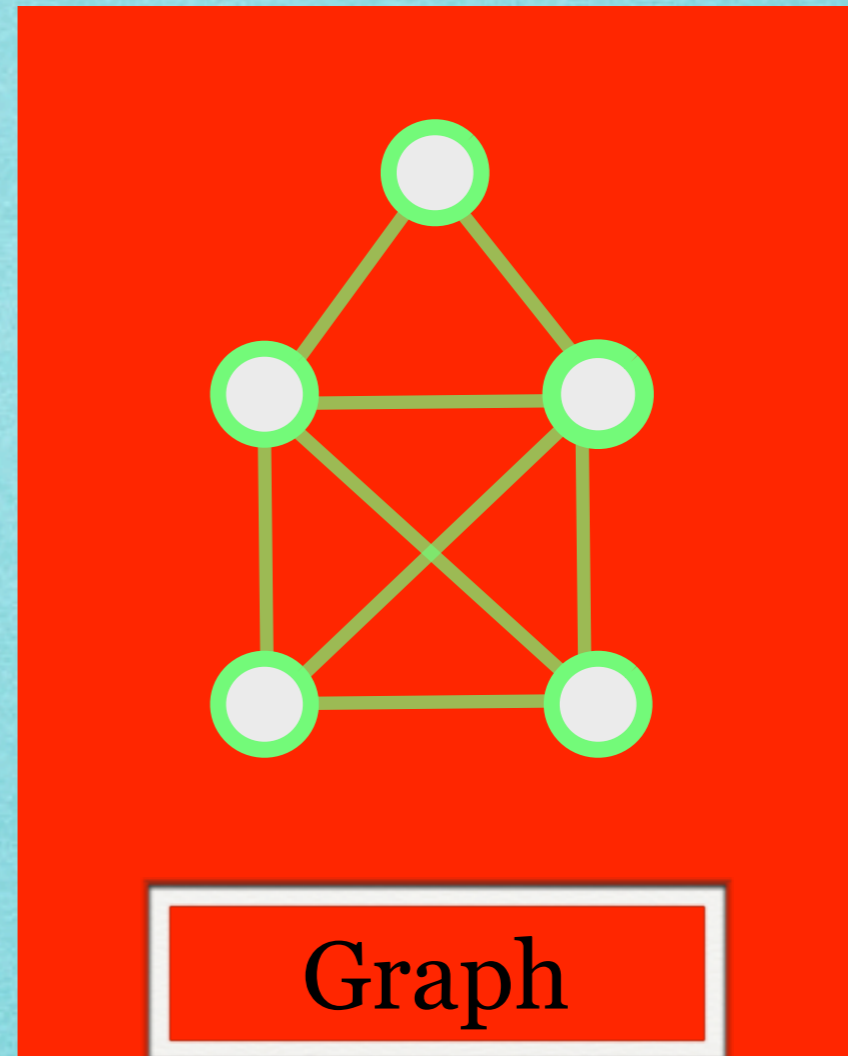
Wie kommt man zur TU?



Gestatten, Graph!



Graf



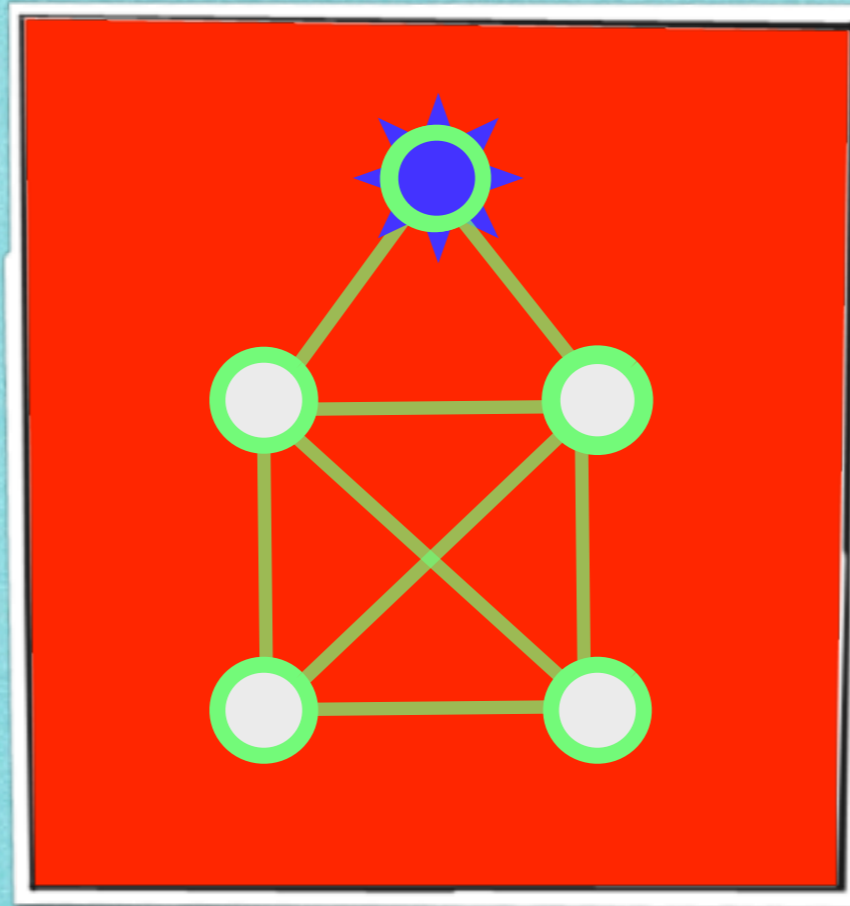
Graph

Graph: Ein Gebilde aus Knoten (Haltestellen) und Kanten (Verbindungen)

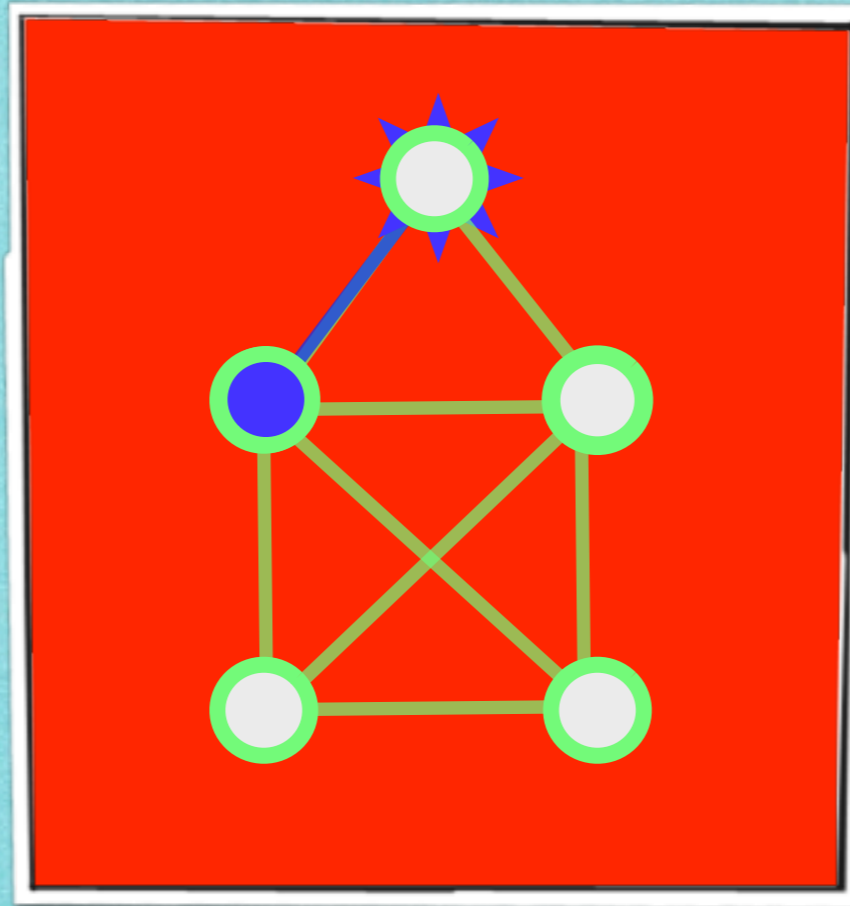
Das Haus des Nikolaus



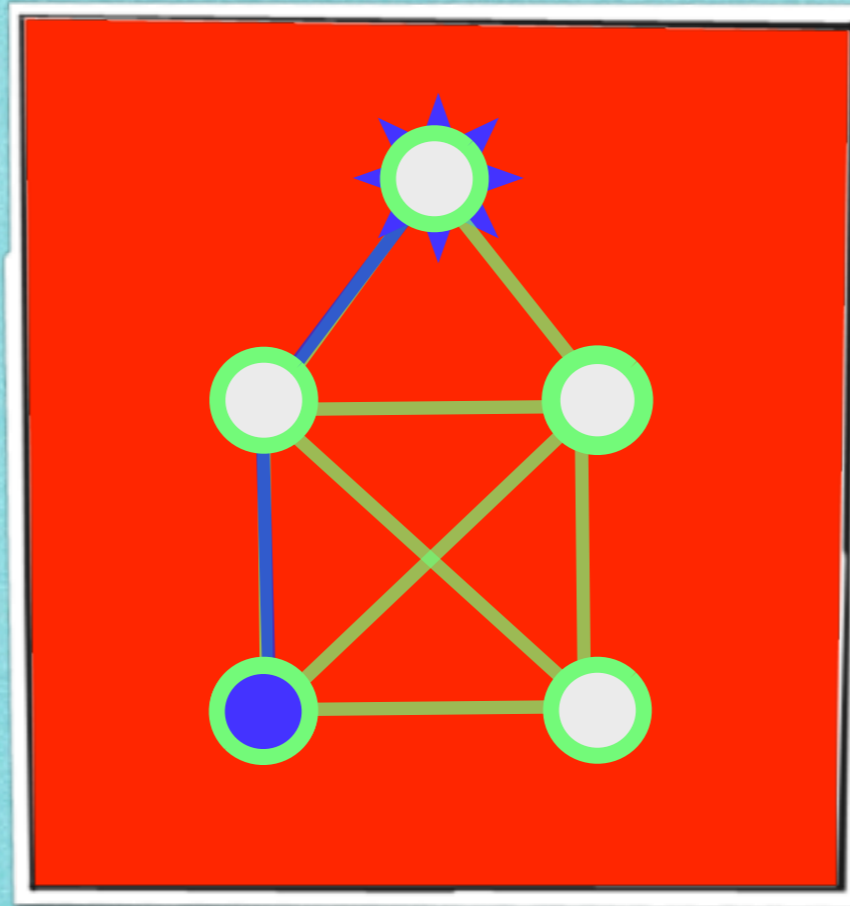
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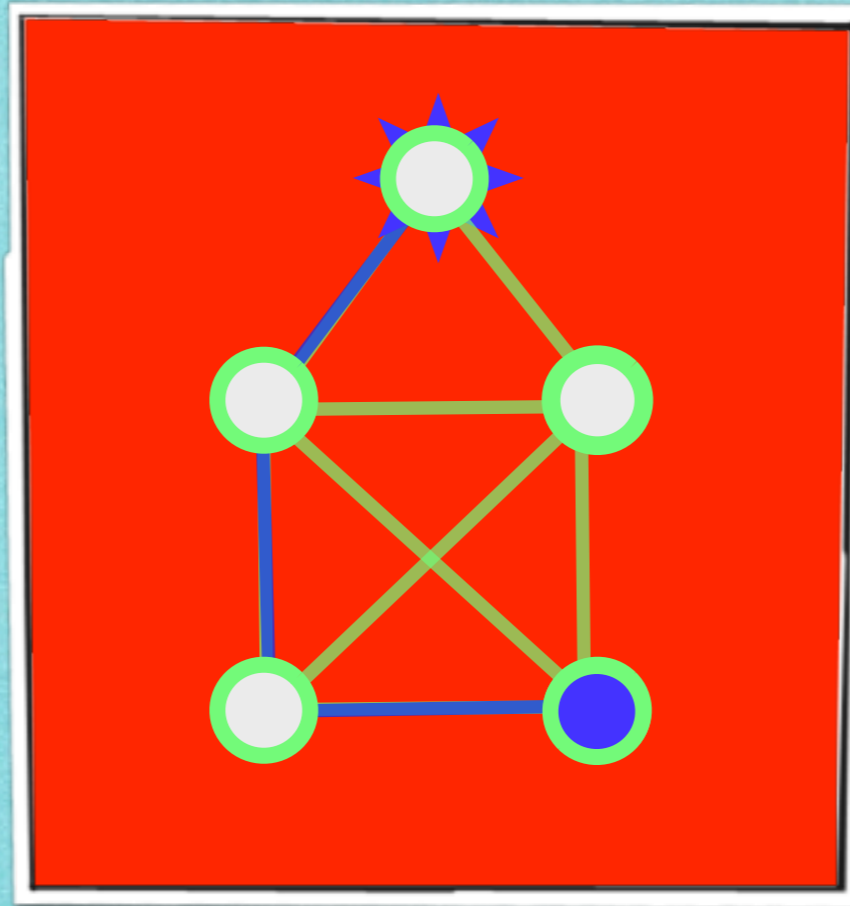
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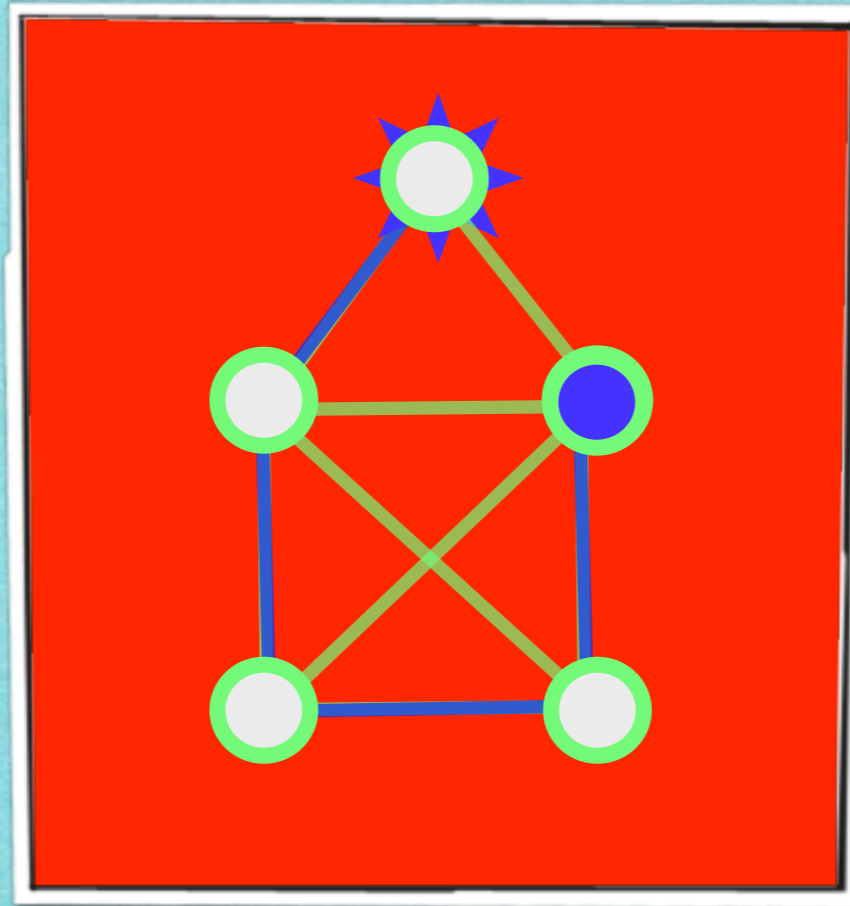
Das Haus des Nikolaus



