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# Collaborative transmission in wireless sensor networks

Alternative algorithmic approaches

Stephan Sigg

Institute of Distributed and Ubiquitous Systems  
Technische Universität Braunschweig

December 6, 2010

# Overview and Structure

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- Introduction to context aware computing
- Wireless sensor networks
- Wireless communications
- Basics of probability theory
- Randomised search approaches
- Cooperative transmission schemes
- Distributed adaptive beamforming
  - Feedback based approaches
  - Asymptotic bounds on the synchronisation time
  - Alternative algorithmic approaches
  - Alternative Optimisation environments

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  - Asymptotic bounds on the synchronisation time
  - **Alternative algorithmic approaches**
  - Alternative Optimisation environments

# Outline

## Alternative beamforming approaches

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- 1 Hierarchical clustering
- 2 Local random search
- 3 An asymptotically optimal algorithm
- 4 Environmental changes
  - Velocity of nodes
  - Multiple receiver nodes
  - Increased population size
  - Receive beamforming

# Alternative algorithmic approaches

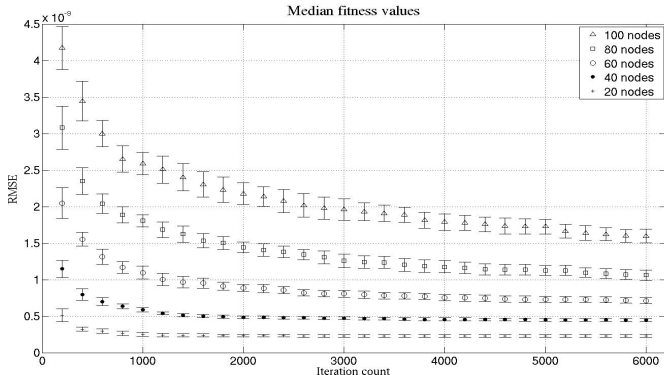
## Hierarchical clustering

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- For feedback based distributed adaptive transmit beamforming:
  - $RSS_{\text{sum}}$  changes linear with the network size  $n$ .
  - Bound on the synchronisation time is more than linear in  $n$

# Alternative algorithmic approaches

## Hierarchical clustering



$$E[T_{\mathcal{P}}] = \Theta(n \cdot k \cdot \log(n))$$

# Alternative algorithmic approaches

## Hierarchical clustering

- Hierarchical clustering
  - 1 Determine clusters
  - 2 Synchronise clusters successively (with possibly increased transmit power for nodes)
  - 3 Build and synchronise overlay-cluster of representative nodes from all clusters.
  - 4 Nodes alter carrier phase by phase offset experienced by representative node:

- $\zeta_i = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t (\gamma_i + \phi_i + \psi_i)} \right)$  (before)

- $\zeta'_i = \Re \left( m(t) \text{RSS}_i e^{j2\pi f_c t (\gamma'_i + \phi_i + \psi_i)} \right)$  (after)

Node  $h$  from same cluster alters carrier signal

- $\zeta_h = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t (\gamma_h + \phi_h + \psi_h)} \right)$  to

- $\zeta'_h = \Re \left( m(t) \text{RSS}_h e^{j2\pi f_c t (\gamma_h + \phi_h + \psi_h + \gamma_i - \gamma'_i)} \right)$

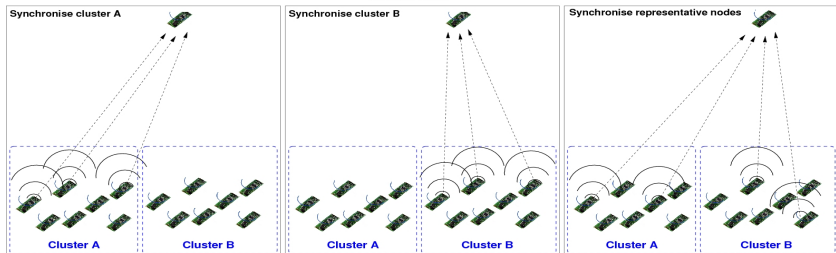
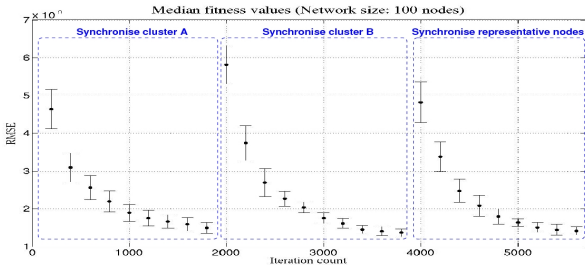
**Ideal conditions:** All nodes should now in phase

- 5 Final synchronisation among all nodes



# Alternative algorithmic approaches

## Hierarchical clustering



# Alternative algorithmic approaches

## Hierarchical clustering

---

Potential problem : Phase noise

- Only one cluster synchronised at a time
- Due to practical properties of oscillators, phases of nodes in the inactive clusters experience phase noise and start drifting out of phase
- Sufficient synchronisation possible in the order of milliseconds

Positive :

- No inter-node communication required

Open Issue :

- More than one hierarchy stage might be optimal
- for optimisation time
- for energy consumption
- Optimum hierarchy depth and cluster size derived by integer programming in time  $\mathcal{O}(n^2)$

# Alternative algorithmic approaches

## Hierarchical clustering

---

Determine optimum cluster size and hierarchy depth :

