**Abteilung Algorithmik**
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**WS 14/15**

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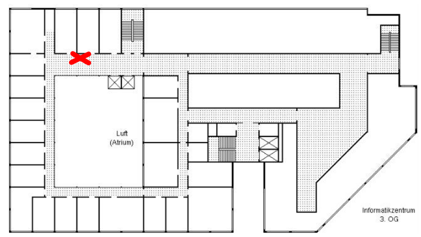
# Computational Geometry
# Homework Set 1, 04.11.2014

Solutions are due Tuesday, November 11, 2014, until 9:45 in the mailbox for homework sheets or at the beginning of the lecture. **Please put your name on all pages!**




**Exercise 1 (Minimum guards vs. minimal guards):**
When we consider the Art Gallery Problem, we ask for the minimum number of guards that are sufficient to monitor a specific given polygon $P$.
A set of guards is *minimal* if we cannot delete one of these guards without losing the complete coverage property (the set is minimal with respect to inclusion).

a) Give an example of a simple polygon $P$ and a set of 5 guards that cover it such that deletion of any one guard causes part of the gallery $P$ to be unseen (i.e., the set of 5 guards is minimal), but the guard number, $G(P)$, for $P$ is less than 5: $(G(P) < 5)$.

b) Give an example that the ratio between the number of guards in a minimal set and the number of guards in a minimum set cannot be bounded by a constant.

(4+9 points)

**Exercise 2 (Guarding the boundary vs. guarding the polygon):**

a) Given a simple polygon $P$ that needs 2 guards for complete coverage of $P$'s boundary. Are these two guards also sufficient to cover $P$ completely?

b) Given a simple polygon $P$ that needs 3 guards for complete coverage of $P$'s boundary. Are these three guards also sufficient to cover $P$ completely?

c) Give an example of a simple polygon $P$ and a placement of some number of guards in $P$ such that the guards see every point of the boundary, but there is at least one point interior to the region $P$ that is not seen by any guard. Try to make your example as small as possible (having the fewest number of vertices in P). Optional: Can you argue that your example is the smallest possible?

(6+6+6 points)

**Exercise 3 (Number of triangulations):** Find the number of distinct triangulations for the polygon in Figure 1.

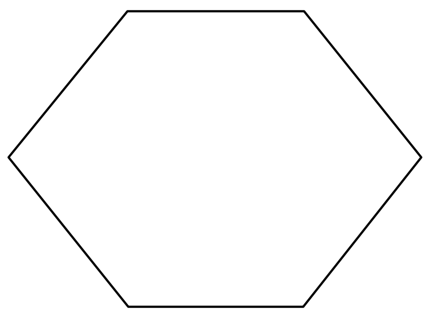


Figure 1: A polygon.

(7 points)

**Exercise 4 (Number of reflex vertices):** Prove the following theorem: In an orthogonal simple polygon of $n$ vertices, $r$ of which are reflex, $n = 2r+4$. Advice: First, show that the sum of interior angles of a simple polygon is $(n-2)\pi$.

(12 points)

**Exercise 5 (Exterior point guards):** Prove the following statement: $\lceil \frac{n+1}{3} \rceil$ point guards suffice to cover the exterior of any simple polygon with $n$ vertices.
Hint: For $n \in \mathbb{N}$, $\lceil \frac{n+1}{3} \rceil = \lfloor \frac{n+3}{3} \rfloor$.

(15 points)