Algorithms for context prediction in Ubiquitous Systems

Exercise 3 – Markov prediction approaches – 08.12.2008

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Algorithms for context prediction – Exercise 3

Exercise 3.1)

To solve the evaluation task for a HMM, the forward algorithm can be utilised:

```
1 initialise t ← 0, p_ij, b_jk, V^T, α_j(o)
2     for t ← t + 1
3         α_j(t) ← b_jk v(t) Σ_{i=1}^{c} α_i(t − 1) p_ij
4     until t = T
5 return P(V^T) ← α_0(T) for the final state
6 end
```

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Exercise 3.1)

Assume a HMM with the following transition probabilities:

$$a_{ij} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0.2 & 0.3 & 0.1 & 0.4 \\ 0.2 & 0.5 & 0.2 & 0.1 \\ 0.8 & 0.1 & 0 & 0.1 \end{pmatrix} \tag{1}$$

$$b_{jk} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.3 & 0.4 & 0.1 & 0.2 \\ 0 & 0.1 & 0.1 & 0.7 & 0.1 \\ 0 & 0.5 & 0.2 & 0.1 & 0.2 \end{pmatrix} \tag{2}$$

- calculate the probability that this model generates the sequence $V^T = \{v_1, v_3, v_2, v_0\}$, supposing that the initial state is ω_1.

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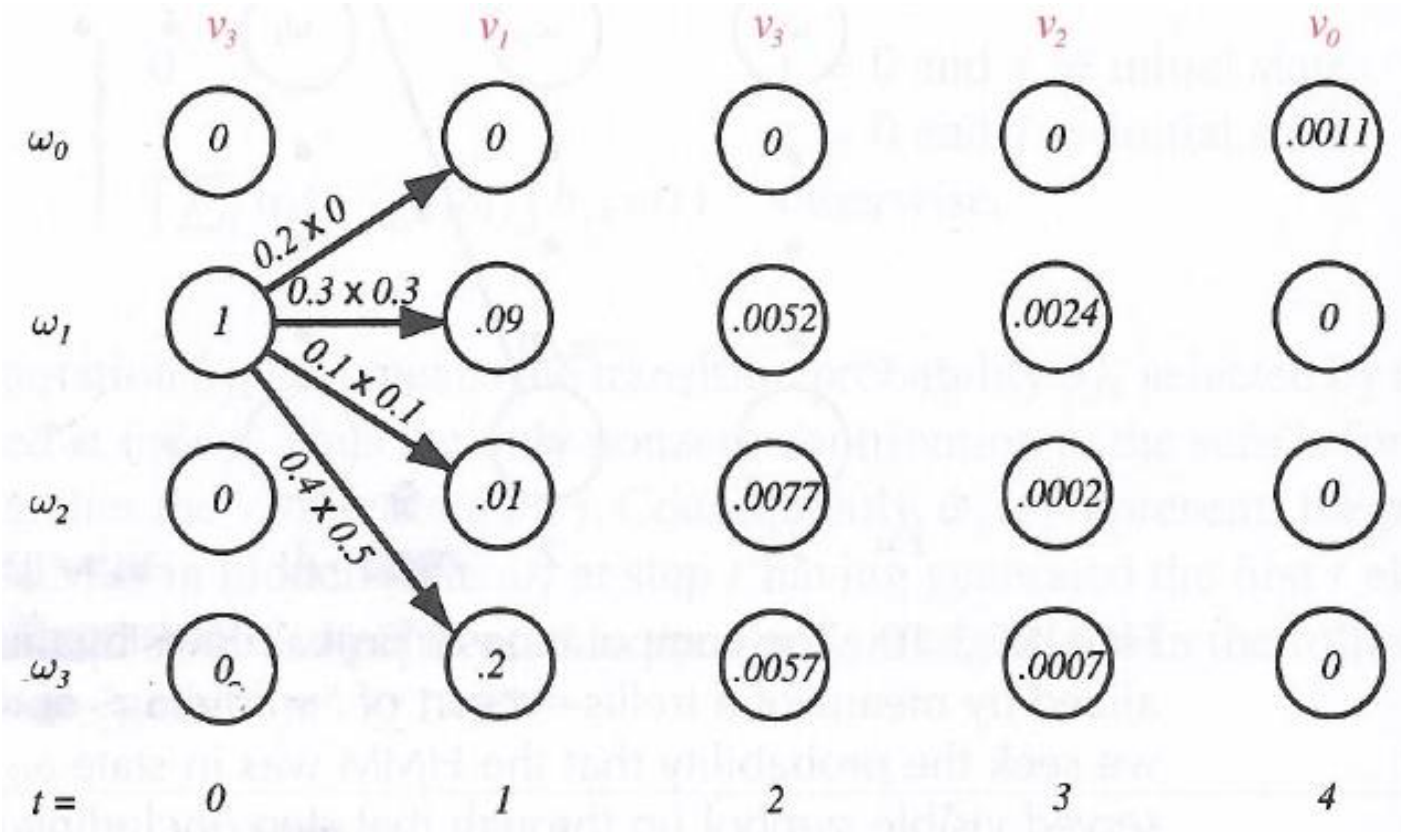
Exercise 3.1) – Solution