- Expected optimisation time:

$$E[T_{\mathcal{P}_n}] = c \cdot k \cdot n \cdot \log(n)$$

- Expected energy consumption:

$$E[\mathcal{E}_{\mathcal{P}_n}] = c \cdot k \cdot n \cdot \log(n) \cdot \overline{\mathcal{E}_{\mathcal{P}_n}}$$

Hierarchy and cluster structure that minimises these formulae optimal

# Alternative algorithmic approaches

## Hierarchical clustering

Opt. cluster size and hierarchy depth (integer programming) :

- For a cluster size of  $m$ :

$$E[T_{\mathcal{P}_n}] = E[T_{\mathcal{P}_{\frac{n}{m}}}] \cdot \frac{n}{m} \cdot E[T_{\mathcal{P}_m}]$$

$$E[\mathcal{E}_{\mathcal{P}_n}] = E[\mathcal{E}_{\mathcal{P}_{\frac{n}{m}}}] \cdot \frac{n}{m} \cdot E[\mathcal{E}_{\mathcal{P}_m}].$$

- Define recursion by

$$E_{\text{opt}}[T_{\mathcal{P}_n}] = \min_m \left[ E_{\text{opt}}[T_{\mathcal{P}_{\frac{n}{m}}}] \cdot \frac{n}{m} \cdot E_{\text{opt}}[T_{\mathcal{P}_m}] \right]$$

$$E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_n}] = \min_m \left[ E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_{\frac{n}{m}}}] \cdot \frac{n}{m} \cdot E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_m}] \right]$$

- Start of recursion ( $\eta$  min feasible cluster size):
  - $E_{\text{opt}}[T_{\mathcal{P}_\eta}]$
  - $E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_\eta}]$

# Alternative algorithmic approaches

## Hierarchical clustering

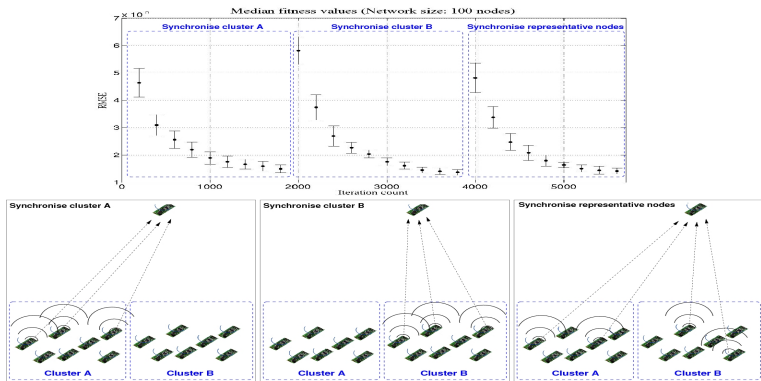
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Opt. cluster size and hierarchy depth (integer programming) :

- Time required for calculation is quadratic.
  - With a network of  $n$  nodes, at most  $n^2$  distinct terms
    - $E_{\text{opt}}[T_{\mathcal{P}_i}]$
    - $E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_i}]$
- Start calculation at
  - $E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_\eta}]$
  - $E_{\text{opt}}[T_{\mathcal{P}_\eta}]$
- All other values by table loop-up in time  $\mathcal{O}(n^2)$  according to
  - $E_{\text{opt}}[T_{\mathcal{P}_n}]$
  - $E_{\text{opt}}[\mathcal{E}_{\mathcal{P}_n}]$  in time  $\mathcal{O}(n^2)$

# Alternative algorithmic approaches

## Hierarchical clustering



- Reduction of synchronisation time and transmission power
- Calculation of optimum cluster size and depth in  $\mathcal{O}(n^2)$

# Outline

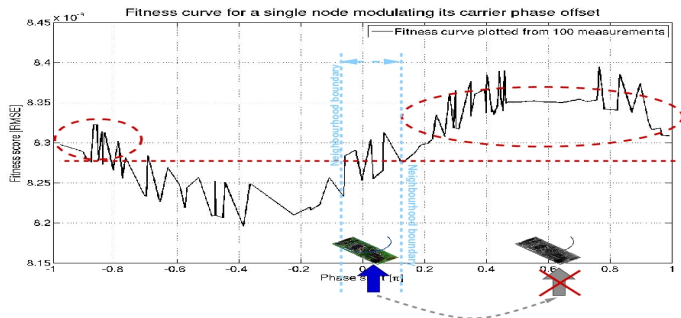
## Alternative beamforming approaches

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# Scenario analysis and algorithmic improvement

## Local random search



- Global random search:
  - Synchronisation performance might deteriorate when the optimum is near
- With small local search space:
  - Majority of worse points excluded



# Local random search

## An upper bound on the synchronisation performance

Assumptions :

Mutation probability:  $n^{-1}$

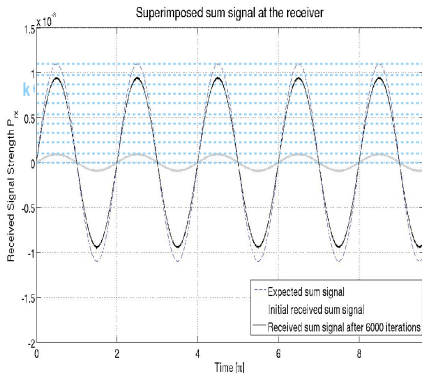
Uniform phase alteration

Initial distance to the optimum :

$$\geq \frac{n \cdot \log(k)}{2} \text{ (Chernoff)}$$

Technical assumption :

