Exercise 1 (The Fortress Problem):
For the Fortress Problem we are interested in the number of guards (vertex guards for the purposes of this exercise) that are needed to see the exterior of a polygon of $n$ vertices. Here an exterior point $y$ is seen by a guard at vertex $z$ iff the segment $zy$ does not intersect the interior of the polygon. O’Rourke and Wood gave the following theorem:

Theorem 1. $\lceil \frac{n}{2} \rceil$ vertex guards are sometimes necessary and always sufficient to see the exterior of a polygon of $n$ vertices.

Prove the necessity of $\lceil \frac{n}{2} \rceil$.

(10 Punkte)
Exercise 2 (Triangulation of monotone Polygons):
Use the algorithm of Garey, Johnson, Preparata and Tarjan to triangulate the polygon $P$ in Figure 1. For each iteration give the stack and diagonals that are drawn. Give the final triangulation of $P$.

Figure 1: Polygon $P$. (10 Punkte)
Exercise 3 (Scissors Congruence):
A dissection of a polygon $P$ cuts $P$ into a finite number of smaller polygons. Triangulations can be viewed as a constrained form of dissections.
Given a dissection of a polygon $P$, we can rearrange its smaller polygonal pieces to create a new polygon $Q$ of the same area. We say two polygons $P$ and $Q$ are scissors congruent if $P$ can be cut into polygons $P_1, \ldots, P_n$ which then can be reassembled by rotations and translations to obtain $Q$.

a) Is the Greek cross from Figure 2 scissors congruent to a square?

b) Prove: Every triangle is scissors congruent with some rectangle.

c) Prove: Any two rectangles of the same area are scissors congruent.

d) Let polygon $P_1$ be scissors congruent to polygon $P_2$, and let polygon $P_2$ be scissors congruent to polygon $P_3$. Show that polygon $P_1$ is scissors congruent to polygon $P_3$. In other words, show that scissors congruence is transitive.

Figure 2: The Greek cross.