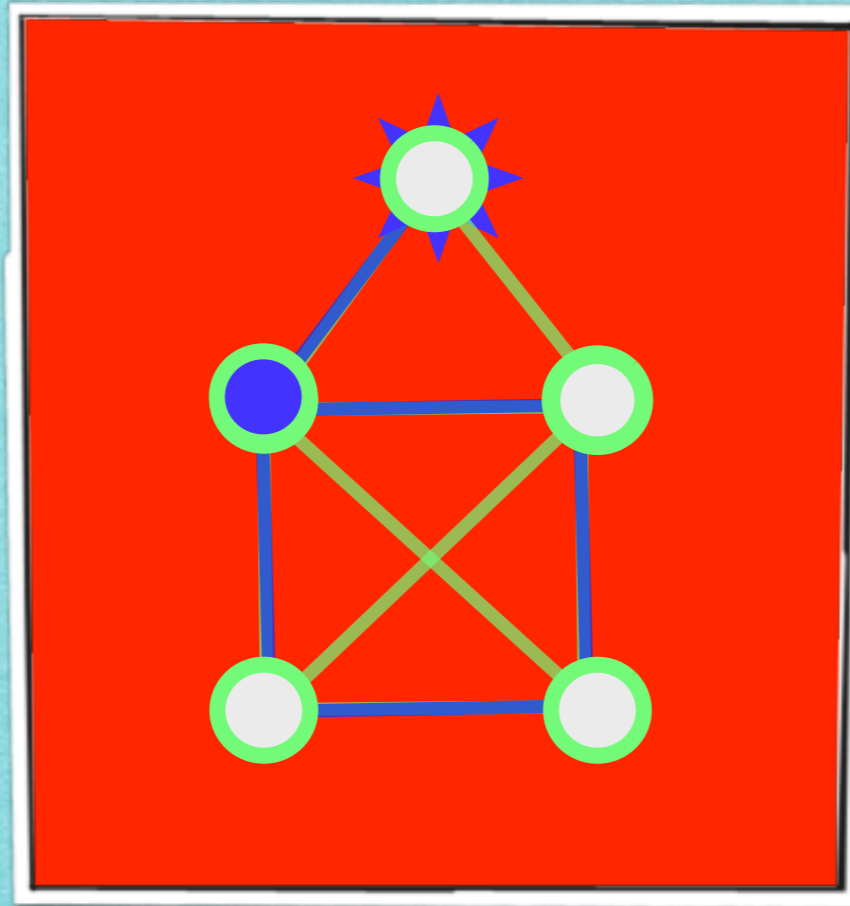
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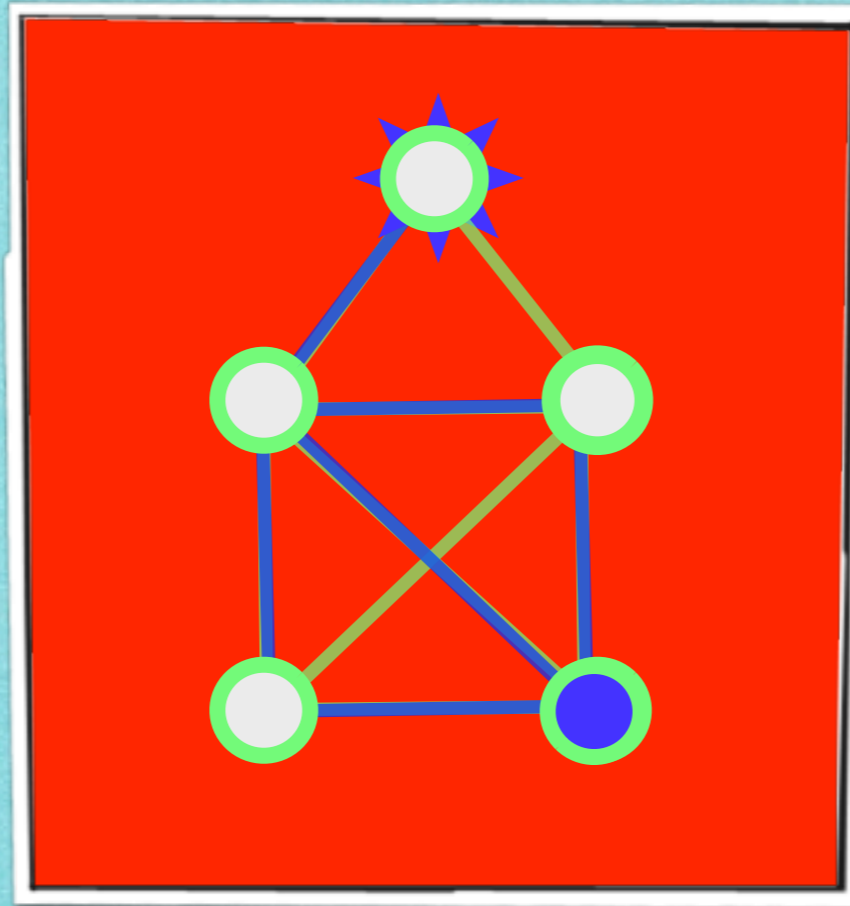
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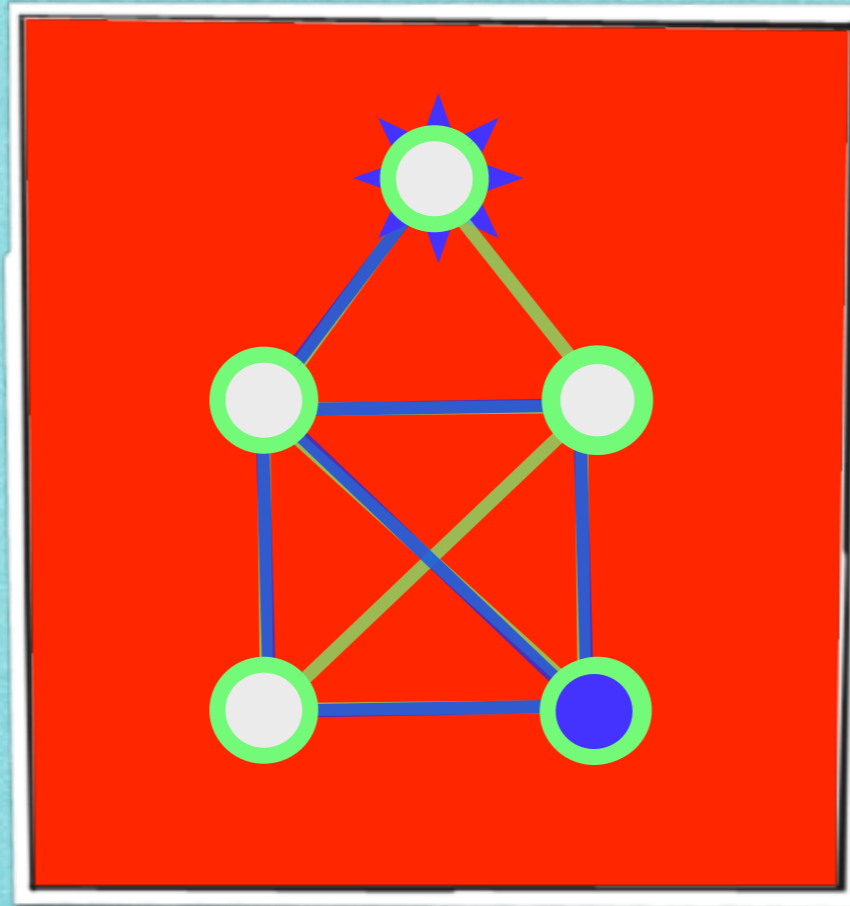
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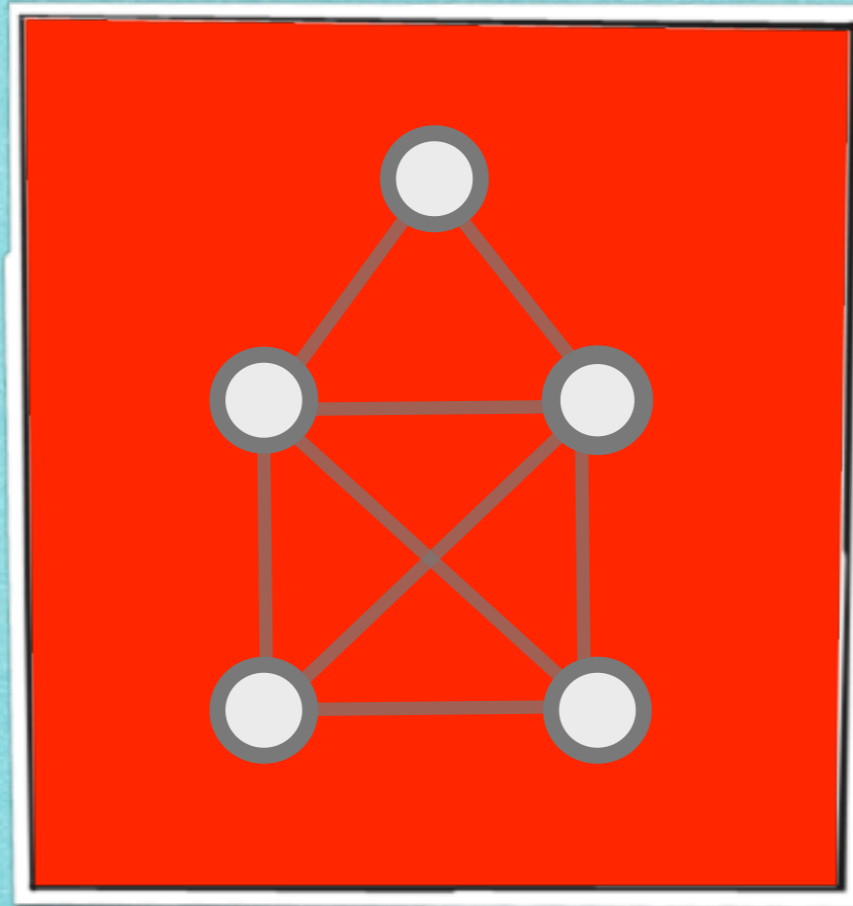