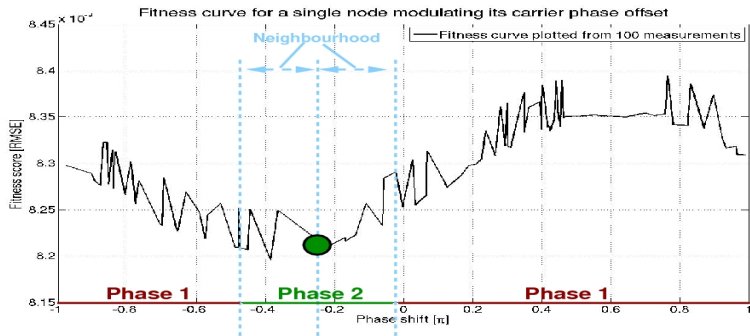
Fitness space divided in  $k$  slices of identical width



# Local random search

## An upper bound on the synchronisation performance

Analysis in two phases for the synchronisation process



**Phase 1:** Optimum outside search neighbourhood for at least one node

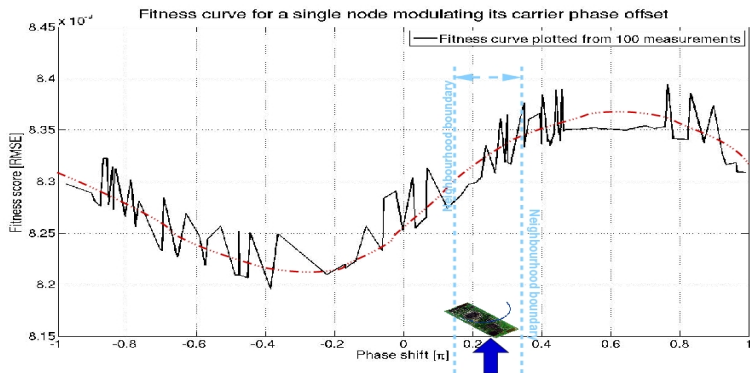
**Phase 2:** Optimum within search neighbourhood for all nodes

# Local random search

## An upper bound on the synchronisation performance

Phase 1: Optimum is outside the neighbourhood

- Reach search point with improved fitness:  $\geq \frac{1}{2}$



# Local random search

## An upper bound on the synchronisation performance

When  $i$  signals synchronised:

- Improve  $n - i$  non-optimal signals
- $i$  already optimal ones unchanged:

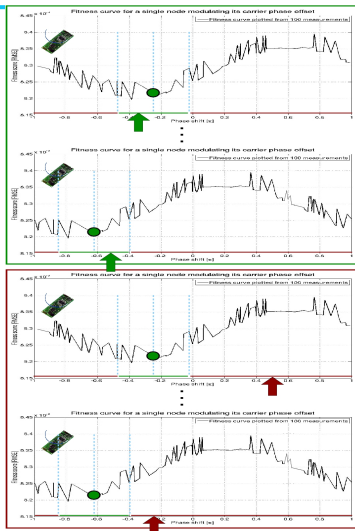
$$(n - i) \cdot \frac{1}{n} \cdot \frac{1}{2} \cdot \left(1 - \frac{1}{n}\right)^i$$

$$= \frac{n-i}{2n} \cdot \left(1 - \frac{1}{n}\right)^i$$

- since  $\left(1 - \frac{1}{n}\right)^n < e < \left(1 - \frac{1}{n}\right)^{n-1}$

$$s_i \geq \frac{n-i}{2en}$$

- Expected number of mutations to increase fitness bounded by  $s_i^{-1}$ .

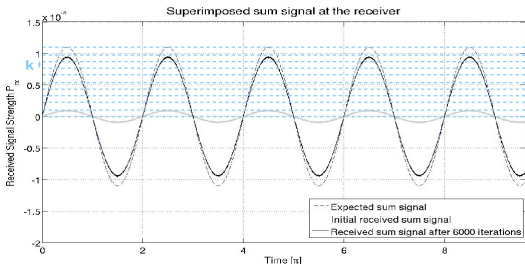


$n-i$

# Local random search

## An upper bound on the synchronisation performance

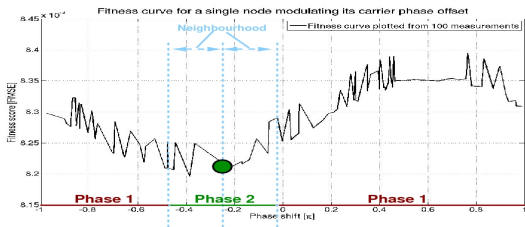
- Time until optimum is within the neighbourhood?
  - Constant time to leave slice
  - $k$  distinct slices



$$\begin{aligned} E[T_P] &\leq c \cdot \sum_{i=0}^k \frac{2en}{n-i} = 2cen \cdot \sum_{i=1}^{k+1} i^{-1} \\ &< 2cen \cdot \ln(k+1) = \mathcal{O}(n \cdot \log(k)) \end{aligned}$$

# Local random search

## An upper bound on the synchronisation performance



Phase 2: Optimum within search neighbourhood

- Worst case: Increase fitness with probability  $\frac{1}{N}$
- Similar to consideration above:

