Approximation Algorithms
Chapter 6: Review and Relay Placement

Sándor P. Fekete

Algorithms Division
Department of Computer Science
TU Braunschweig

1. Introduction
2. Review
3. Extra Packing: Dispersion
4. Extra Tours: Lawn Mowing
5. Relay Placement
6. Coordinated Motion Planning

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Review: Matching & Vertex Cover

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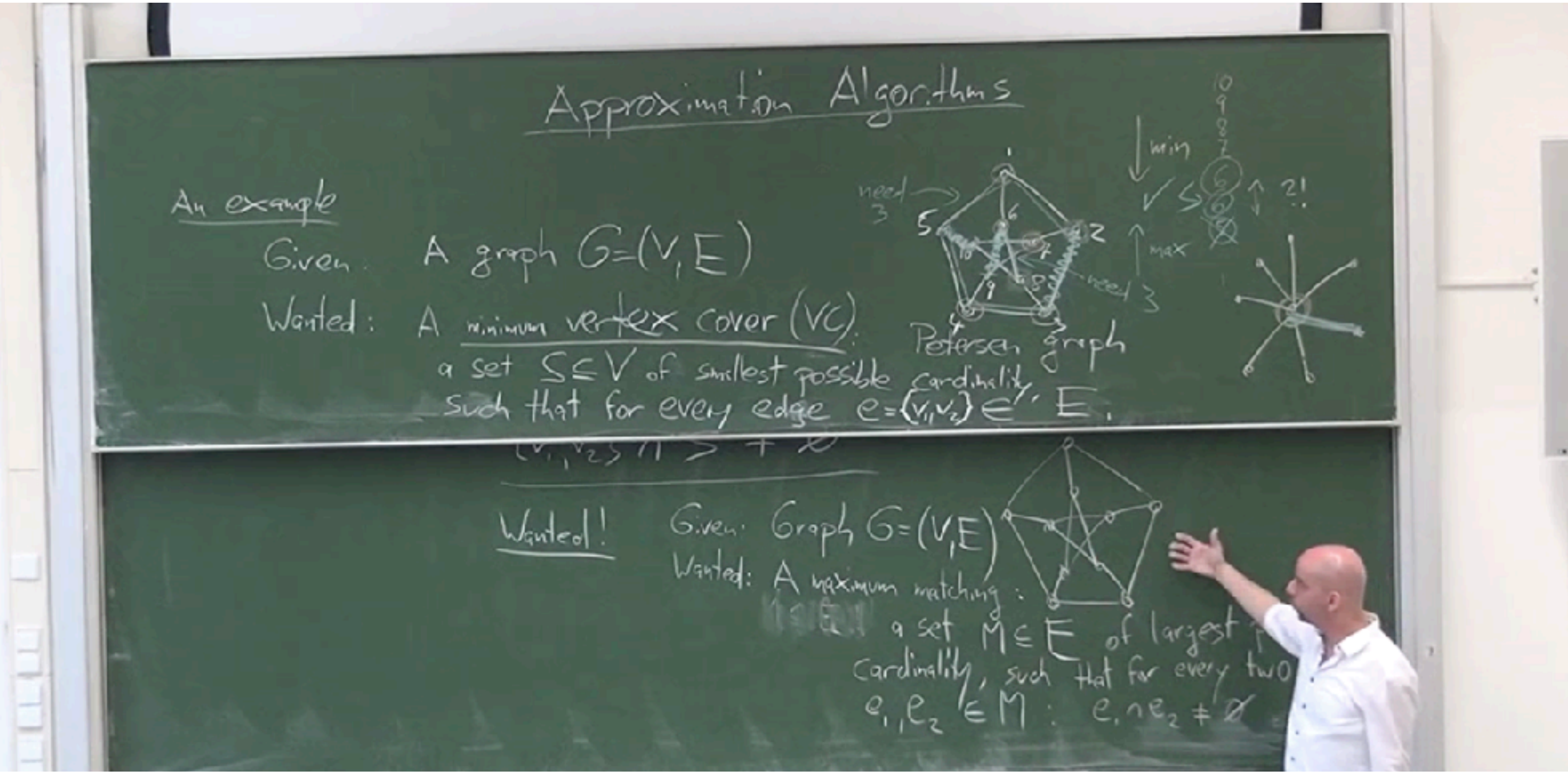

