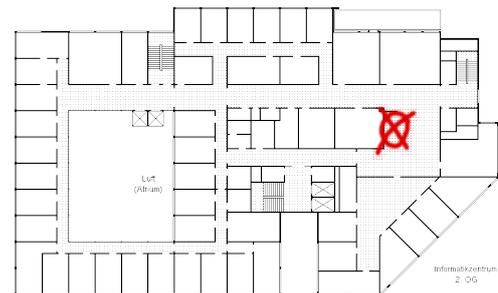


Dr. Alexander Kröller  
 Christiane Schmidt

## Combinatorial Algorithms homework set #1, 04. 11. 2010

Solutions are due thursday, November 18th,  
 2010, either

- at the beginning of the tutorial in room IZ161 or
- until 16:40 in the cupboard for handing in practice sheets.



**Please put your name on all pages!**

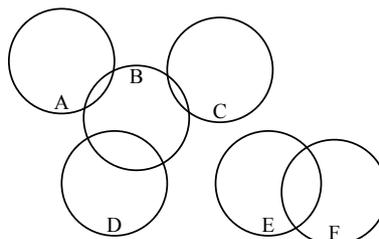
**Exercise 1 (Independence Systems):** Let  $E = \{1, \dots, 10\}$  and  
 $\mathcal{I}_1 = \{\{1, 2, 3, 4\}, \{2, 3, 4, 5\}, \{3, 4, 5, 6\}, \{4, 5, 6, 7\}, \{5, 6, 7, 8\}, \{6, 7, 8, 9\},$   
 $\{7, 8, 9, 10\}, \{1, 2, 3\}, \{2, 3, 4\}, \{3, 4, 5\}, \{4, 5, 6\}, \{5, 6, 7\}, \{6, 7, 8\},$   
 $\{7, 8, 9\}, \{8, 9, 10\}, \{1, 2\}, \{2, 3\}, \{3, 4\}, \{4, 5\}, \{5, 6\}, \{6, 7\}, \{7, 8\},$   
 $\{8, 9\}, \{9, 10\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}, \emptyset\}$   
 and

$\mathcal{I}_2 = \{\{1, 2, 3\}, \{6, 7, 9\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{6, 7\}, \{6, 9\}, \{7, 9\}, \{1\}, \{2\}, \{3\}, \{6\}, \{7\}, \{9\}\}$

- a) Is  $(E, \mathcal{I}_1)$  an independence system?
- b) Is  $(E, \mathcal{I}_2)$  an independence system?

**(2+2 P.)**

**Exercise 2:** Consider the following system: We are given a ground set, consisting of circles with uniform radius in the plane. For an example:



We say that a selection of some of these circles is *independent*, iff no two of them intersect. For example,  $\{C, D\}$  is independent, but  $\{E, F\}$  is dependent.

- a) Prove that this system is an independence system for any given ground set of circles.
- b) Find a nonempty example of circles for which the system is a matroid (and prove it).
- c) Find an example of circles for which the system fulfills these criteria:
- It is **not** a matroid (prove it).
  - All bases have the same size  $k$ , with  $k \geq 3$ .

**(2+2+2 P.)**