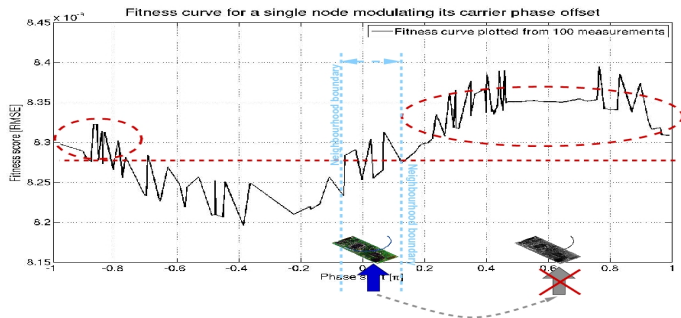
$$\mathcal{O}(N \cdot n \cdot \log(k))$$

Overall synchronisation time :

$$\mathcal{O}(N \cdot n \cdot \log(k)).$$

# Scenario analysis and algorithmic improvement

## Local random search



A lower bound on the synchronisation time :

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size  $n$  by neighbourhood size  $N$

# Scenario analysis and algorithmic improvement

## Local random search

---

A lower bound on the synchronisation time :

- Method of the expected progress
- Similar to estimation for global random search
- Basically: Substitute network size  $n$  by neighbourhood size  $N$ 
  - Probability to alter individual bit

$$\frac{1}{N \cdot \log(k)}$$

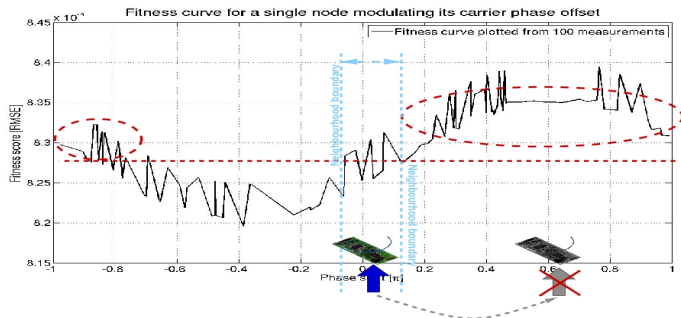
- Instead of

$$\frac{1}{n \cdot \log(k)}$$



# Scenario analysis and algorithmic improvement

## Local random search



A lower bound on the synchronisation time :

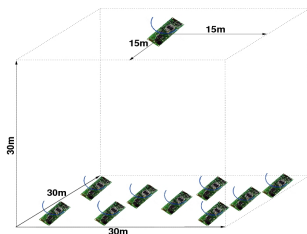
- With similar arguments as for global random search, lower bound

$$\Omega(N \cdot \log(k) \cdot \Delta)$$

# Mathematical simulation environment

## Impact of the node choice

Property	Value
Node distribution area	30m × 30m
Location of the receiver	(15m, 15m, 30m)
Mobility	stationary nodes
Base band frequency	$f_{base} = 2.4 \text{ GHz}$
Transmission power of nodes	$P_{tX} = 1 \text{ mW}$
Gain of the transmit antenna	$G_{tX} = 0 \text{ dB}$
Gain of the receive antenna	$G_{rX} = 0 \text{ dB}$
Iterations per simulations	6000
Identical simulation runs	10
Random noise power [46]	-103 dBm
Pathloss calculation ( $P_{rX}$ )	$P_{tX} \left( \frac{\lambda}{2\pi d} \right)^2 G_{tX} G_{rX}$

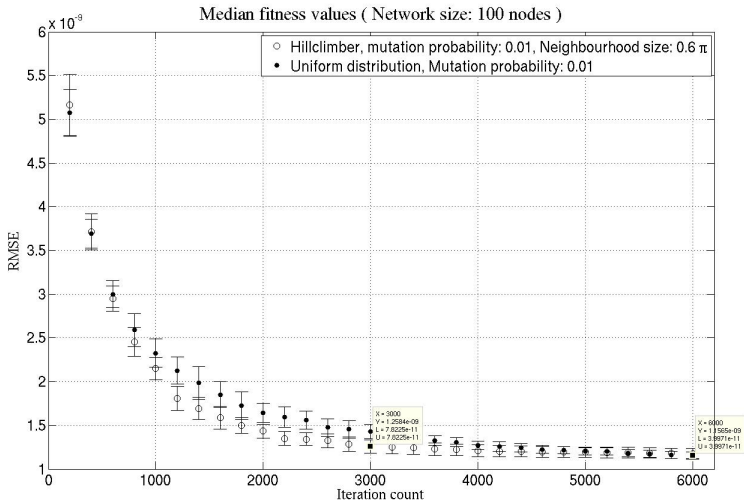


- Fitness measure:

$$RMSE = \sqrt{\sum_{t=0}^{\tau} \frac{(\sum_{i=1}^n S_i + S_{noise}(i) - s^*)^2}{n}}$$

# Scenario analysis and algorithmic improvement

## Local random search



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# Multivariable equations

## Received sum signal

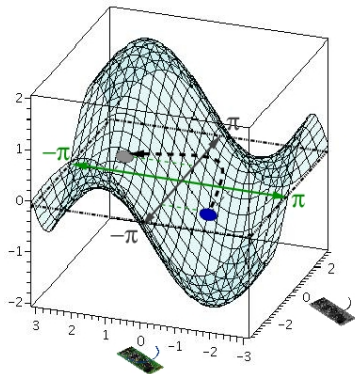
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- Reduce the amount of randomness in the optimisation
- Improve the synchronisation performance
- Improve the synchronisation quality

# Scenario analysis and algorithmic improvements

## Search space

- Search space:
  - Spanned by all Configurations of carrier phase offsets  $\gamma_i$
- Search point / Configuration:
  - One possible configuration of carrier phase offsets