Review: Approximation Algorithms

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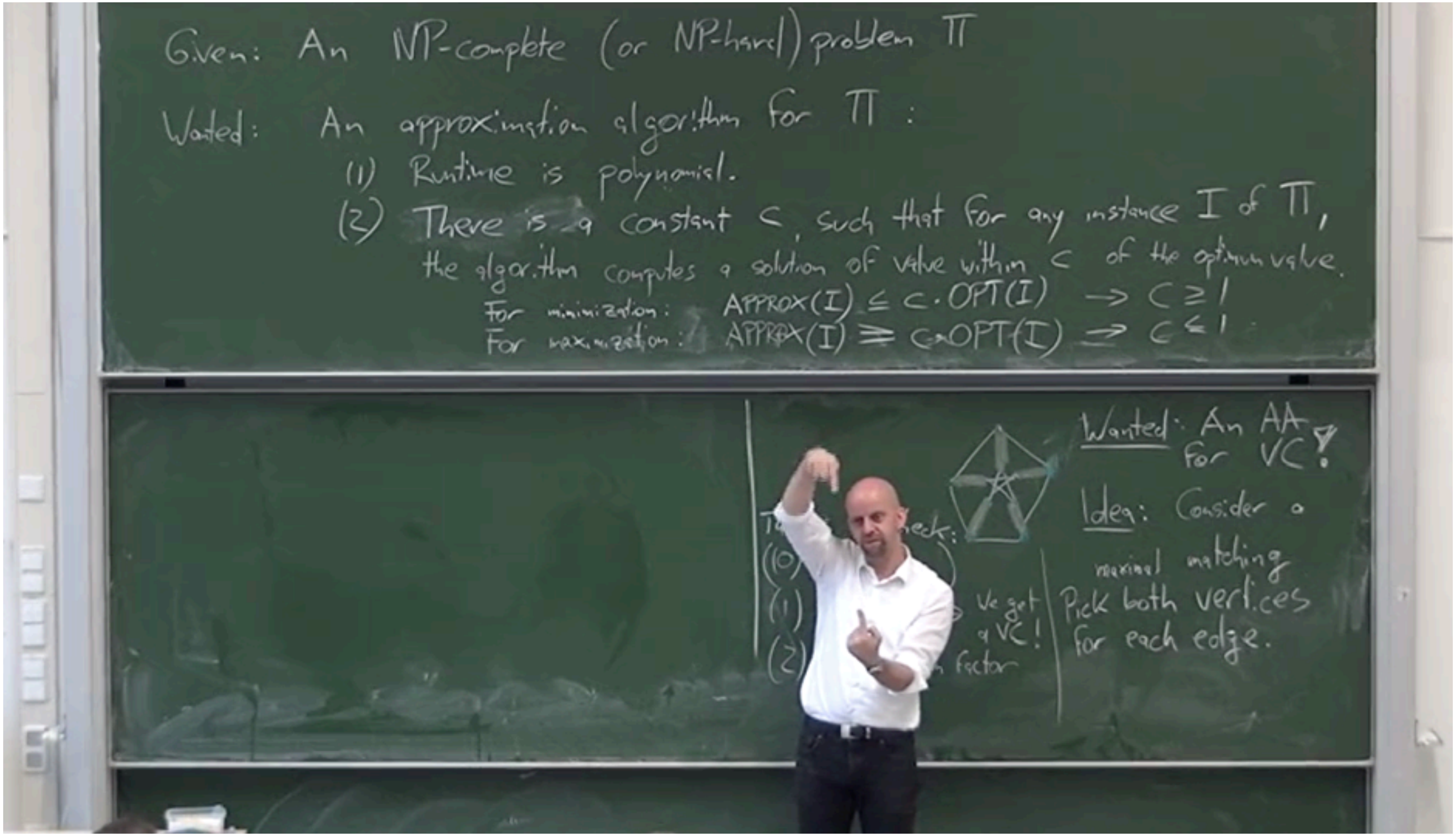
Review: Approximation Algorithms