- Following the forward algorithm $(\alpha_j(t) \leftarrow b_{jk} v(t) \sum_{i=1}^{c} \alpha_i(t-1) p_{ij})$, the probability to create the sequence V^T is 0.001:



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Exercise 3.1) – Solution

$$
\begin{aligned}
P(v_1|\Theta) &= (0.2 \cdot 0 + 0.3 \cdot 0.3 + 0.1 \cdot 0.1 + 0.4 \cdot 0.5) = 0.3 \\
P(v_1 v_3|\Theta) &= 0.9 \cdot (0.2 \cdot 0 + 0.3 \cdot 0.3 + 0.1 \cdot 0.1 + 0.4 \cdot 0.5) + \\
&\quad 0.01 \cdot (0.2 \cdot 0 + 0.5 \cdot 0.4 + 0.2 \cdot 0.1 + 0.1 \cdot 0.2) + \\
&\quad 0.2 \cdot (0.8 \cdot 0 + 0.1 \cdot 0.4 + 0 \cdot 0.1 + 0.1 \cdot 0.2) \\
&= 0.2844 \\
P(v_1 v_3 v_2|\Theta) &= \ldots \\
P(v_1 v_3 v_2 v_0|\Theta) &= 0.001
\end{aligned} \tag{3}
$$

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Exercise 3.2)

For the decoding problem we have introduced the HMM decoding algorithm:

```
1  initialise path ← {}, t ← 0
2      for t ← t + 1
3          j ← j + 1
4          for j ← j + 1
5              α_j(t) ← b_jk v(t) Σ_{i=1}^{c} α_i(t − 1) p_ij
6          until j = c
7          j' ← arg max_j α_j(t)
8          append ω_j' to path
9      until t = T
10 return path
11 end
```

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Exercise 3.2)

- Calculate the path generated by the deconding algorithm.
- Is the path a legal path in the HMM?