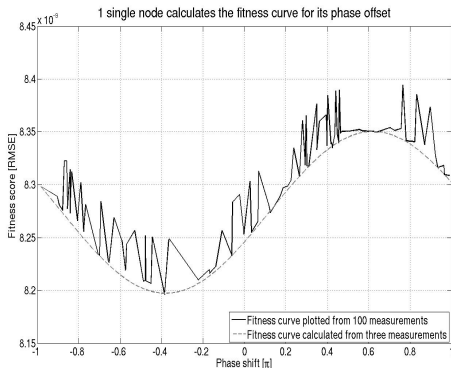


# Multivariable equations

## Received sum signal

- Fitness function observed by single node
- Constant carrier phase offset for  $n - 1$  nodes
- Fitness function:

$$\mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c$$

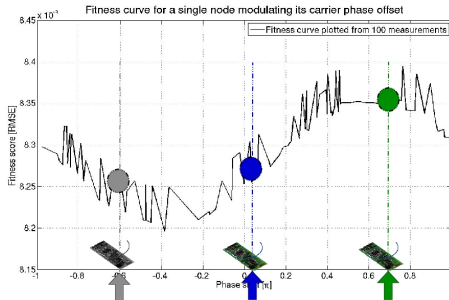


# Multivariable equations

## Received sum signal

Approach:

- Measure feedback at 3 points
- Solve multivariable equations
- Apply optimum phase offset calculated



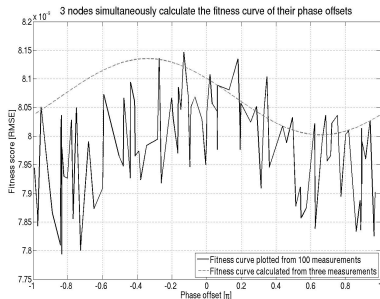
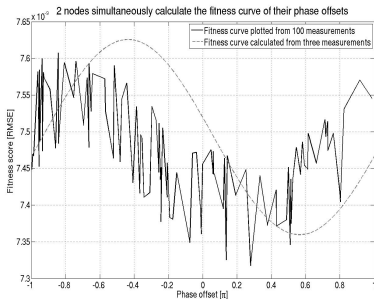
$$\mathcal{F}(\Phi_i) = A \sin(\Phi_i + \phi) + c$$



# Multivariable equations

## Received sum signal

- Problem:
  - Calculation not accurate when two or more nodes alter the phase of their transmit signals



# Multivariable equations

## Solution

---

An active node will :

- 1 Transmit with three distinct phase offsets  $\gamma_1 \neq \gamma_2 \neq \gamma_3$  and measure feedback.
- 2 From these three feedback values and phase offsets, estimate feedback function and optimum phase offset  $\gamma_i^*$ .
- 3 Transmit a fourth time with  $\gamma_4 = \gamma_i^*$ .
- 4 If the deviation is less than 1% save  $\gamma_i^*$  as optimal phase offset, otherwise discard it.

A passive node will :

- 1 Transmit 4 times with identical phase offset  $\gamma_i$ .

# Multivariable equations

## Solution

- Node estimates the quality of the function estimation itself
- Transmit with optimum phase offset and measure channel again
- When Expected fitness deviates significantly from measured fitness, discard altered phase offset
- Deviation:
  - 1 node:  $\approx 0.6\%$
  - 2 nodes:  $\approx 1.5\%$
  - 3 nodes:  $> 3\%$



# Multivariable equations

## Synchronisation process



- 1 Transmit with phase offsets  $\gamma_1 \neq \gamma_2 \neq \gamma_3$ ; measure feedback
- 2 Estimate feedback function and calculate  $\gamma_i^*$
- 3 Transmit with  $\gamma_4 = \gamma_i^*$
- 4 If deviation smaller 1% finished, otherwise discard  $\gamma_i^*$

# Multivariable equations

## Received sum signal

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- Asymptotic synchronisation time:

$$\mathcal{O}(n)$$

- Classic approach:<sup>1</sup>

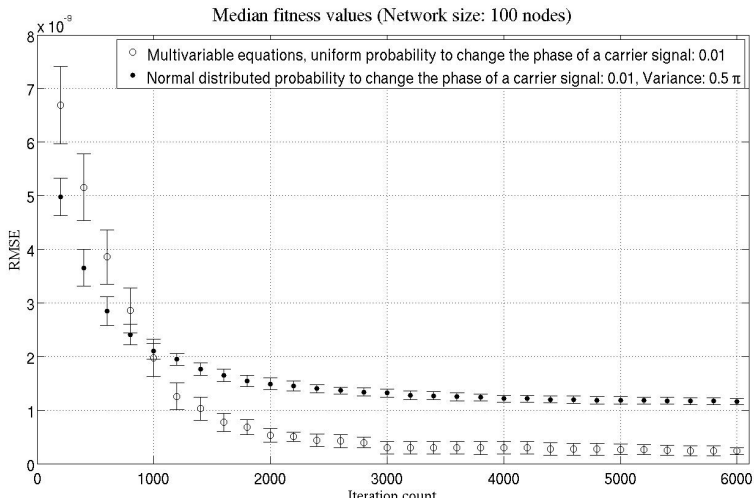
$$\Theta(n \cdot k \cdot \log(n))$$

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<sup>1</sup>Sigg, El Masri and Beigl, A sharp asymptotic bound for feedback based closed-loop distributed adaptive beamforming in wireless sensor networks (submitted to Transactions on Mobile Computing)