Review: Set Cover

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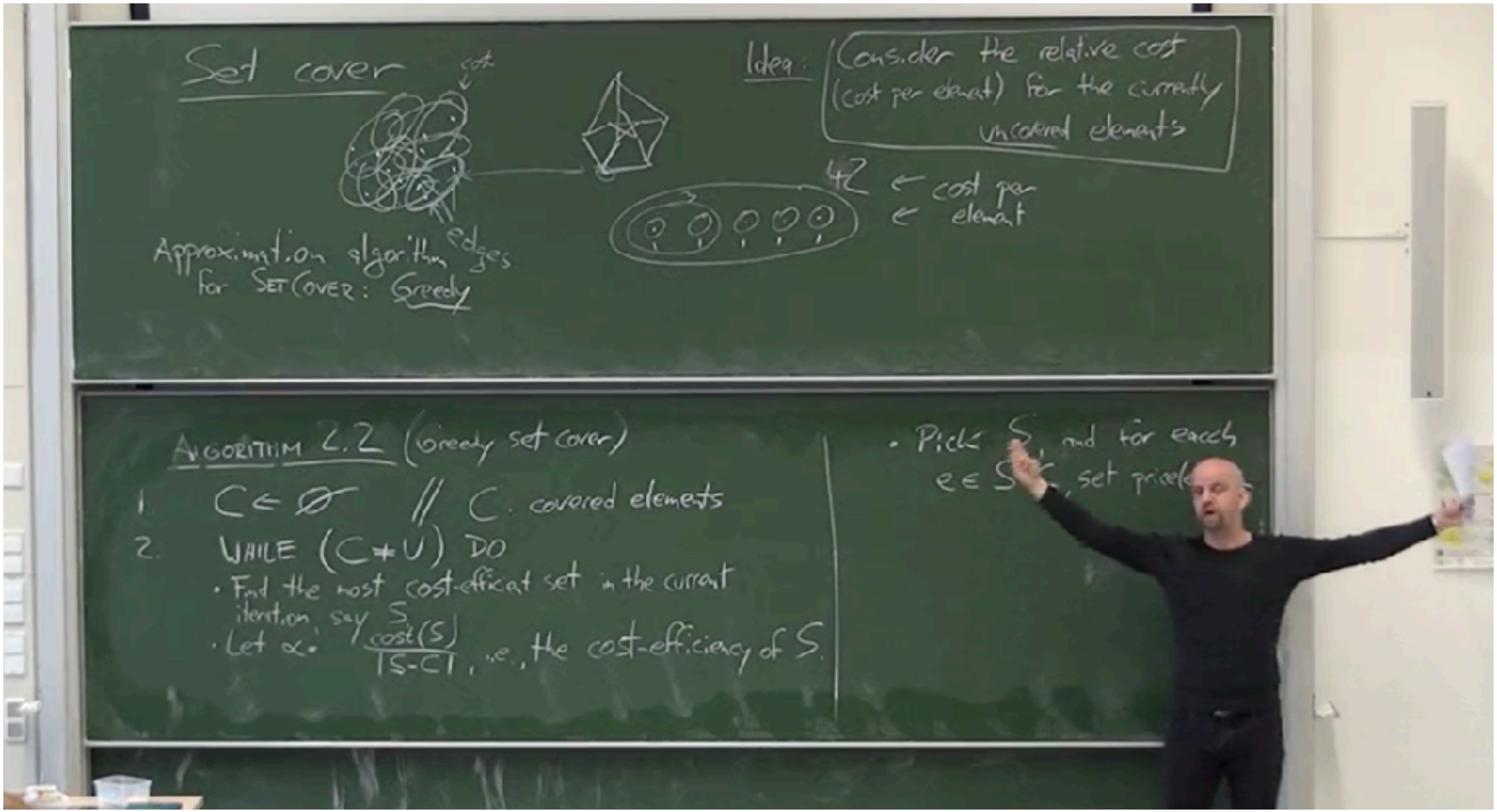