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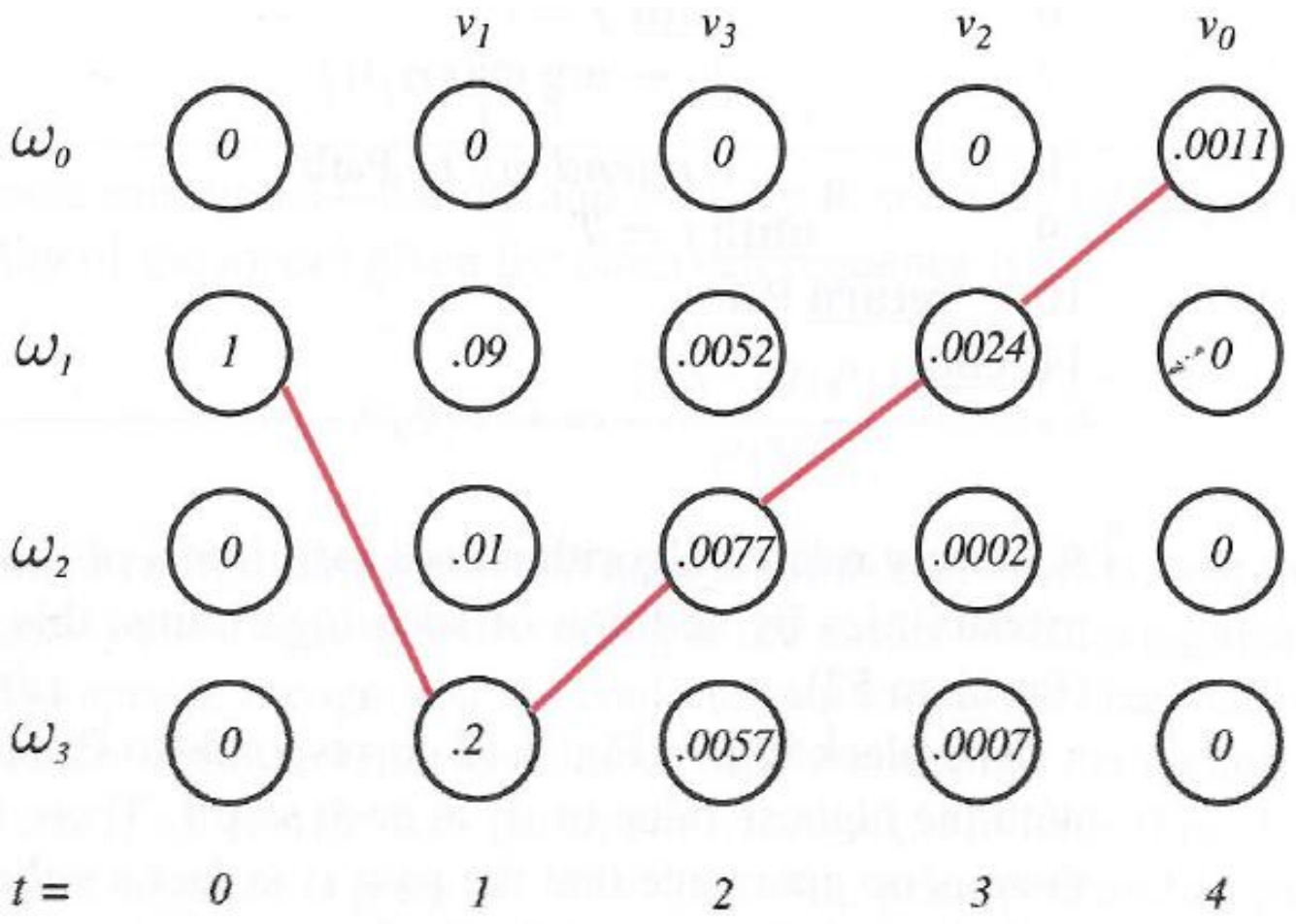
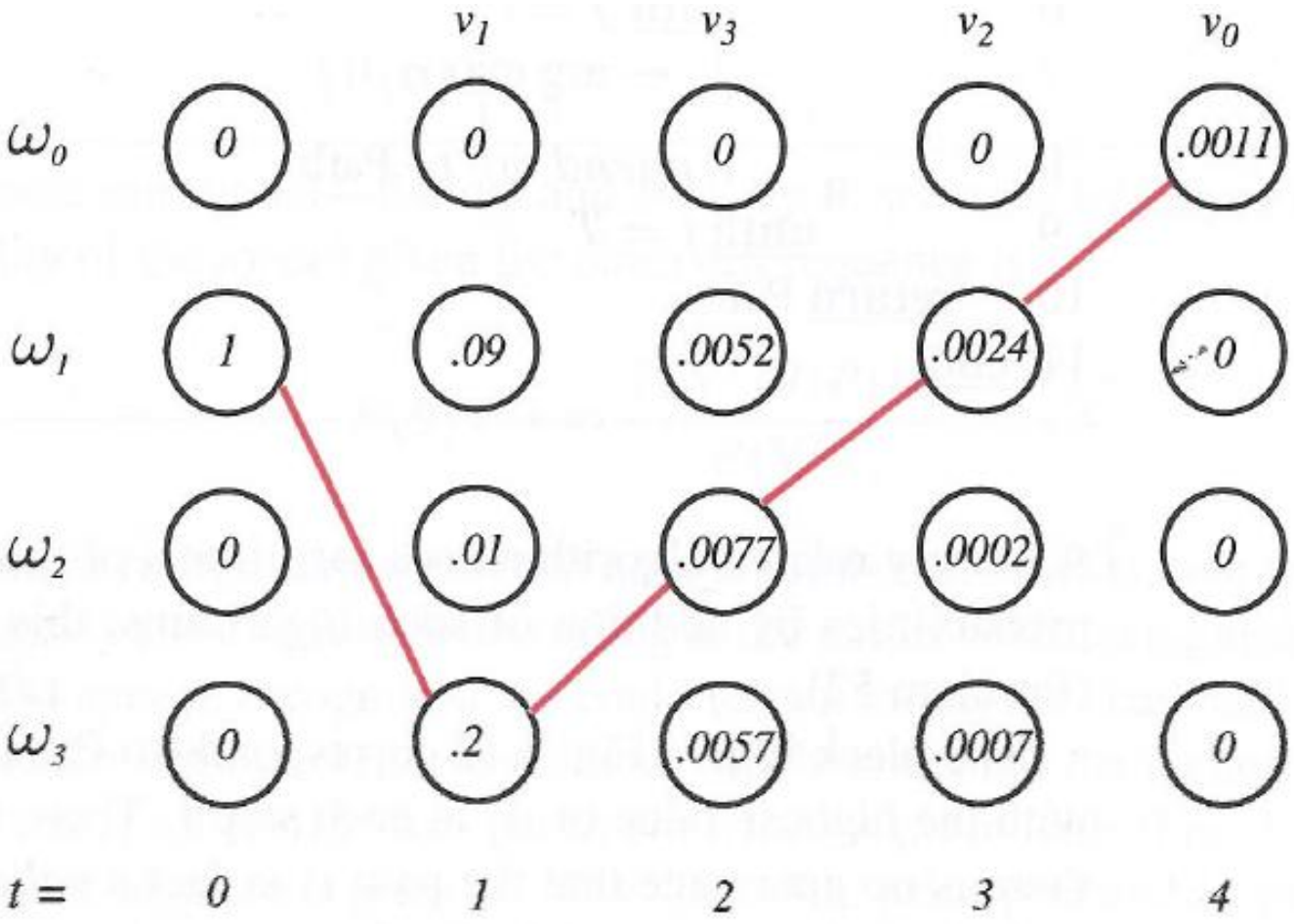
Exercise 3.2) – solution