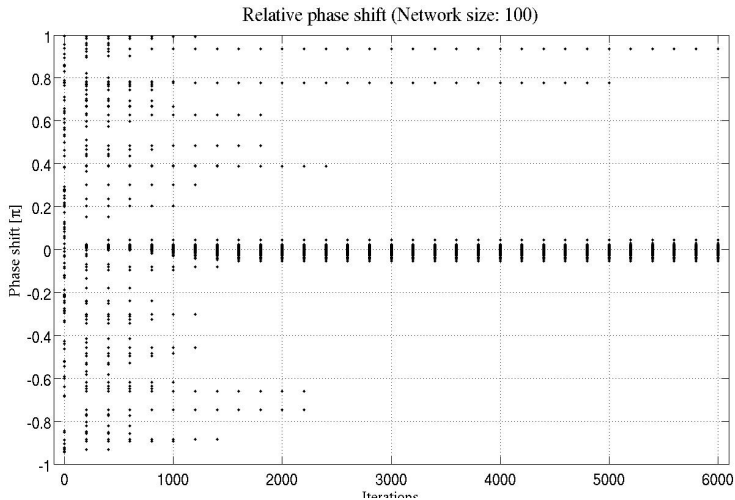
# Multivariable equations

## Performance estimation



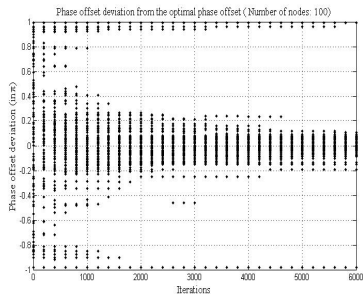
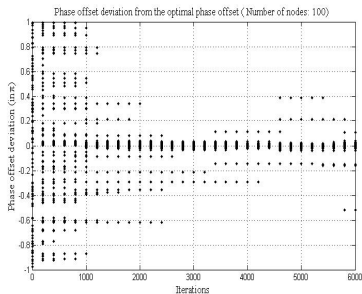
# Multivariable equations

## Performance estimation



# Multivariable equations

## Performance estimation

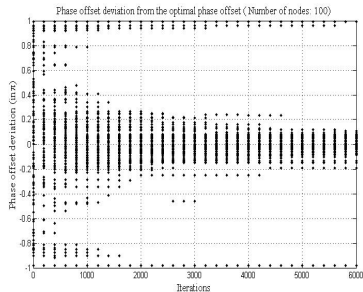
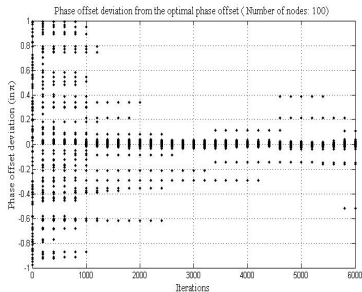


- Phase offset of distinct nodes is within  $+/- 0.05\pi$  for up to 99% of all nodes.



# Multivariable equations

## Performance estimation



- Asymptotically optimal synchronisation time
- Simulations:  $\approx 12n$
- Further improvement:
  - 3 iterations per turn
  - Utilise last transmission from previous iteration

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  - Feedback based approaches
  - Asymptotic bounds on the synchronisation time
  - Alternative algorithmic approaches
  - **Alternative Optimisation environments**

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## Environmental changes

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# Environmental changes

## Introduction

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- Velocity of nodes
- Multiple receiver nodes
- Increased population size
- Receive beamforming

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# Environmental changes

## Velocity of nodes

---

### Moving receiver :

- Straight line
- Random walk

### Moving transmitter :

Straight line

Random walk

# Environmental changes

## Velocity of nodes

---

### Moving receiver :

- Straight line
- Random walk

### Aspects :

Only one moving node

Simple case

Also applicable when all transmitters move identically

# Environmental changes

## Velocity of nodes

---

### Moving transmit nodes :

- Straight line
- Random walk

### Aspects :

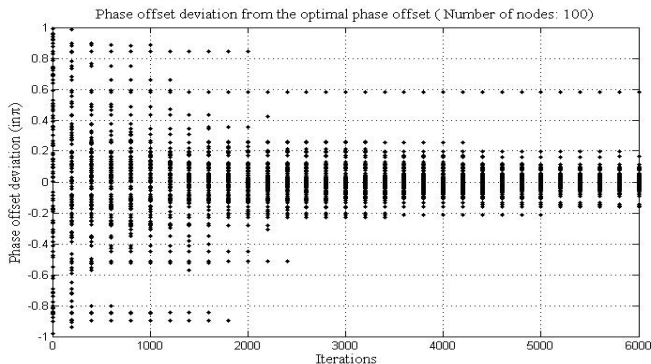
Multiple nodes moving

Hard case



# Environmental changes

## Velocity of nodes

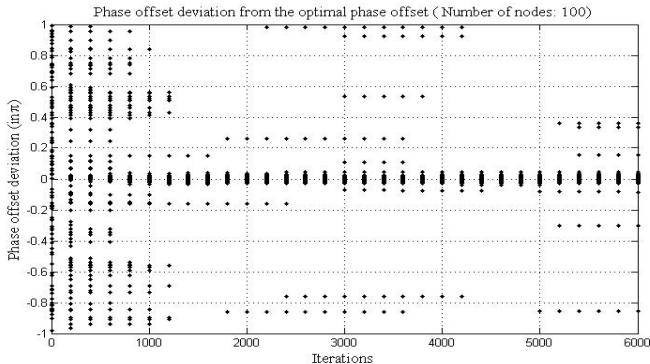


Random walk – receiver :