Review: Maximum Coverage

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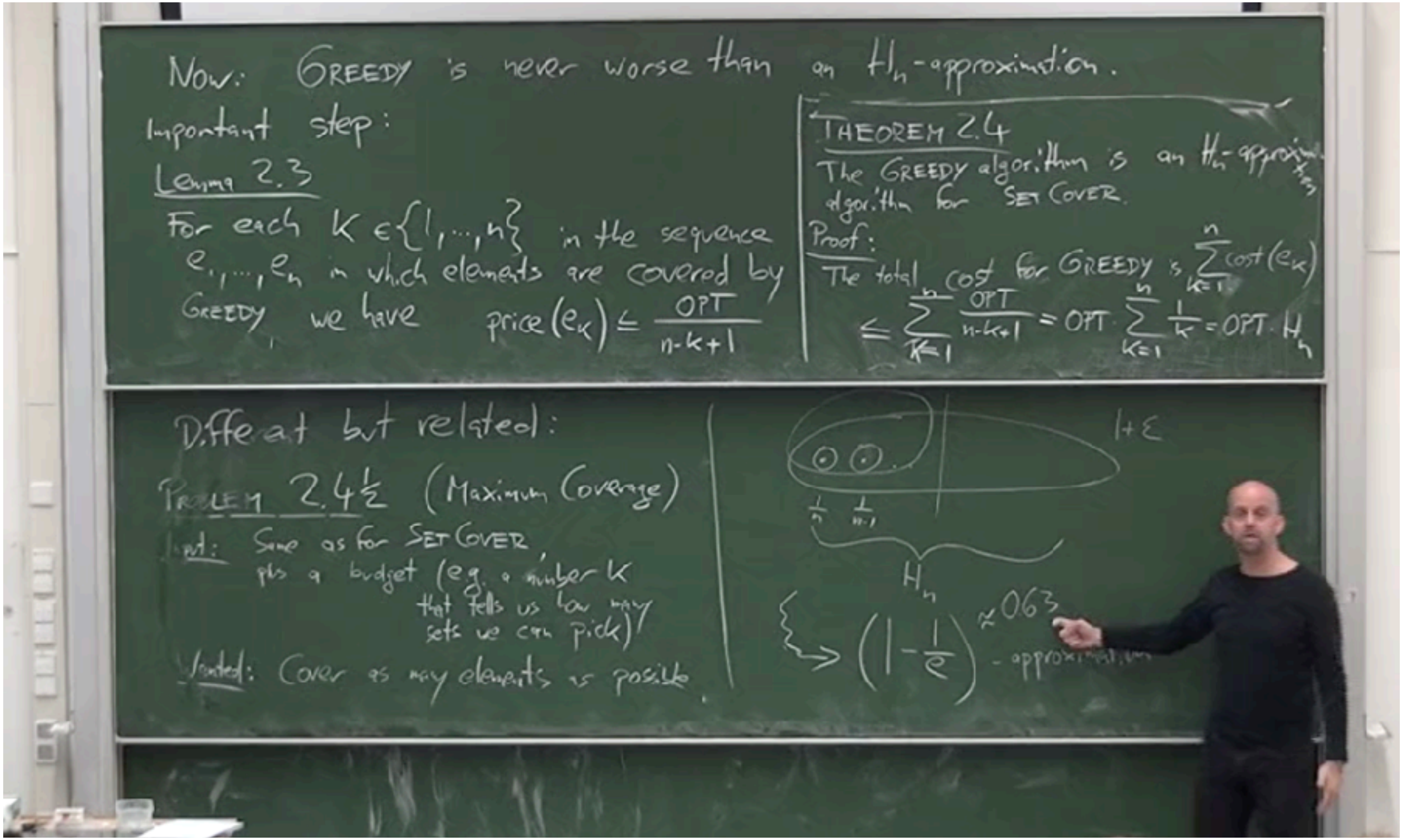