Algorithms for context prediction – Exercise 3

Exercise 3.2) – solution



- The path from ω_3 to ω_2 is impossible.

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Exercise 3.3)

- Derive and explain the properties of the CRF prediction approach in the table

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Exercise 3.3)

	IPAM	ONISI	Markov	CRF
Numeric Contexts	yes	no	yes	
Non-numeric Contexts	yes	yes	yes	
Complexity	$O(k)$	( )	$O(C^2)$	
Learning ability	(no)	yes	yes	
Approximate matching	no	no	no	
Multi-dim. TS	(no)	(no)	(no)	
Discrete data	yes	yes	yes	
Variable length patterns	no	yes	no	
Multi-type TS	yes	no	(no)	
Continuous data	no	no	no	
Pre-processing	$O(k)$	–	$O(k)$	
Context durations	no	no	no	
Continuous time	no	no	yes	

Algorithms for context prediction – Exercise 3

Exercise 3.3) – solution

	IPAM	ONISI	Markov	CRF
Numeric Contexts	yes	no	no	no
Non-numeric Contexts	yes	yes	yes	yes
Complexity	$O(k)$	( )	$O(C^2)$	$O(C^2)$
Learning ability	(no)	yes	yes	yes
Approximate matching	no	no	no	no
Multi-dim. TS	(no)	(no)	(no)	(no)
Discrete data	yes	yes	yes	yes
Variable length patterns	no	yes	no	(yes)
Multi-type TS	yes	no	(no)	(no)
Continuous data	no	no	no	no
Pre-processing	$O(k)$	–	$O(k)$	$O(k)$
Context durations	no	no	no	no
Continuous time	no	no	yes	yes