Das Haus des Nikolaus

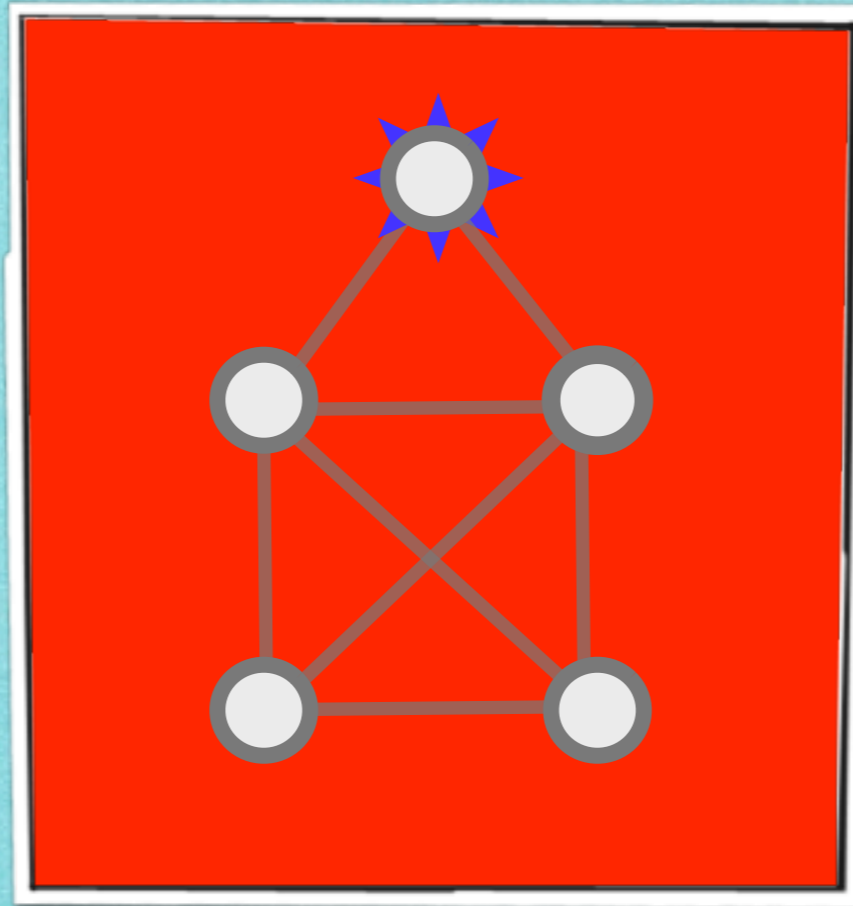


Klappt so nicht...