Review: Shortest Superstring

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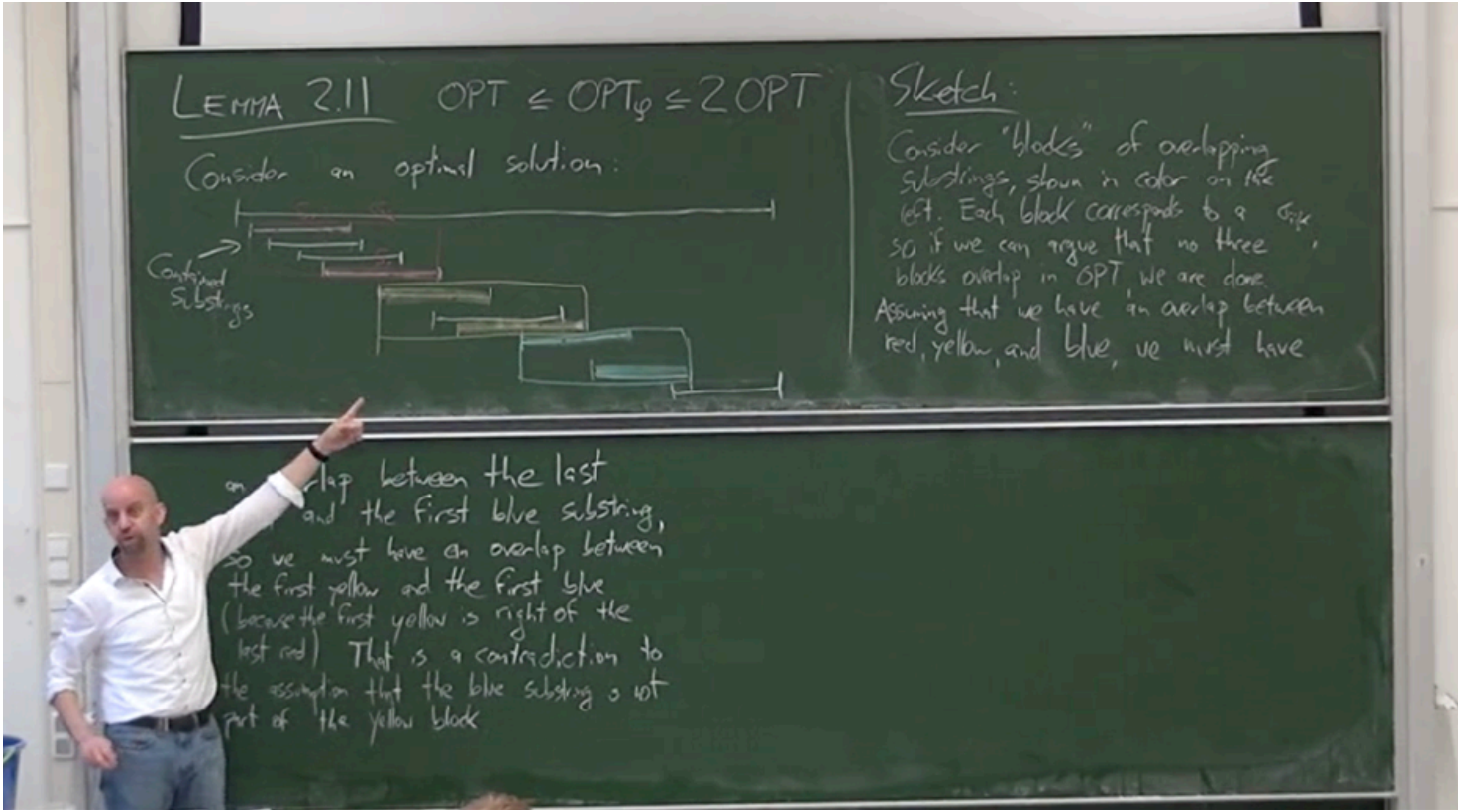

