Exercise 1 (Independence Systems): Let $E = \{1, \ldots, 10\}$ and
$I_1 = \{\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{3, 4, 5, 6\}, \{4, 5, 6, 7\}, \{5, 6, 7, 8\}, \{6, 7, 8, 9\},
\{7, 8, 9, 10\}, \{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\}, \{4, 5, 6\}, \{5, 6, 7\}, \{6, 7, 8\}, \{7, 8\},
\{8, 9\}, \{9, 10\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}, \emptyset\}$
and
$I_2 = \{\{1, 2, 3\}, \{6, 7, 9\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{6, 7\}, \{6, 9\}, \{7, 9\}, \{1\}, \{2\}, \{3\}, \{6\}, \{7\}, \{9\}\}$

a) Is $(E, I_1)$ an independence system?
b) Is $(E, I_2)$ an independence system?

(2+2 P.)

Exercise 2: Consider the following system: We are given a ground set, consisting of
circles with uniform radius in the plane. For an example:

We say that a selection of some of these circles is independent, iff no two of them intersect.
For example, $\{C, D\}$ is independent, but $\{E, F\}$ is dependent.
a) Prove that this system is an independence system for any given ground set of circles.

b) Find a nonempty example of circles for which the system is a matroid (and prove it).

c) Find an example of circles for which the system fulfills these criteria:
   • It is not a matroid (prove it).
   • All bases have the same size $k$, with $k \geq 3$. 

   (2+2+2 P.)