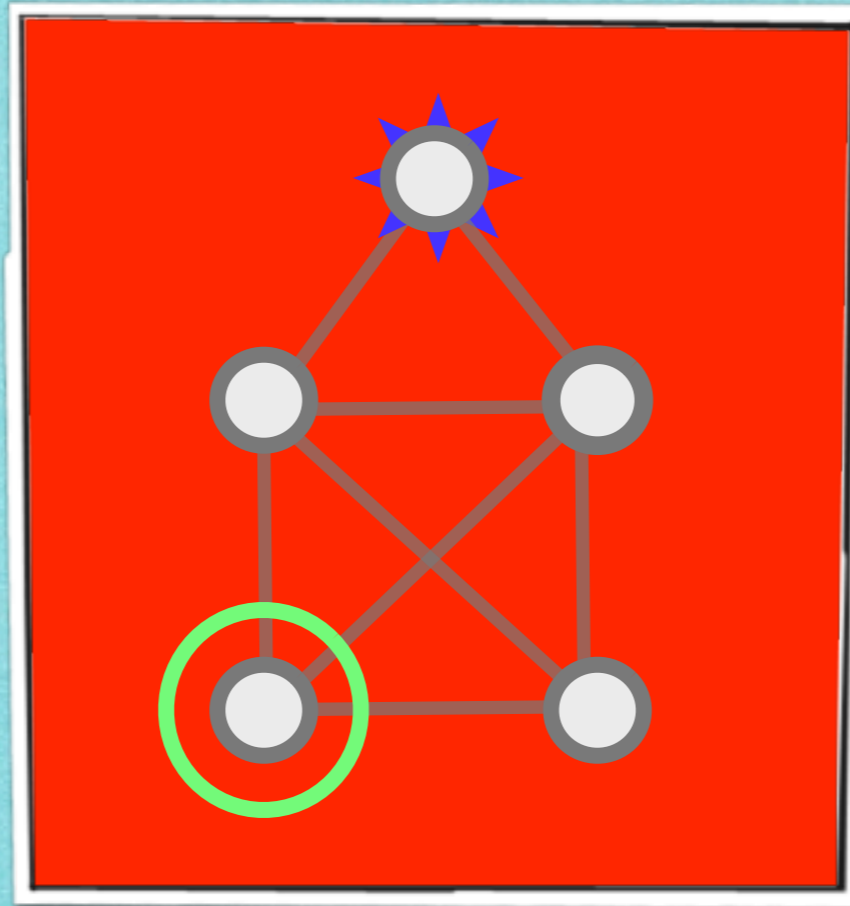
Das Haus des Nikolaus



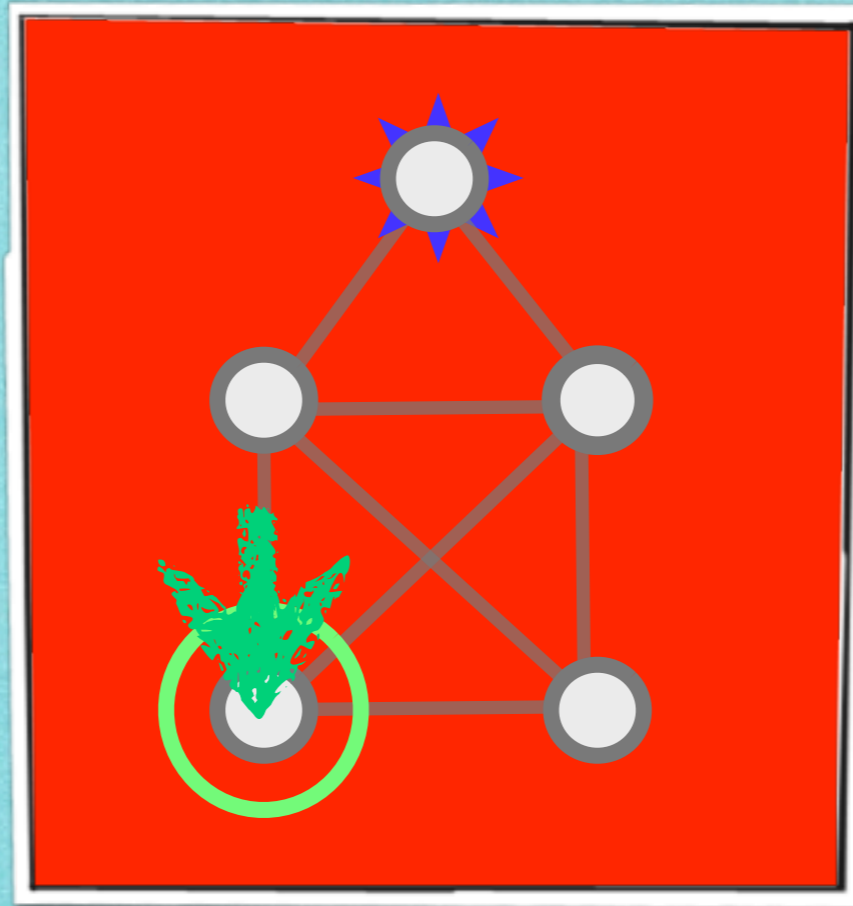
Das Haus des Nikolaus



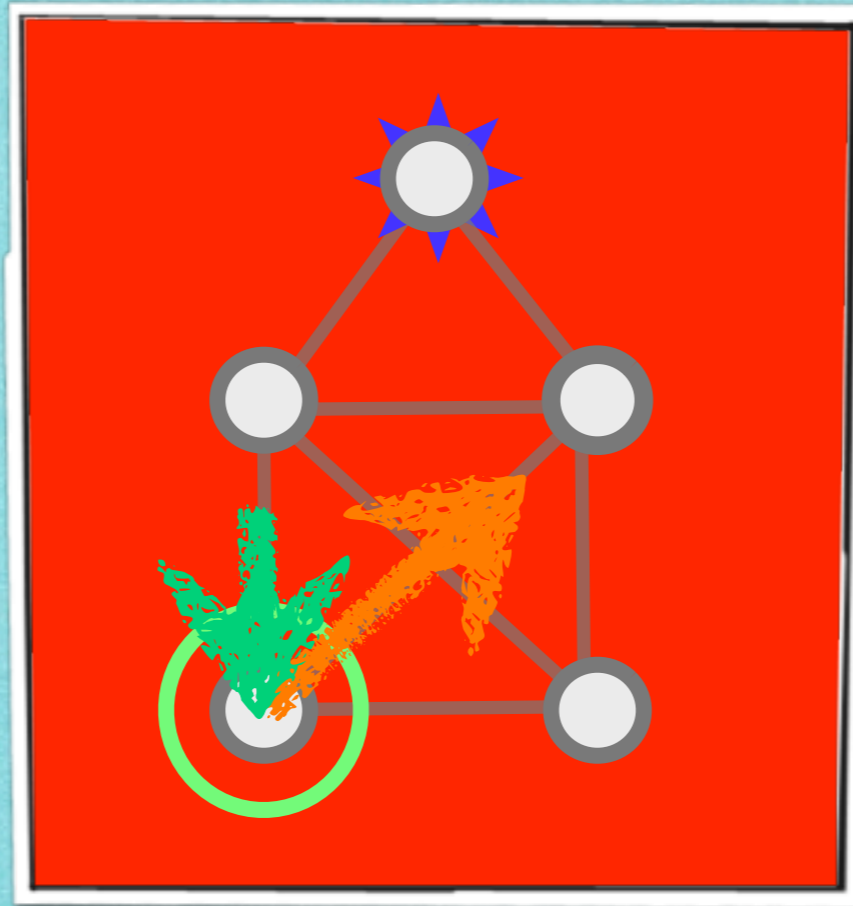
Das Haus des Nikolaus



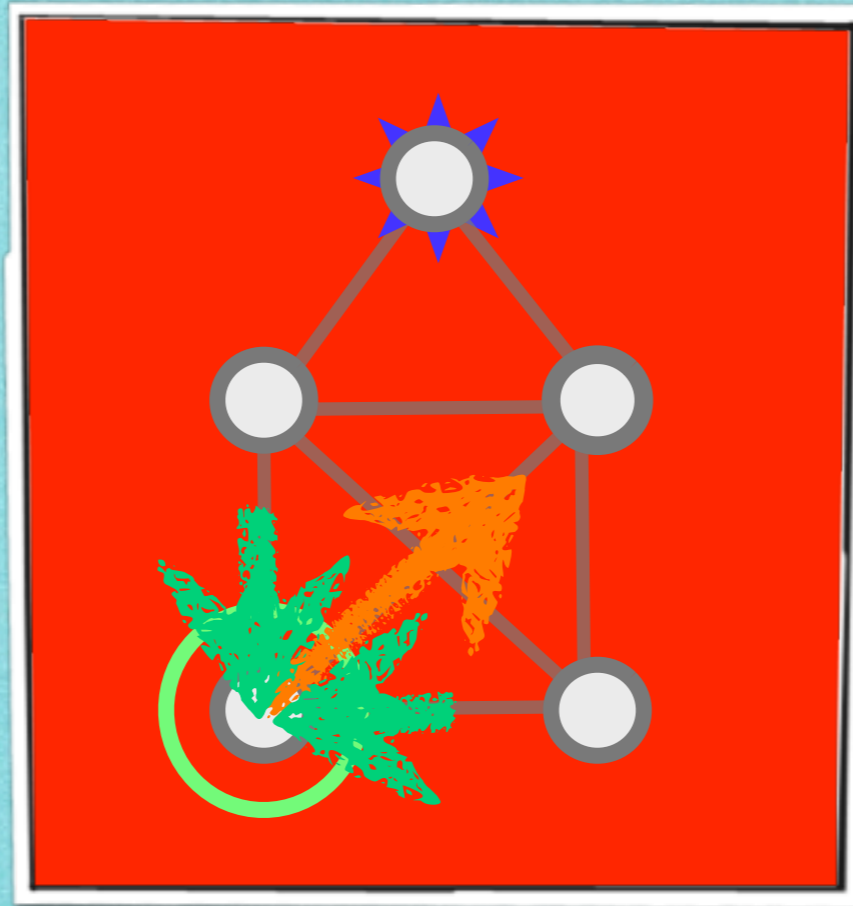
Das Haus des Nikolaus



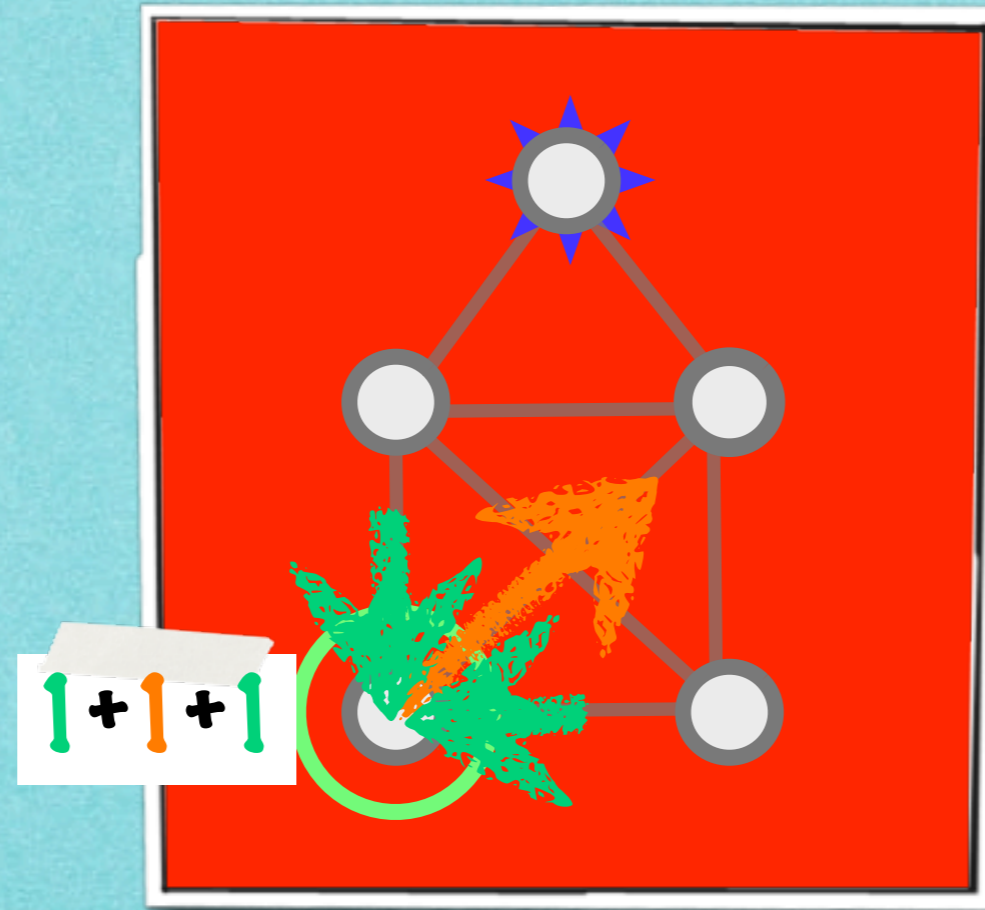
Das Haus des Nikolaus



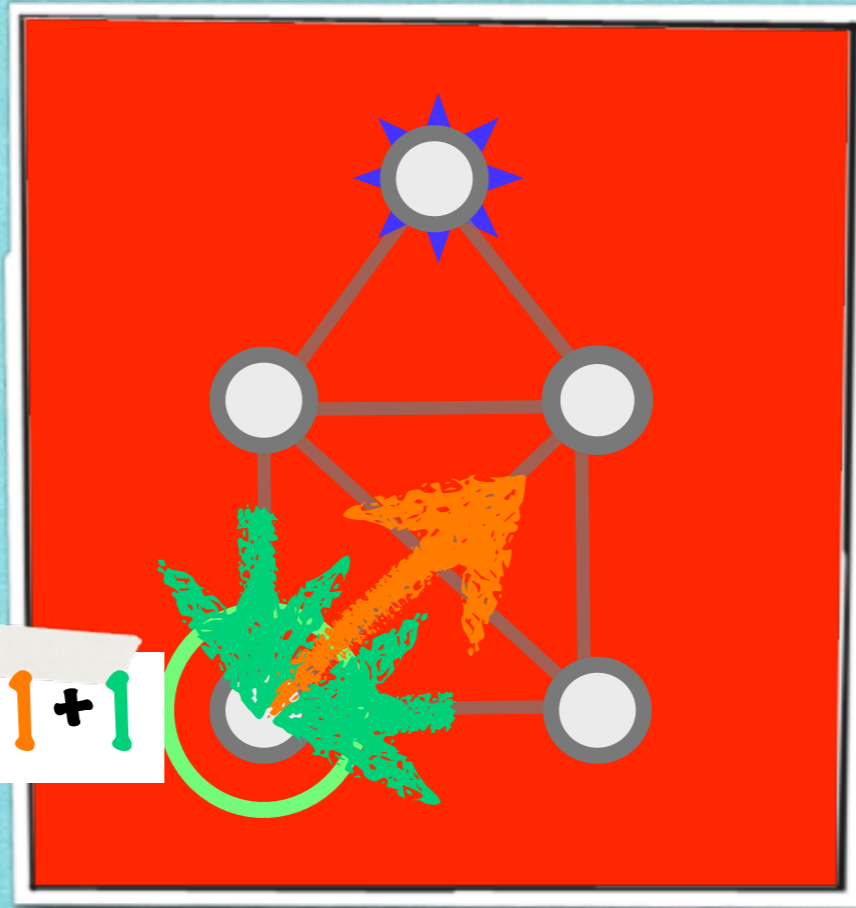
Das Haus des Nikolaus



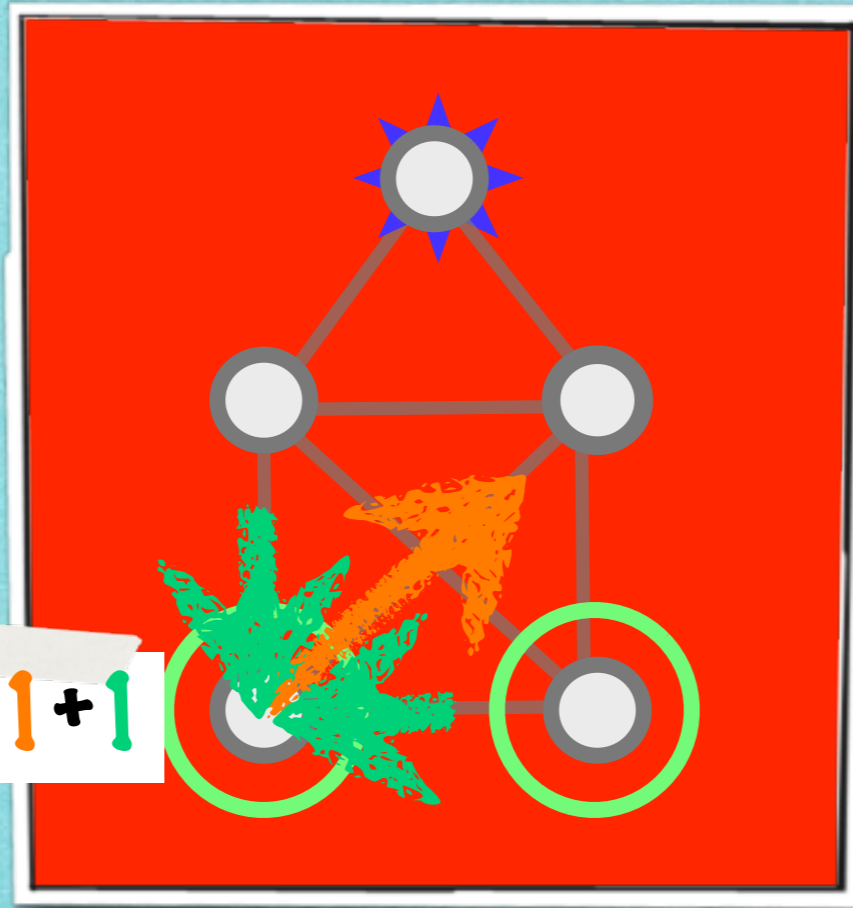