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Review: Packing

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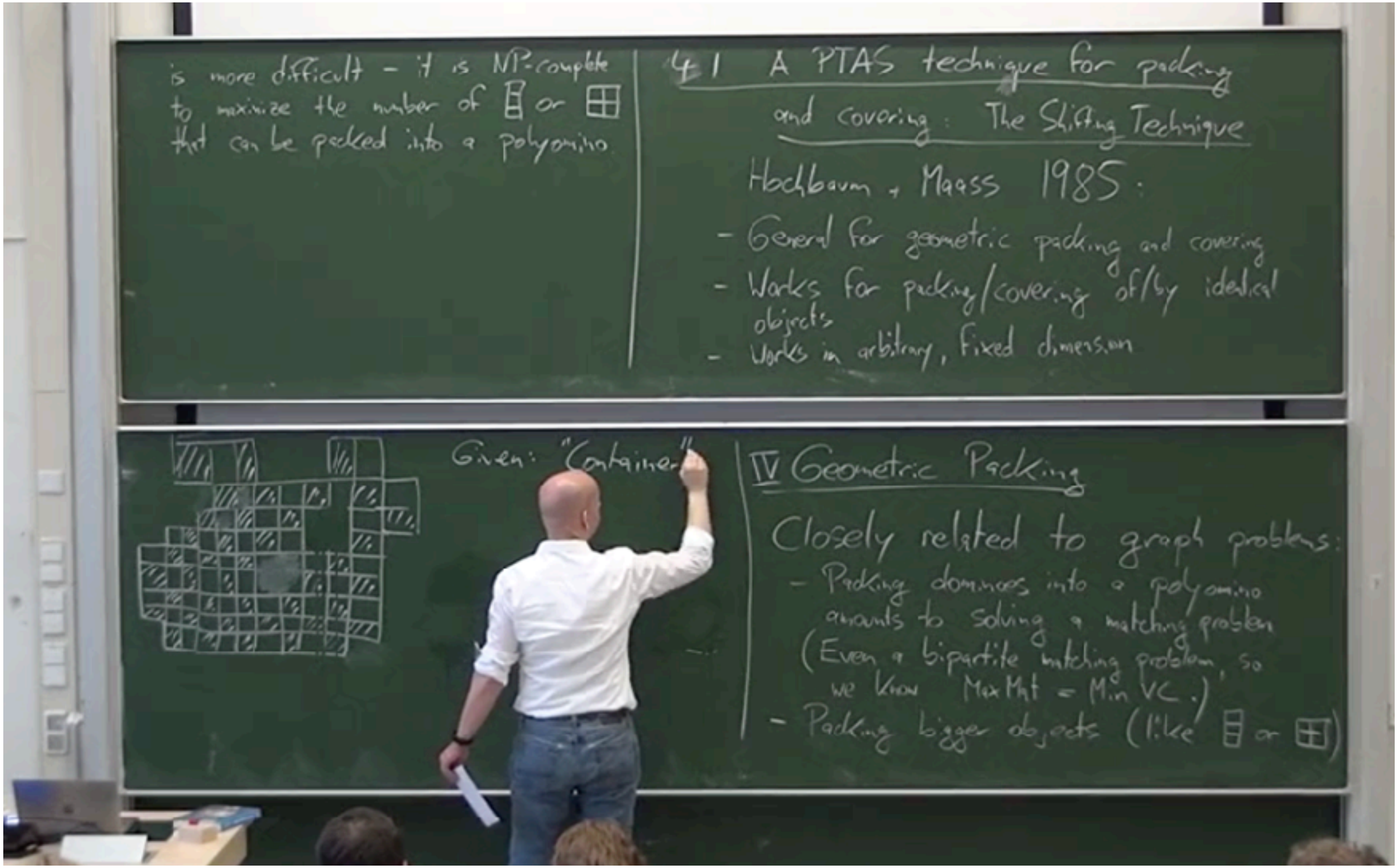

