Abteilung Algorithmik
Institut für Betriebssysteme
und Rechnerverbund
TU Braunschweig

WS 14/15

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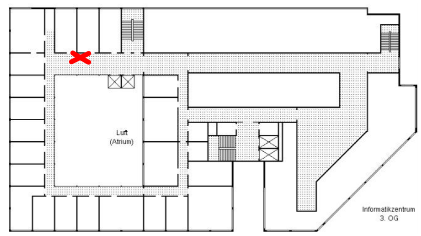
Computational Geometry
Homework Set 4, 21.12.2014

Solutions are due Tuesday, January 6, 2014, until 9:45 in the mailbox for homework sheets or at the beginning of the lecture. **Please put your name on all pages!**



Exercise 1 (Triangle splitting without general position):
Extend the triangle splitting algorithm (Alg. 3.3) from the lecture so that it works for point sets that may include three or more collinear points (but not all points are collinear!).

(10 points)

Exercise 2 (Flip graphs):

a) Give the flip graph of a convex hexagon. Denote the triangulations represented in each node.

b) Show that all nodes of the flip graph of a convex n-gon have degree $n-3$.

(10+10 points)

Exercise 3 (Voronoi Diagrams):
Show:

a) Let S be the set of input sites. For each $p \in S$, $\mathcal{V}(p) = \bigcap_{q \in S \setminus \{p\}} H(p,q)$.

b) All Voronoi regions are convex.

(5+5 points)

Exercise 4 (Convex Hull):
Let $P = \{p_1, p_2, \ldots, p_n\}$ be a point set of n points in $\mathbb{R}^d$. Show: $x \in \operatorname{conv}(P) \Leftrightarrow x = \lambda_1 p_1 + \lambda_2 p_2 + \ldots + \lambda_n p_n$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_n = 1$ with only $(d+1)$ $\lambda_i \neq 0$ (i.e., each point can be expressed as a convex combination of $d+1$ "original" points).

(10 points)

Exercise 5 (Optimal present wrapping):
To help Mr. Claus with the yearly chaos, one of his elves, Timothy, has invented an incredibly efficient and automatic procedure for gift wrapping. However, it still needs a subroutine that does the following:
Given a convex 2-dimensional present with n corners, and a candy cane, fixed by a screw at one end (see Figure 1), calculate in $O(\log n)$ which corner of the present is touched when rotating the candy cane counterclockwise.
Assume that Mr. Claus can retrieve the coordinates of a corner in constant time and that the candy cane is infinitely long (yum!).

(10 points)

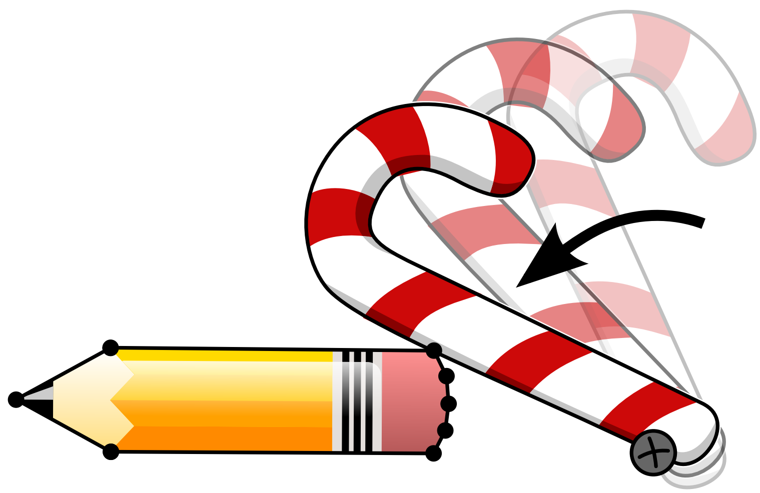


Figure 1: A convex present and a candy cane.

Enjoy your semester break!