Computational Geometry
Homework Set 3, 02.12.2014

Solutions are due Tuesday, December 9, 2014, until 9:45 in the mailbox for homework sheets or at the beginning of the lecture. Please put your name on all pages!

Exercise 1 (Convex hulls):
Let \( W \) be a simple closed path that encloses the convex hull of a point set \( S \). Show that the boundary of \( ch(S) \) is at most as long as \( W \).

(10 points)

Exercise 2 (Triangulations and convex hull):
Show that each triangulation of a point set \( S \) contains the edges of the points’ convex hull.

(5 points)

Exercise 3 (Convex vertices):
Show that any polygon must have at least three convex vertices.

(10 points)

Exercise 4 (Monotonicity):

a) Give an algorithm that tests in \( O(n) \) steps whether a simple polygon \( P \) is monotone with respect to a line \( g \).

You may assume \( P \) to be given either as a Doubly-Connected Edge List (DCEL) or simply as a list of vertices and edges.

Hint: You may assume that no edge of the polygon is perpendicular to \( g \).
b) Given a simple polygon $P$, give an algorithm that decides in $O(n)$ steps whether there exists a line $g$, such that $P$ is monotone with respect to $g$. Hint: Consider the interior angles at potential saddle points.

Of course it is possible/allowed to present an algorithm that solves both problems.

(10+10 points)

**Exercise 5 (Triangulation):**

a) Triangulate the polygon shown in Figure 1 using the algorithms from the lecture.

b) Give an algorithm that triangulates a polygon with holes in $O(n \log n)$.

(10+5 points)

![Figure 1: A Polygon.](image-url)