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Review: Tour Problems

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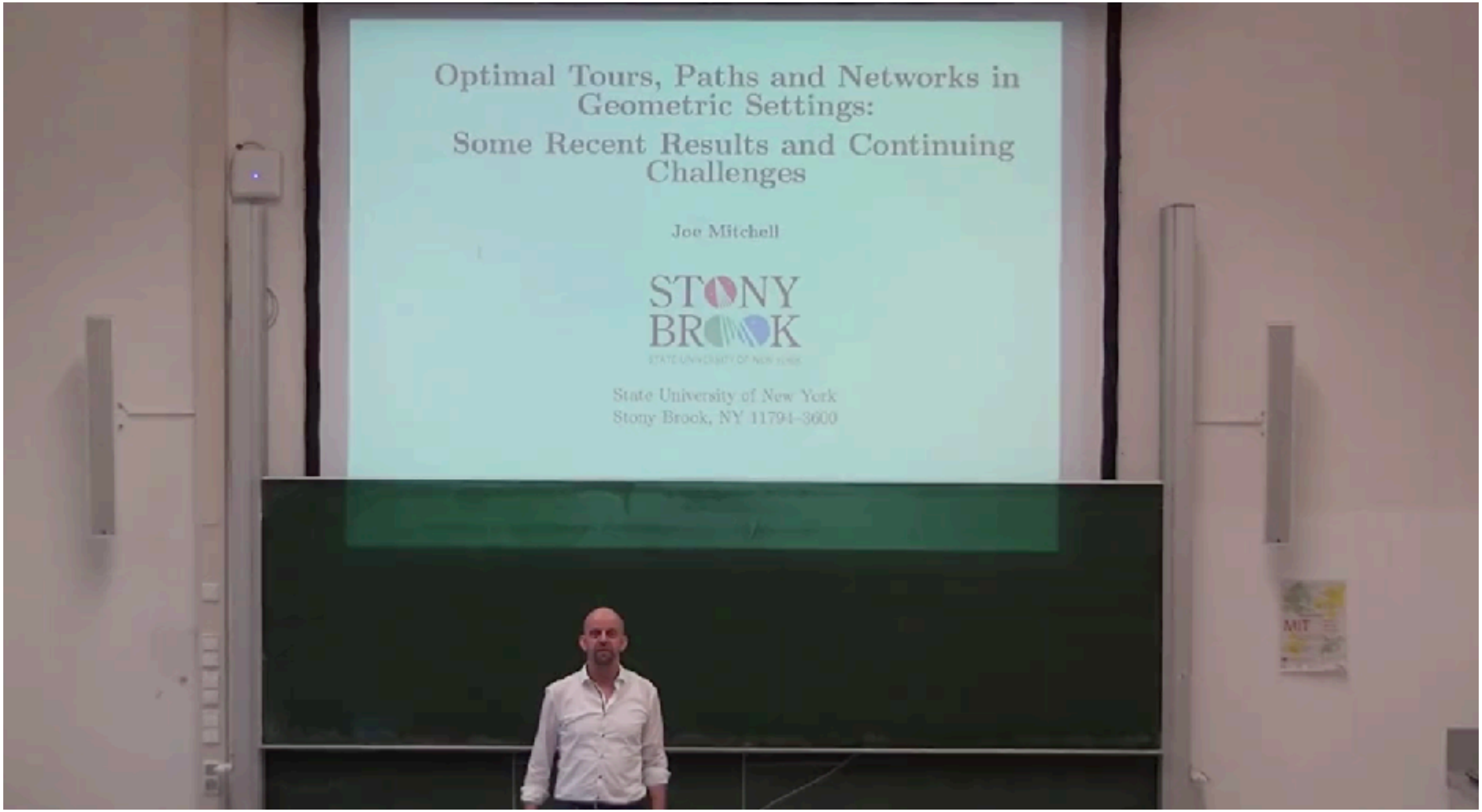