- Maximum velocity for classic algorithm: 5m/sec

# Environmental changes

## Velocity of nodes

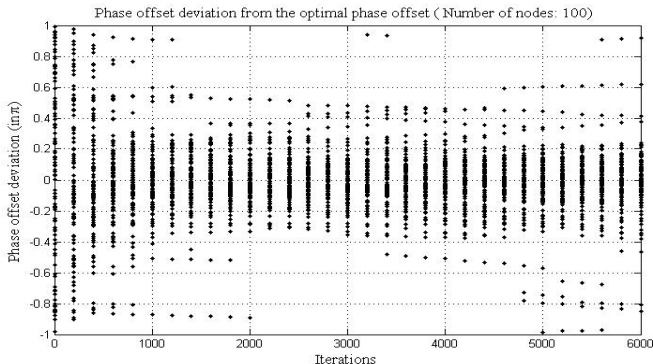


Random walk – receiver :

- Max. velocity for Multivariable equations:  
5m/sec easily supported

# Environmental changes

## Velocity of nodes

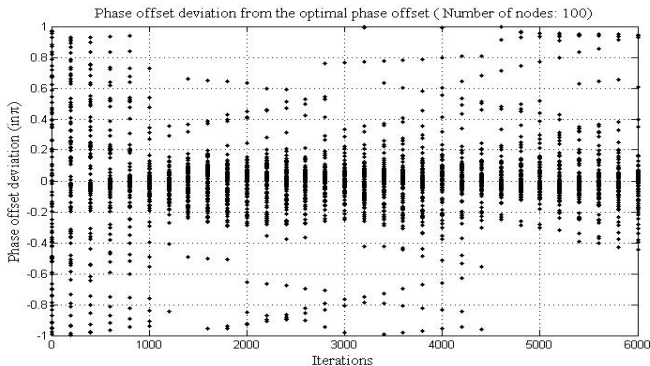


Random walk – transmitter :

- Maximum velocity for classic algorithm: 2m/sec

# Environmental changes

## Velocity of nodes

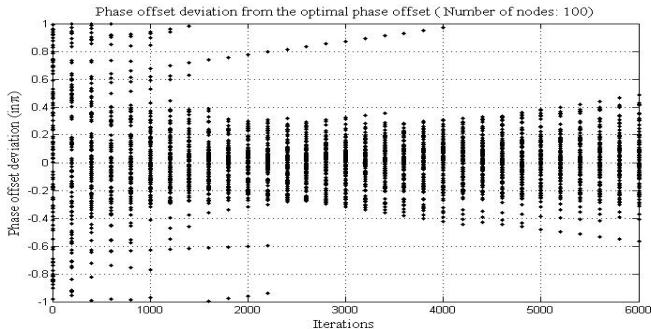


Random walk – transmitter :

- Max. velocity for Multivariable equations:  
5m/sec supported

# Environmental changes

## Velocity of nodes

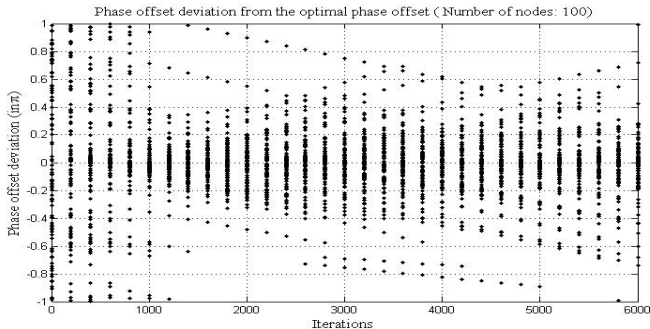


straight line – maximum relative speed :

- Maximum velocity for classic algorithm: 30m/sec
- Regardless if transmitter or receiver move

# Environmental changes

## Velocity of nodes



straight line – maximum relative speed :

- Maximum velocity for Multivariable equations algorithm: 60m/sec
- Regardless if transmitter or receiver move

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# Environmental changes

## Multiple receiver nodes

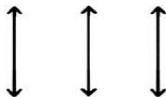
Runde 1:  
Erstes Netzwerk optimiert  
sein Trägersignal, zweites  
Netzwerk gibt Feedback.



(Summensignal)

Phasenanpassung wie beim  
Szenario mit digitalem Feedback.

Feedback wird durch  
Vorhandensein von einem Signal  
gegeben.



(Einzelsignal)

Runde 2:  
Zweites Netzwerk optimiert  
sein Trägersignal, erstes  
Netzwerk gibt Feedback.



(Summensignal)



# Environmental changes

## Multiple receiver nodes

### 3m

<b>Knoten</b>	<i>n0</i>	<i>n1</i>	<i>n2</i>	<i>m0</i>	<i>m1</i>
Gain zur Anfangsamplitude (Median) [dB]	0,96	2,39	1,40	1,46	1,10
Gain zu einem Knoten (Median) [dB]	2,33	2,32	2,37	3,50	4,05
Anzahl letztes Feedback	5/11	3/11	3/11	8/11	7/11
Amplitude nach Synchronisation [%]	92,4	51,4	65,3	91,0	90,7

### 12m

<b>Knoten</b>	<i>n0</i>	<i>n1</i>	<i>n2</i>	<i>m0</i>	<i>m1</i>
Gain zur Anfangsamplitude (Median) [dB]	1,24	0,63	1,39	2,06	1,47
Gain zu einem Knoten (Median) [dB]	2,53	1,09	2,00	2,74	4,18
Anzahl letztes Feedback	2/10	4/10	4/10	5/10	5/10
Amplitude nach Synchronisation [%]	57,1	92,0	86,5	86,4	86,6