Das Haus des Nikolaus



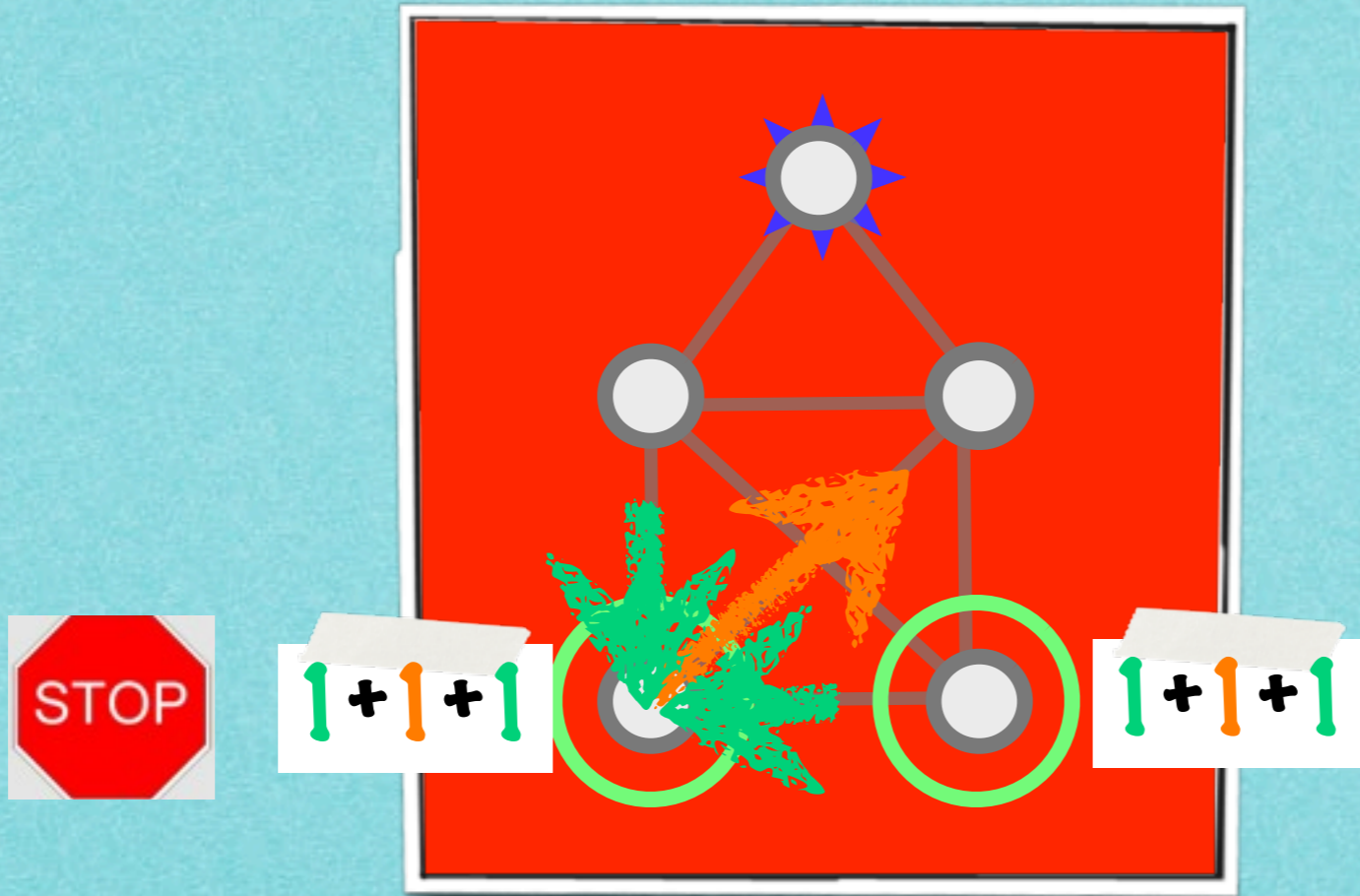
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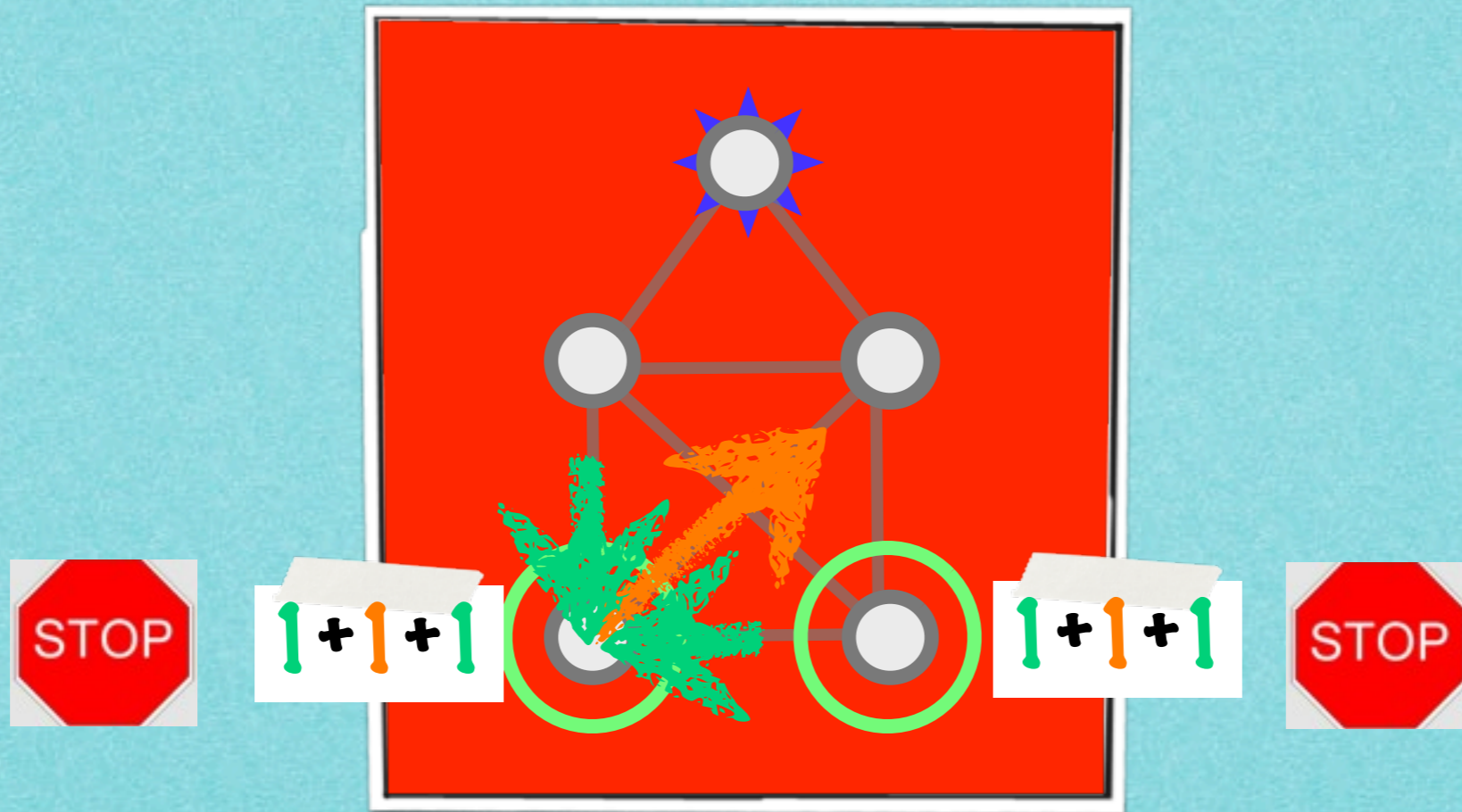
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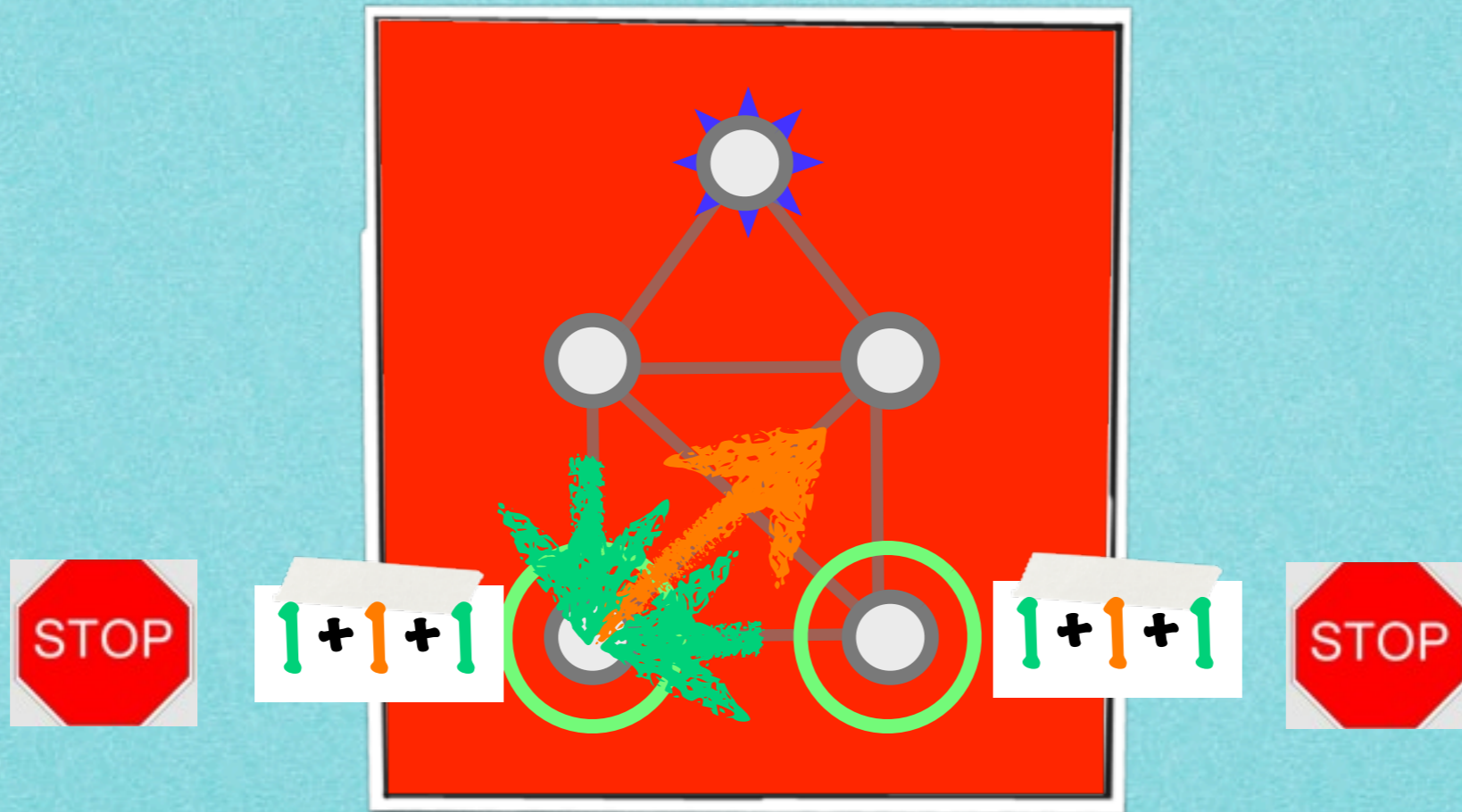
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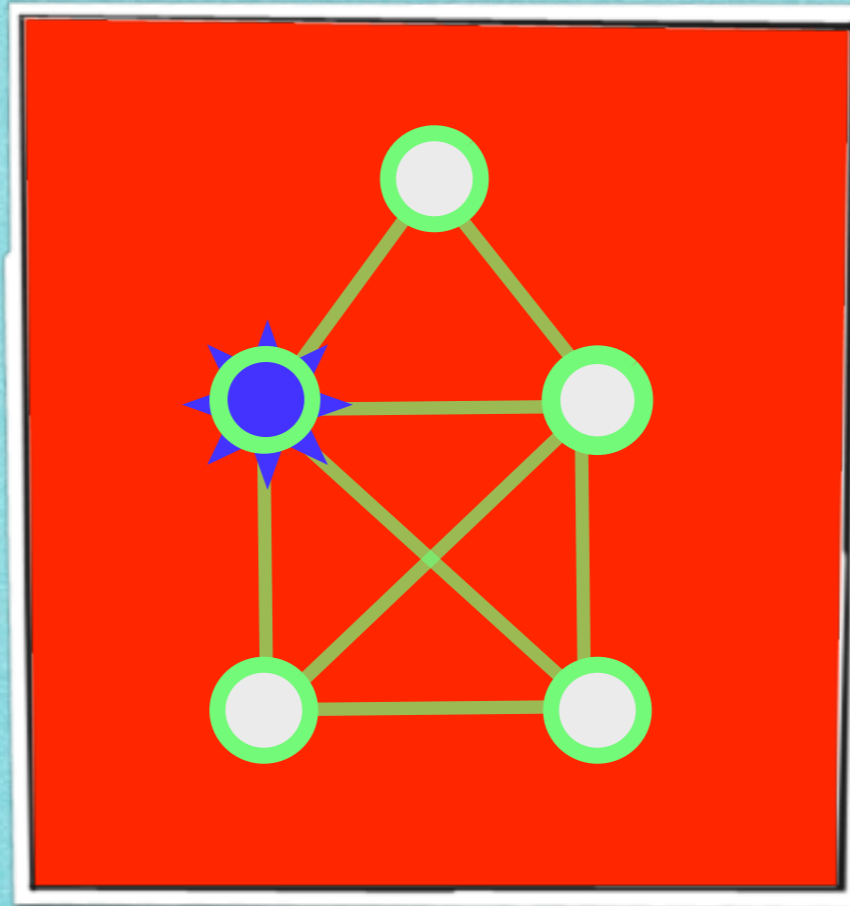


Das Haus des Nikolaus

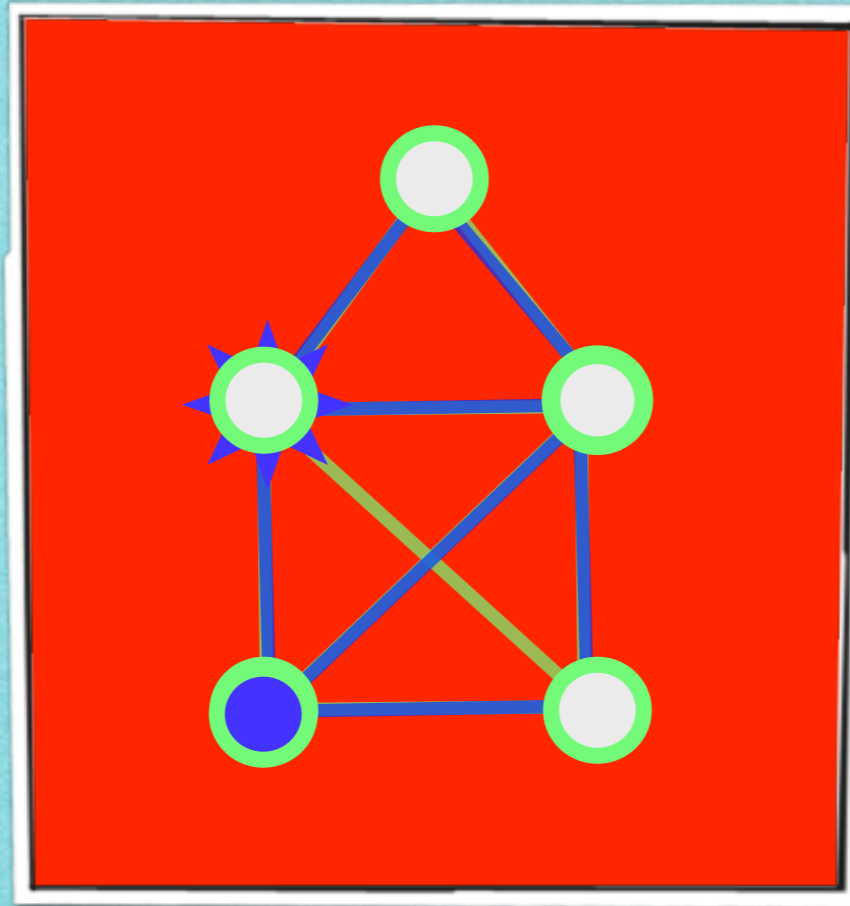


Нмммм...

