Exercise 1 (Lower Bound for Voronoi):
Assume that an algorithm $ALG$ is able to compute the Voronoi diagram of a point set $P$ with $|P| = n$ in $O(f(n))$ with $f(n) \in o(n \log n)$ (that is, faster than $O(n \log n)$).
Prove that this is not possible by showing that you could use $ALG$ to sort $n$ numbers in $O(n + f(n))$.

(15 Punkte)

Exercise 2 (Voronoi Lookup):
Given: A point set $P$ with $n \in \mathbb{N}$ points and its Voronoi diagram $Vor(P)$.

Give a method for the construction of a data structure that allows finding the nearest site for an arbitrary point $q \in \mathbb{R}^2$ (that is, to determine in which cell $q$ lies).

Generating the data structure can be arbitrarily complex; once it is established, it should allow finding the next site for arbitrary $q$ in time $O(\log n)$.
Explain why your method complies with a lookup time of $O(\log n)$.

Hint: In case $q$ is located on a Voronoi edge or a Voronoi vertex, the next site is not uniquely defined. Your lookup should simply give one of those next sites.

(15 Punkte)

Exercise 3 (Triangulations and the Convex Hull):
A triangulation of a planar point set $S$ is a subdivision of the plane determined by a maximal set of noncrossing edges whose vertex set is $S$. 


Show: The edges of the convex hull of a point set $S$ will be in every triangulation of $S$.

(15 Punkte)

Exercise 4 (Gift Wrapping for the Convex Hull):
Consider the Gift Wrapping algorithm presented in the tutorial. Prove that the point forming the largest angle to the previous edge must be a hull point.

(15 Punkte)