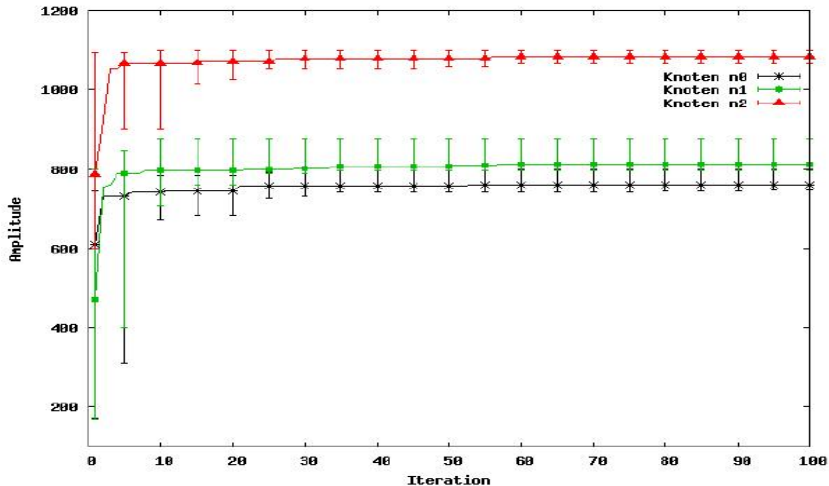
### 24m

<b>Knoten</b>	<i>n0</i>	<i>n1</i>	<i>n2</i>	<i>m0</i>	<i>m1</i>
Gain zur Anfangsamplitude (Median) [dB]	1,12	2,33	2,76	3,61	1,67
Gain zu einem Knoten (Median) [dB]	1,2	2,54	2,03	5,15	3,76
Anzahl letztes Feedback	4/5	0/5	1/5	4/5	3/5
Amplitude nach Synchronisation [%]	94,2	80,0	61,4	95,8	97,9

# Environmental changes

## Multiple receiver nodes

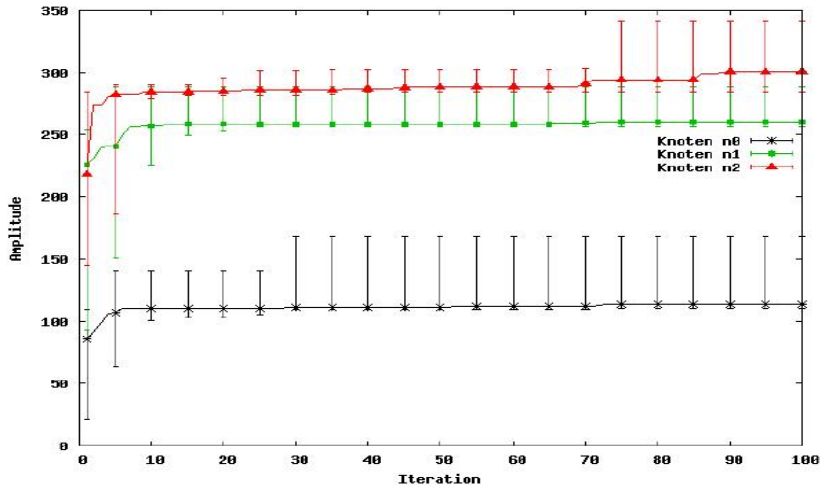
3m



# Environmental changes

## Multiple receiver nodes

12m



# Environmental changes

## Multiple receiver nodes

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### Multiple receiver nodes – issues :

- Only binary feedback value
  - Therefore only classic optimisation approach
- Distance between transmit and receive nodes relative to spatial diversity of nodes in one network
  - Better synchronisation when nodes in one network in spatial proximity
  - When nodes in one network communicate: No issue

# Outline

## Environmental changes

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- 1 Hierarchical clustering
- 2 Local random search
- 3 An asymptotically optimal algorithm
- 4 Environmental changes
  - Velocity of nodes
  - Multiple receiver nodes
  - Increased population size
  - Receive beamforming

# Environmental changes

## Increased population size

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### Increased population size – Discussion :

How to achieve population size greater than one?

- Separate transmit times
- WCDMA
- Distinct frequencies simultaneously

Only separate transmit times feasible for WSN

More time for each iteration

- Initial solution: Random search
- Not clear if performance improvement possible by crossover

# Outline

## Environmental changes

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- 1 Hierarchical clustering
- 2 Local random search
- 3 An asymptotically optimal algorithm
- 4 Environmental changes
  - Velocity of nodes
  - Multiple receiver nodes
  - Increased population size
  - Receive beamforming

# Environmental changes

## Receive beamforming

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### Receive beamforming – Discussion :

- Transmit node transmits only once
- Receiver nodes combine received signal fragments in the network
- Tradeoff:
  - Transmission power for in-network communication
  - Transmission over several iterations with receiver node
- More complex computation of transmit nodes



# Questions?

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- Introduction to context aware computing
- Wireless sensor networks
- Wireless communications
- Basics of probability theory
- Randomised search approaches
- Cooperative transmission schemes
- Distributed adaptive beamforming
  - Feedback based approaches
  - Asymptotic bounds on the synchronisation time
  - Alternative algorithmic approaches
  - Alternative Optimisation environments