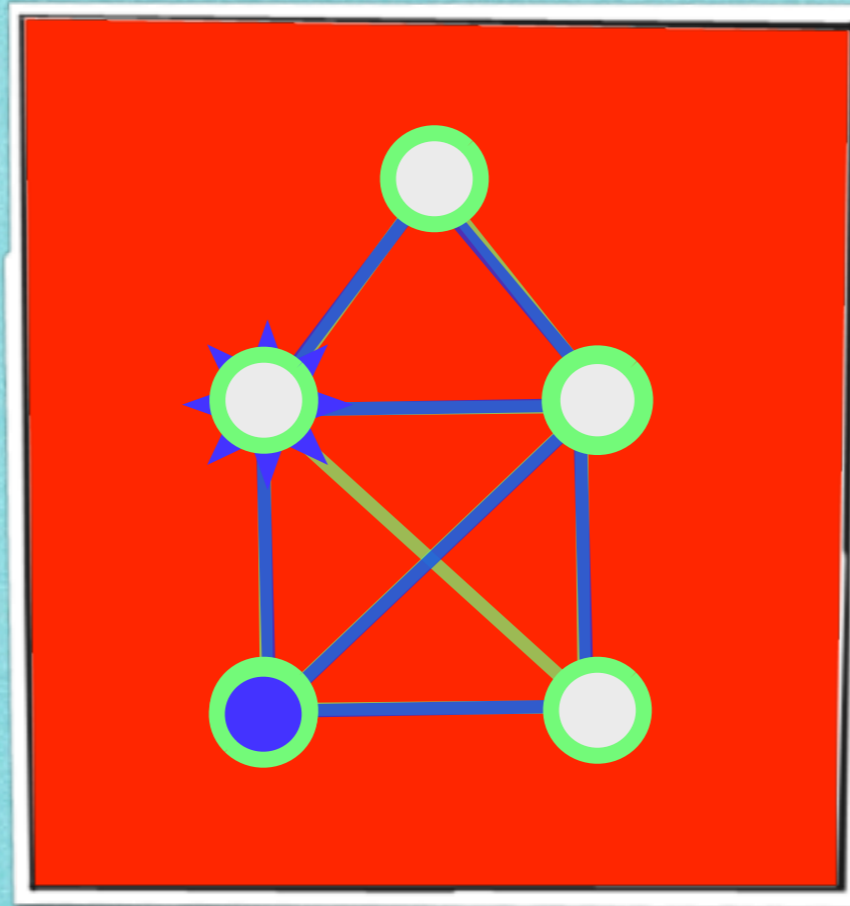
Das Haus des Nikolaus



Das Haus des Nikolaus

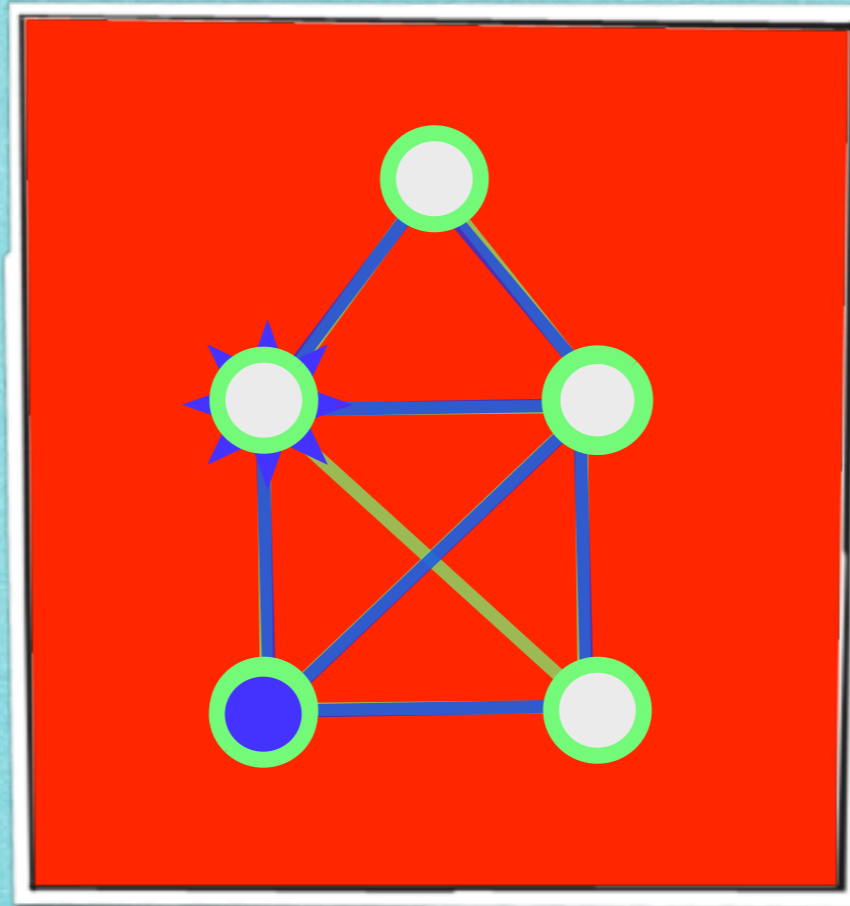


Das Haus des Nikolaus



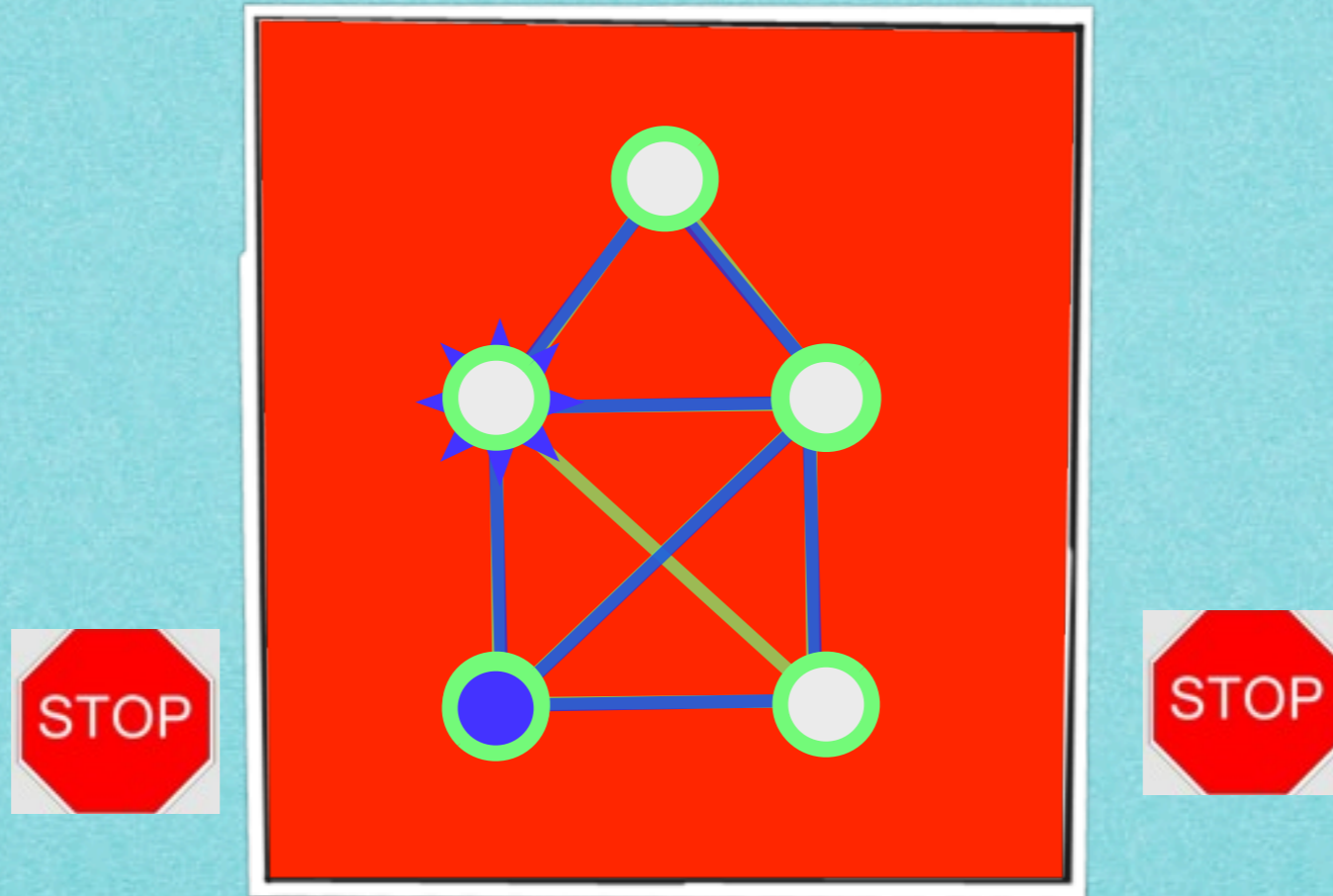
Ohhhhhhhh...

Das Haus des Nikolaus



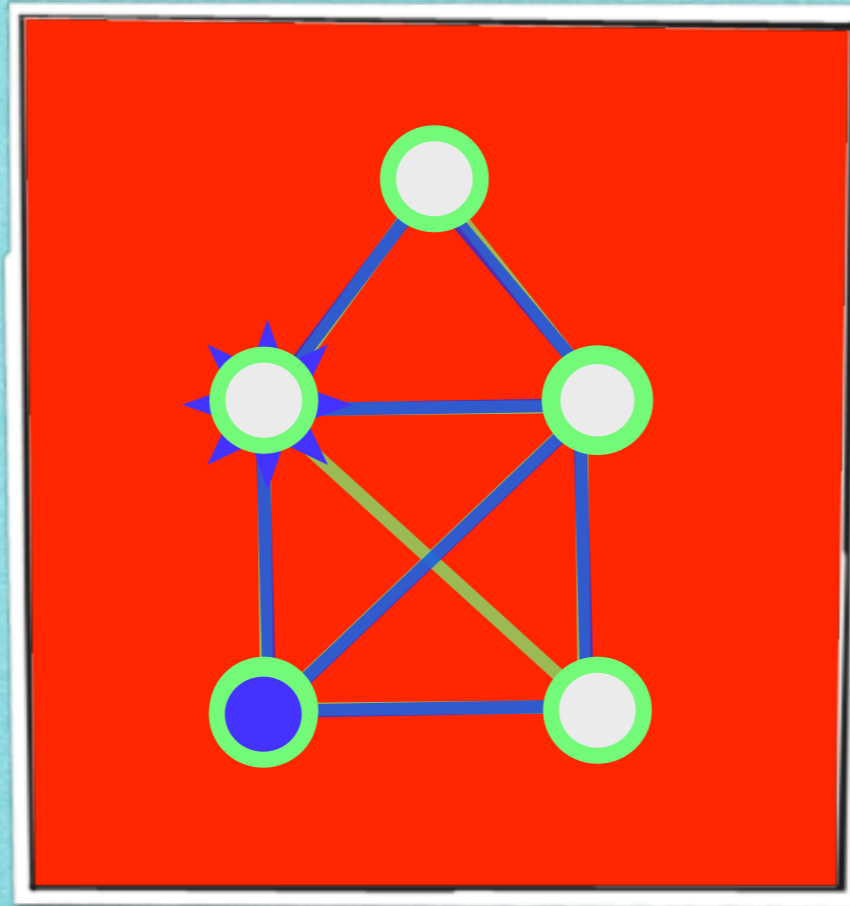
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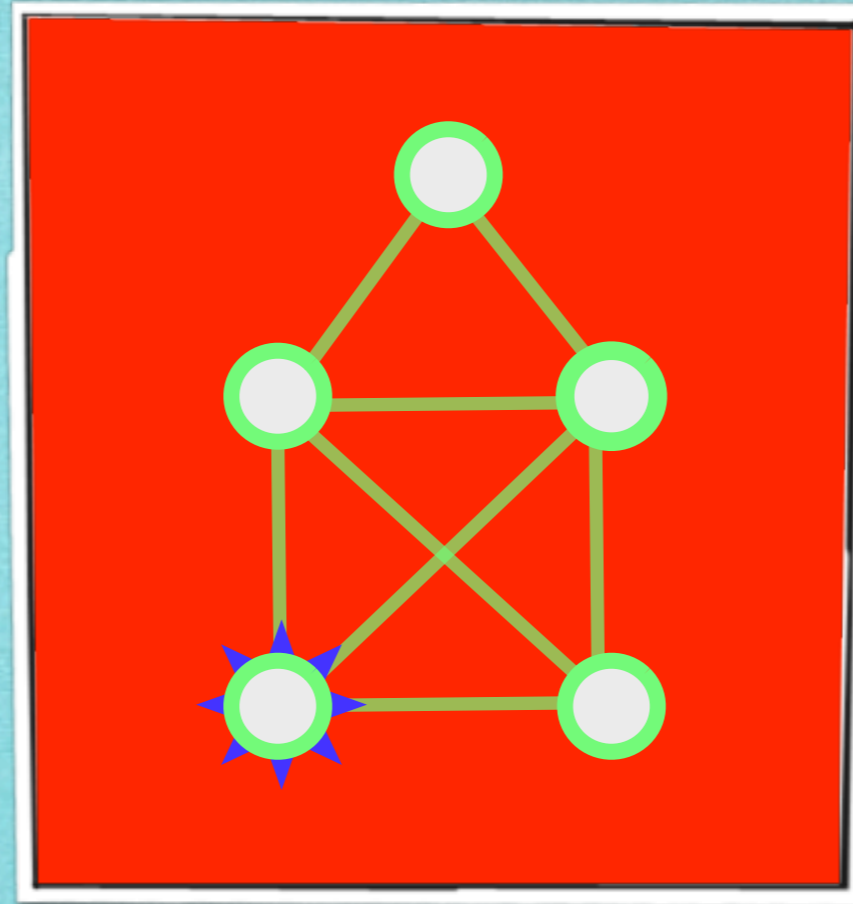


