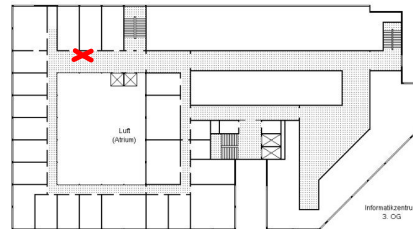


## Computational Geometry Homework Set 3, 02.12.2014

Solutions are due Tuesday, December 9, 2014, until 9:45 in the mailbox for homework sheets or at the beginning of the lecture.  
**Please put your name on all pages!**



### Exercise 1 (Convex hulls):

Let  $W$  be a simple closed path that encloses the convex hull of a point set  $S$ . Show that the boundary of  $ch(S)$  is at most as long as  $W$ .

(10 points)

### Exercise 2 (Triangulations and convex hull):

Show that each triangulation of a point set  $S$  contains the edges of the points' convex hull.

(5 points)

### Exercise 3 (Convex vertices):

Show that any polygon must have at least three convex vertices.

(10 points)

### Exercise 4 (Monotonicity):

- a) Give an algorithm that tests in  $O(n)$  steps whether a simple polygon  $P$  is monotone with respect to a line  $g$ .

You may assume  $P$  to be given either as a Doubly-Connected Edge List (DCEL) or simply as a list of vertices and edges.

Hint: You may assume that no edge of the polygon is perpendicular to  $g$ .

- b) Given a simple polygon  $P$ , give an algorithm that decides in  $O(n)$  steps whether there exists a line  $g$ , such that  $P$  is monotone with respect to  $g$ .  
Hint: Consider the interior angles at potential saddle points.

Of course it is possible/allowed to present an algorithm that solves both problems.

(10+10 points)

**Exercise 5 (Triangulation):**

- a) Triangulate the polygon shown in Figure 1 using the algorithms from the lecture.
- b) Give an algorithm that triangulates a polygon with holes in  $O(n \log n)$ .

(10+5 points)

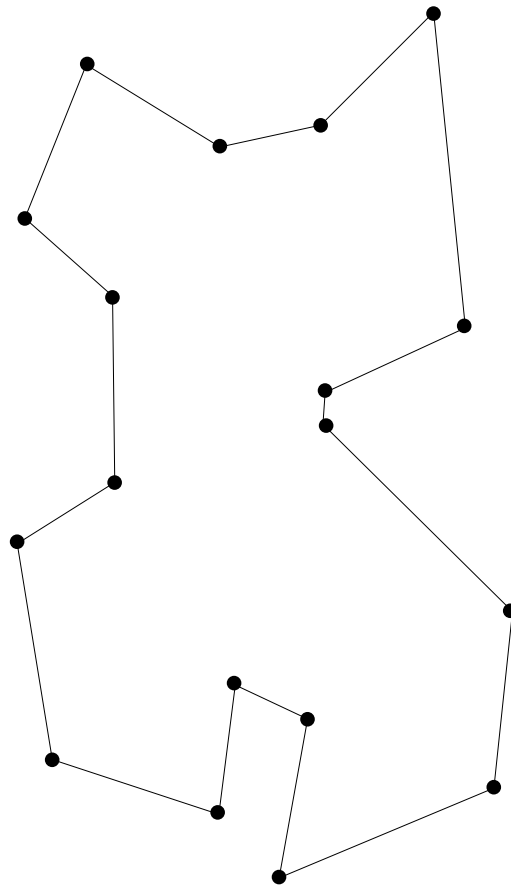


Figure 1: A Polygon.