# Exercises for Geometric Algorithms for Folding and Unfolding WS 2012/13

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1. Consider the additor employed in Kempe's proof. It uses 4 multiplicators. Is it possible to have a simpler design, for example with 2 multiplicators and one rhombus?

2. Consider the left linkage in the figure below. Assume that $|ax| = |by| = 5, |ab| = 2$ The joints x and y are pinned, the others may move freely. What curve is traced by the point c, in the middle of the segment ab?

3. Consider the right linkage in the figure below. You want to achieve an essentially left - right, parallel motion of the two long bars. Which of the joints (if any) have to be pinned (and where), and how do the other joints move to realize this motion?
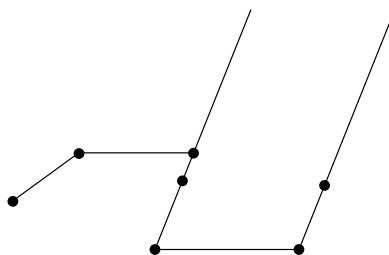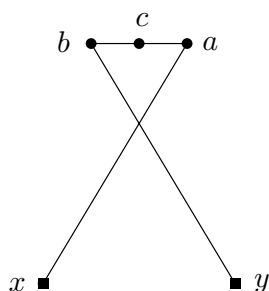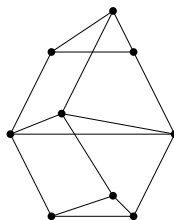

4. Construct a linkage that writes the first letter of your name.

5. Which of the following graphs are generically rigid in the plane? Are those graphs also minimally rigid?
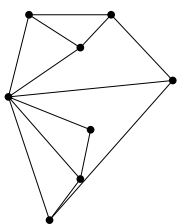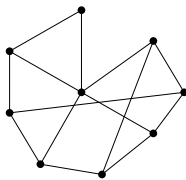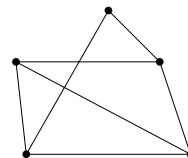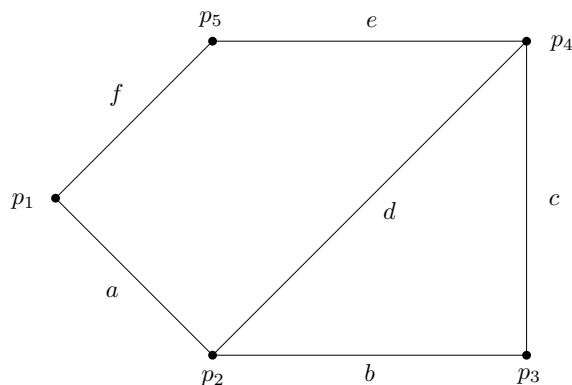

1	2	3	4

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6. Consider the 2D linkage given below. The coordinates of the vertices are: $p_1 = (0,1), p_2 = (1,0), p_3 = (3,0), p_4 = (3,2), p_5 = (1,2)$.

   (a) Give the rigidigy matrix of this linkage.

   (b) Assume the vertices p_2 and p_4 are pinned. Use the rigidity matrix to compute how the other vertices move.

7. Consider the tensegrities below, with bars, cables and struts. Is there an equilibrium stress that is non-zero for these tensegrities?

cables: green dotted
struts: red dashed
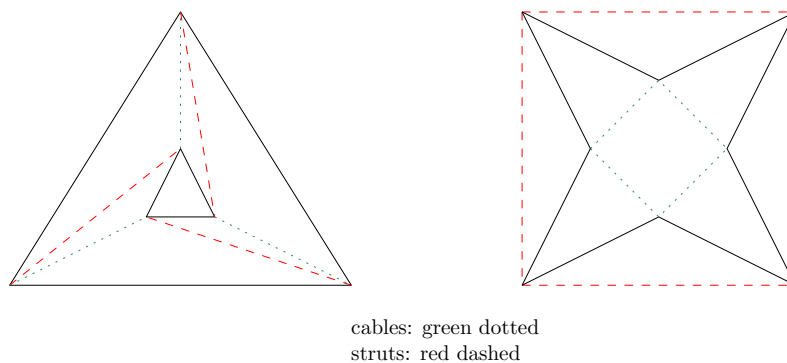
8. Give a formal proof to the observation *Any expansive motion cannot cause crossings.*

9. Consider a tensegrity and planarize it (i.e., add vertices at crossings and split the edges there. If there are multiple copies of one and the same edge, keep only the most restrictive one - often, this is a bar.) Give a proof to the following lemma of the lecture:

   Lemma: *If the original tensegrity (with all bars and struts) has an equilibrium stress that is not everywhere zero, so does the planar tensegrity derived from it.*

10. Give a formal proof of the following lemma presented in the lecture.
    Lemma: *Every flat foldable 1D mountain-valley pattern is mingling.*

11. Consider the 2D map with 3x3 squares given in Appendix A. Cut it out and fold it according to the given mountain-valley pattern such that in the end the whole map is only a single square big.

12. Consider a large square piece of paper. Fold it four times using all-layers simple folds, such that you fold alternatingly in the horizontal and vertical direction. What is the resulting crease pattern of the paper? (Please try this as a thought experiment first, without actually folding any paper.)

13. Consider a square with a single vertex in the center, where an even number of creases meet, which are all assigned either mountain or valley. When is such a mountain-valley pattern around a single vertex flat foldable?

14. Consider the image of the swan in Appendix B. It contains both the outline (black) and the part of the straight skeleton that is inside the polygon (red).

    (a) Construct all perpendiculars of the polygon.
    (b) Label the perpendiculars and construct the shadow tree accordingly.

15. Consider the image of the Christmas tree in Appendix C. Turn it into a fold-and-one cut pattern so that you can cut it out with only one cut across the paper. You will have to do the following steps:

    (a) Construct the straight skeleton of the polygon (recall that it consists of a part inside and a part outside of the polygon).
    (b) Construct all perpendiculars.
    (c) Construct the shadow tree.
    (d) Choose a flat folding of the shadow tree and construct the mountain-valley pattern.
    (e) Fold the tree.
    (f) Cut.
    (g) Merry X-Mas!

16. Special Christmas exercise: You are given a square piece of paper (of normalized side-length one). What is the largest cube that can be wrapped with it?

17. What are the principal curvatures $\mathcal{K}_1, \mathcal{K}_2$ and the Gaussian curvatures $\mathcal{K}$ of the following objects?

    (a) a sphere
    (b) a cylinder
    (c) a flat piece of paper

18. Consider a truncated cube, i.e., a cube with all its corners cut off. What is the face path for such a truncated cube?

19. Recall the following lemma from the lecture and give a formal proof of it:
    If $\mathcal{T}$ has a face path, then $\mathcal{T}$ has a vertex unfolding in which each triangle of the path occupies an otherwise empty vertical strip of the plane.

20. Consider the induction process of the dome unfolding, where you select a triangle, remove it by extending the adjacent faces, etc. Show that for any dome, this extension is always possible.

21. Consider the nets that together make up "Patches the Cat". Can you transform the five pieces of the net into a single one that does not overlap and still fold to "Patches"?

Appendix B

Appendix C