Ohhhhhhhh...

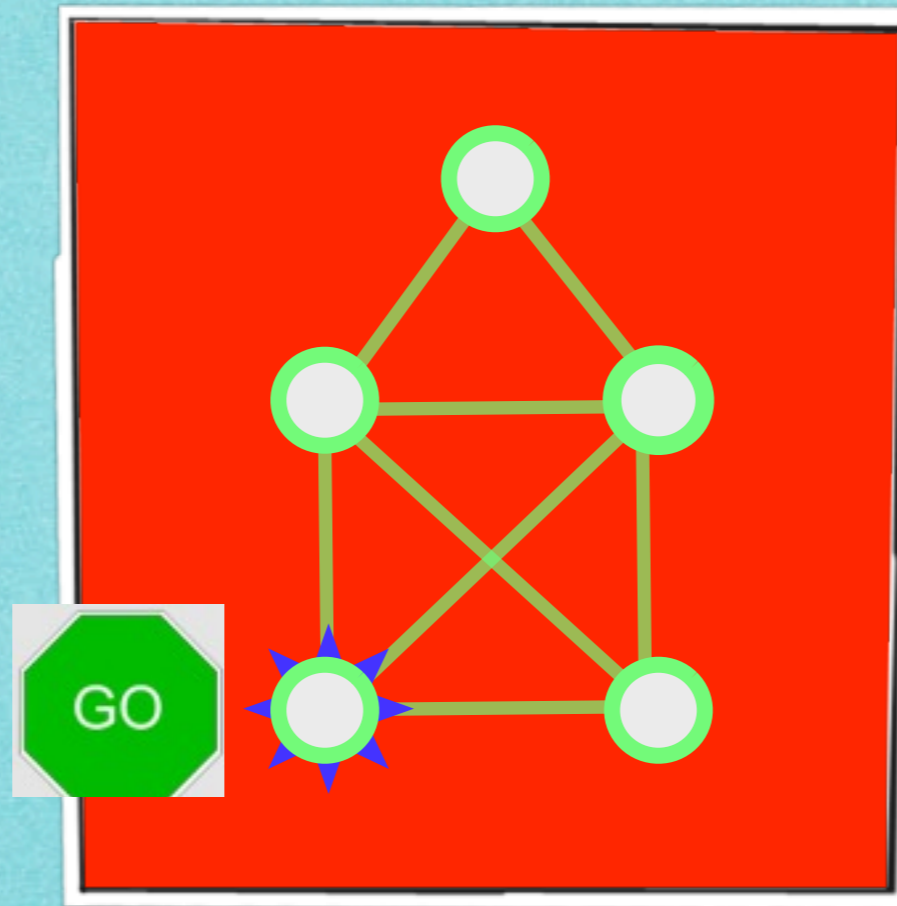
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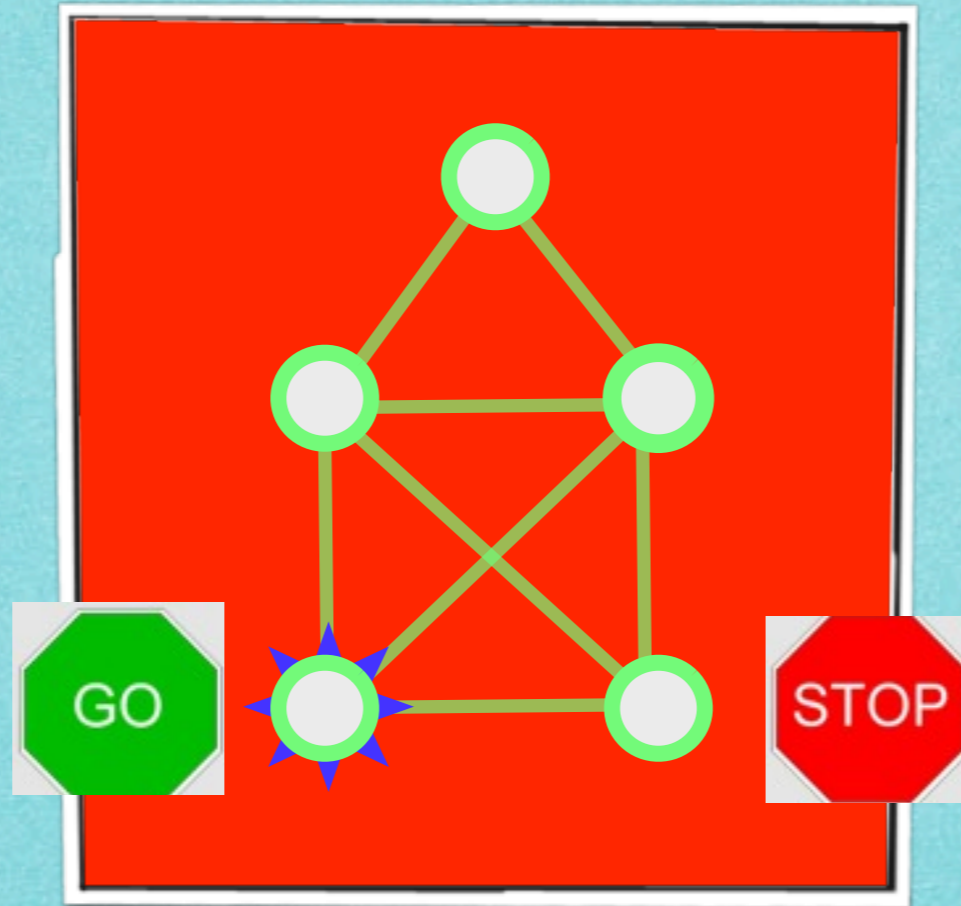
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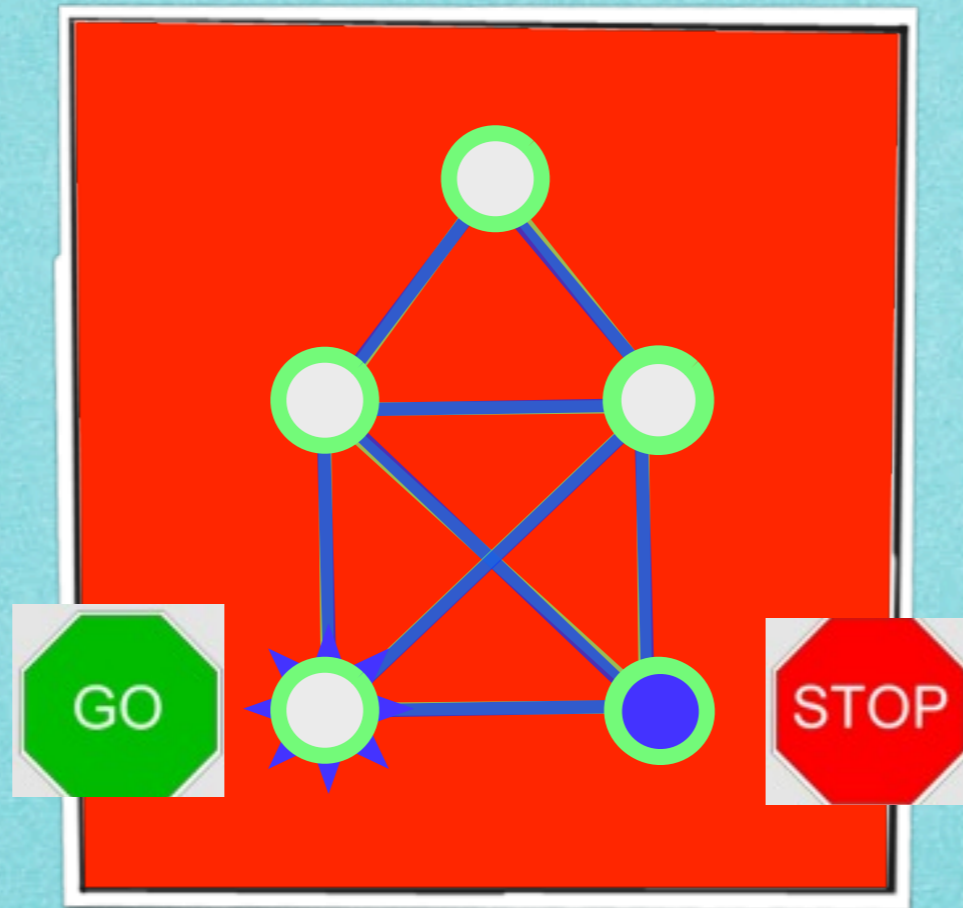
Das Haus des Nikolaus



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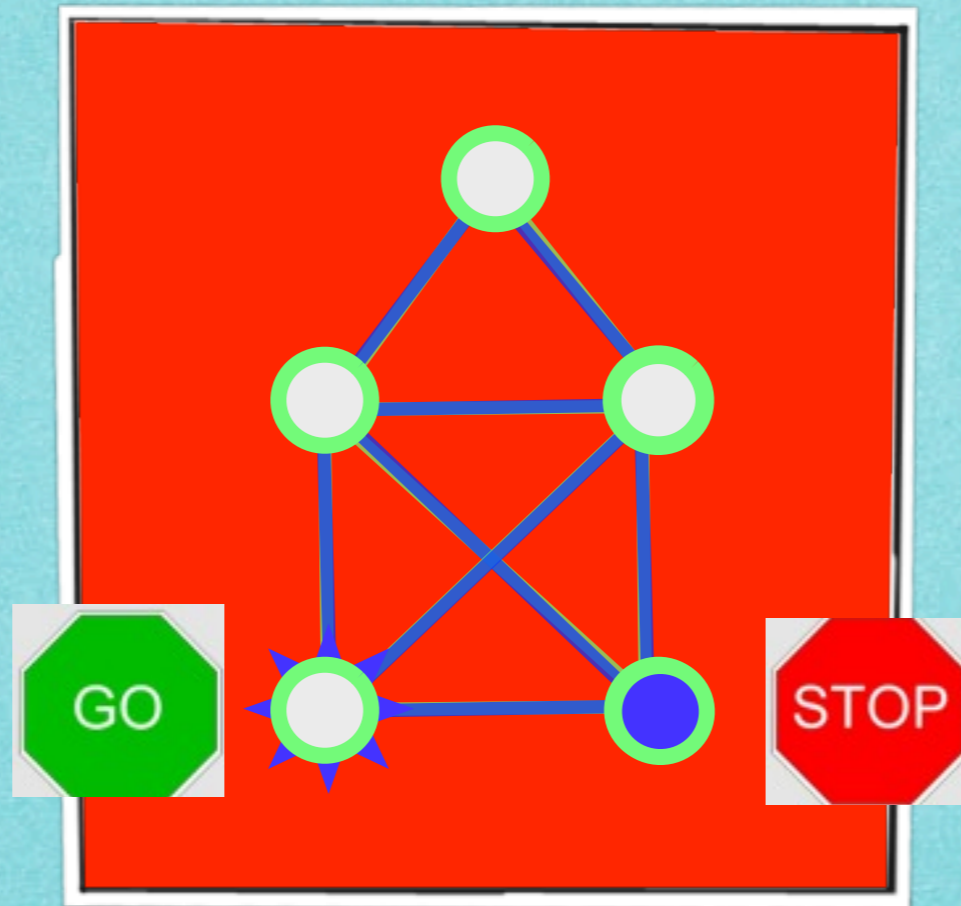


Das Haus des Nikolaus

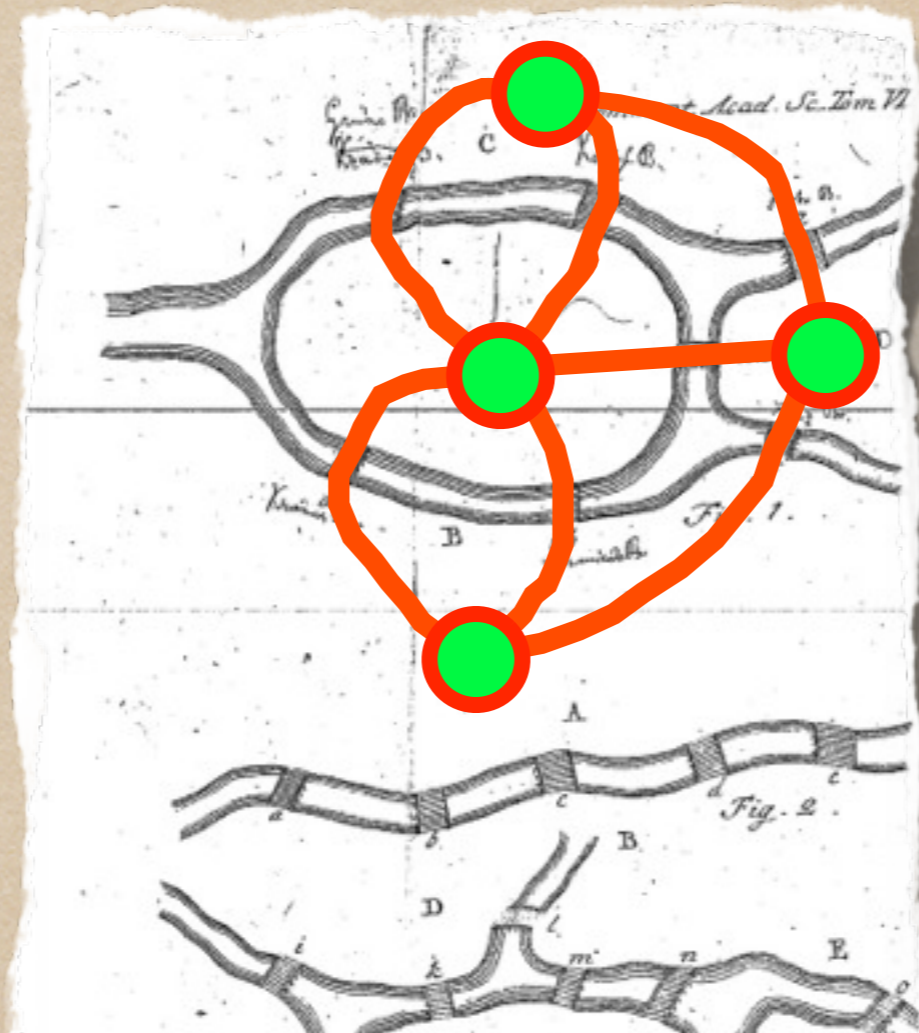


Ahhhhhhhh!

Das Haus des Nikolaus



Wichtig: An einem der Knoten mit drei Kanten anfangen, weil man sonst irgendwann dort nicht mehr weg kommt!



- Alle Knoten sind ungerade?!
- Man müsste an allen anfangen oder aufhören!
- Das geht nicht an einem Stück!

Euler: (1) Das gilt für jede beliebige Instanz: Mit mehr als zwei ungeraden Knoten gibt es keinen solchen Weg.

(2) Man kann auch charakterisieren, unter welchen Bedingungen es einen Weg tatsächlich gibt.

SOLVTIO PROBLEMATIS
AD
GEOMETRIAM SITVS
PERTINENTIS.

AVCTORE

Leonb. Eulero.

§. 1.

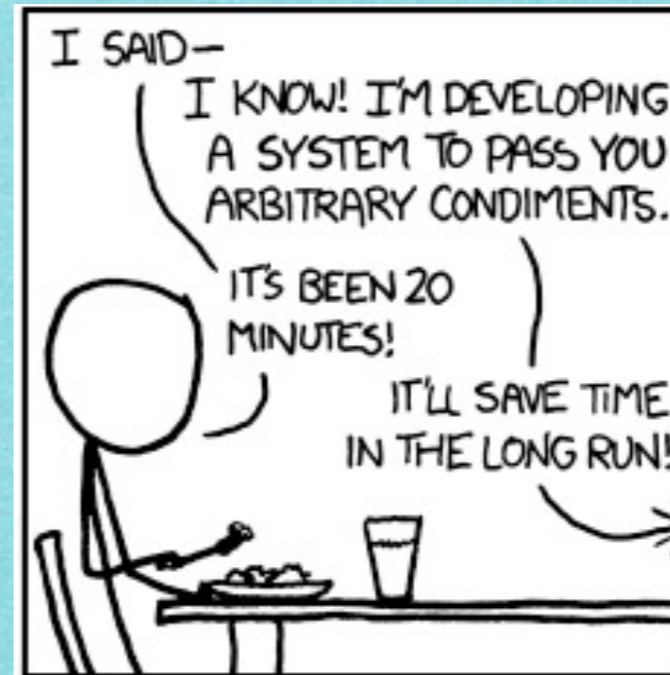
Tabula VIII. **P**raeter illam Geometriae partem, quae circa quantitates versatur, et omni tempore summo studio est exulta, alterius partis etiamnum admodum ignotae primus mentionem fecit *Leibnitzius*, quam Geometriam situs vocauit. Ista pars ab ipso in solo siti determinando, situsque proprietatibus eruendis occupata esse statuitur; in quo negotio neque ad quantitates respiciendum, neque calculo quantitatum vtendum sit. Cuiusmodi autem problemata ad hanc situs Geometriam pertineant, et quali methodo in iis resoluendis vti oporteat, non satis est definitum. Quamobrem, cum nuper problematis cuiusdam mentio esset facta, quod quidem ad geometriam pertinere videbatur, at ita erat comparatum, vt neque determinationem quantitatum requireret, neque solutionem calculi quantitatum ope admitteret, id ad geometriam situs referre haud dubitavi: praesertim quod in eius solutione solus situs in considerationem veniat, calculus vero nullius prorsus sit vti. Methodum ergo meam quam ad huius generis problemata

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2.1 Historie



Euler hat:

- eine Instanz betrachtet
- ein Problem gelöst
- ein Gebiet begründet

2.1 Historie

Leonhard Euler:

1707 Geboren in Basel
1720 Studienbeginn in Basel
1723 Magister
1727 Berufung an Petersburger Akademie
1731 Professur für Physik

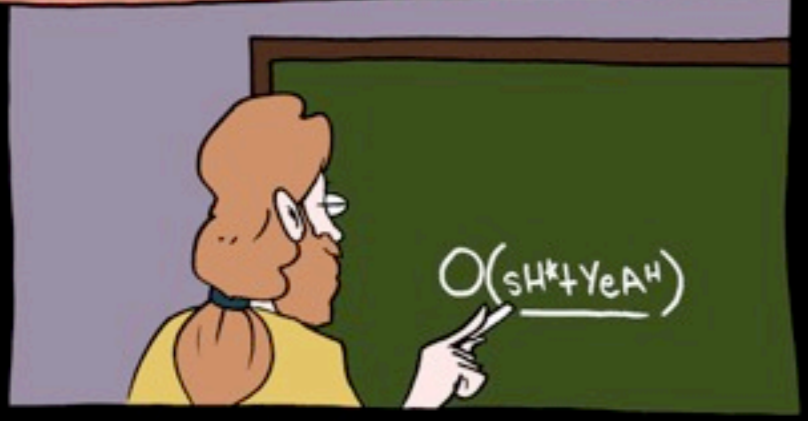


Erik Demaine:

1981 Geboren in Halifax
1993 Studienbeginn in Halifax
1995 Bachelor
1996 Master
2001 Ph.D.
2001 Assistenzprofessor am MIT
2005 Full Professor am MIT



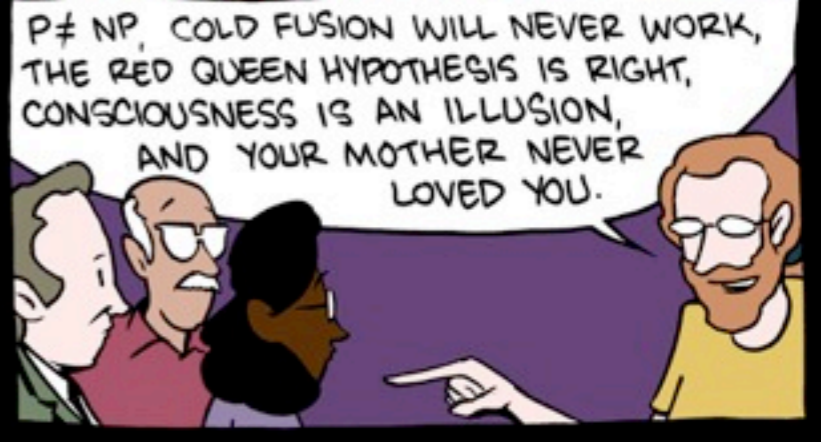
DR. DEMAINÉ CREATED AN ALGORITHM THAT SOLVED ALL MATHEMATICAL THEOREMS.



SOON AFTER, ALL PHYSICS QUESTIONS WERE ANSWERED



THEN ENGINEERING, CHEMISTRY, BIOLOGY, NEUROSCIENCE, PSYCHIATRY...



HAVING COMPLETED SCIENCE, HE MOVED ON TO PHILOSOPHICAL AND LITERARY QUESTIONS.



THEN UNINTERESTING RHETORICAL QUESTIONS



FINALLY, ALL THAT WAS LEFT WAS SENSELESS HALF-CONCEIVED QUESTIONS FROM STONED PHILOSOPHY UNDERGRADS.



Three Colors Suffice: Conflict-Free Coloring of Planar Graphs*

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¹Mathematics Department, MIT, Cambridge, Massachusetts, USA

²Algorithms Group, TU Braunschweig, Braunschweig, Germany

³Computer Science and Artificial Intelligence Laboratory, MIT, Massachusetts, USA

⁴Computer Science and Engineering, IIT Bombay, Mumbai, India

Abstract

A *conflict-free k -coloring* of a graph assigns one of k different colors to some of the vertices such that, for every vertex v , there is a color that is assigned to exactly one vertex among v and v 's neighbors. Such colorings have applications in wireless networking, robotics, and geometry, and are well-studied in graph theory. Here we study the natural problem of the *conflict-free chromatic number* $\chi_{CF}(G)$ (the smallest k for which conflict-free k -colorings exist), with a focus on planar graphs.

For general graphs, we provide a sufficient condi-

1 Introduction.

Coloring the vertices of a graph is one of the fundamental problems in graph theory, both scientifically and historically. Proving that four colors always suffice to color a planar graph [5, 6, 25] was a tantalizing open problem for more than 100 years; the quest for solving this challenge contributed to the development of graph theory, but also to computers in theorem proving [29]. A generalization that is still unsolved is the *Hadwiger Conjecture* [18]: A graph is k -colorable if it has no K_{k+1} minor.

Jetzt wird's genauer!

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