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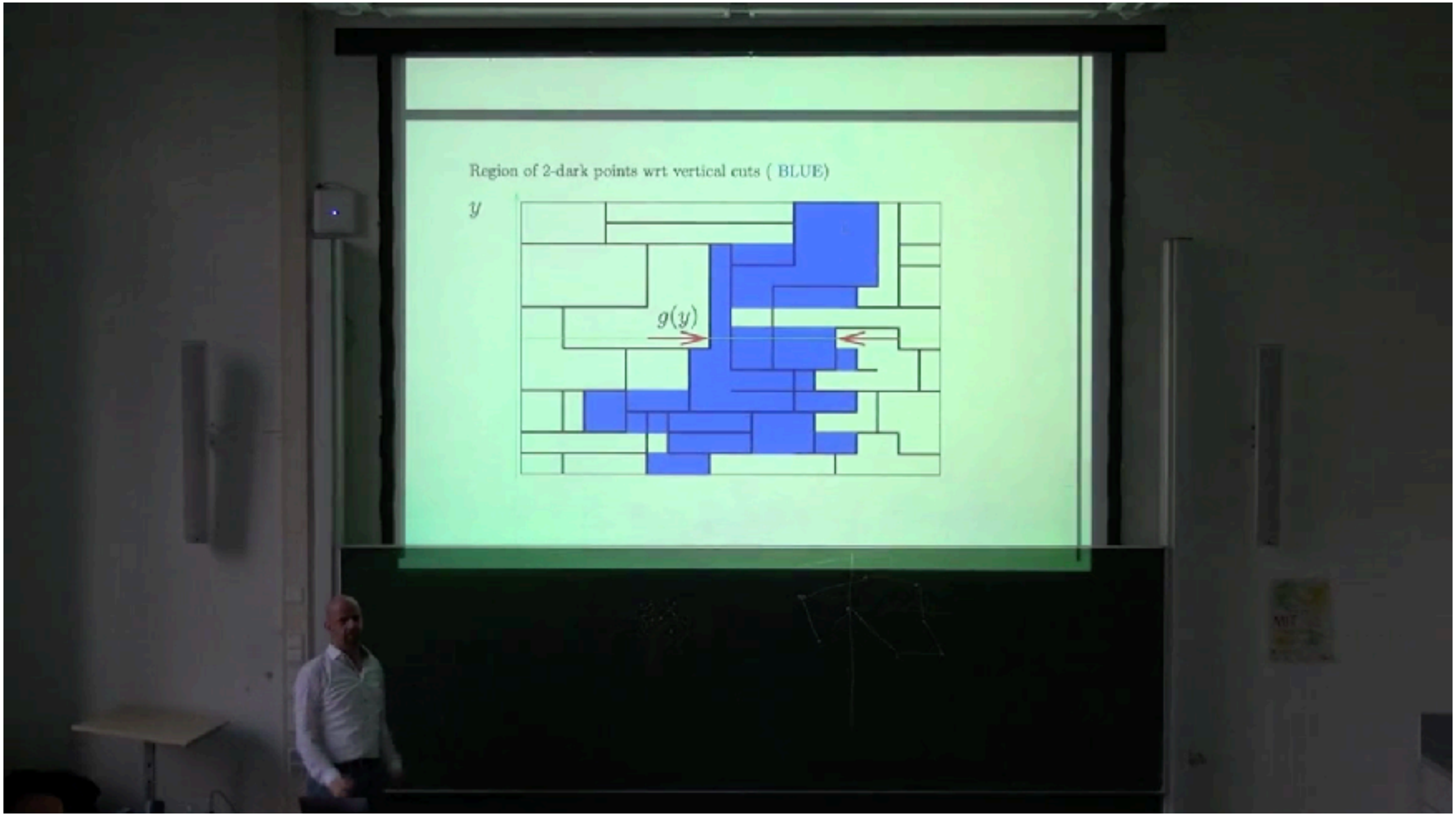

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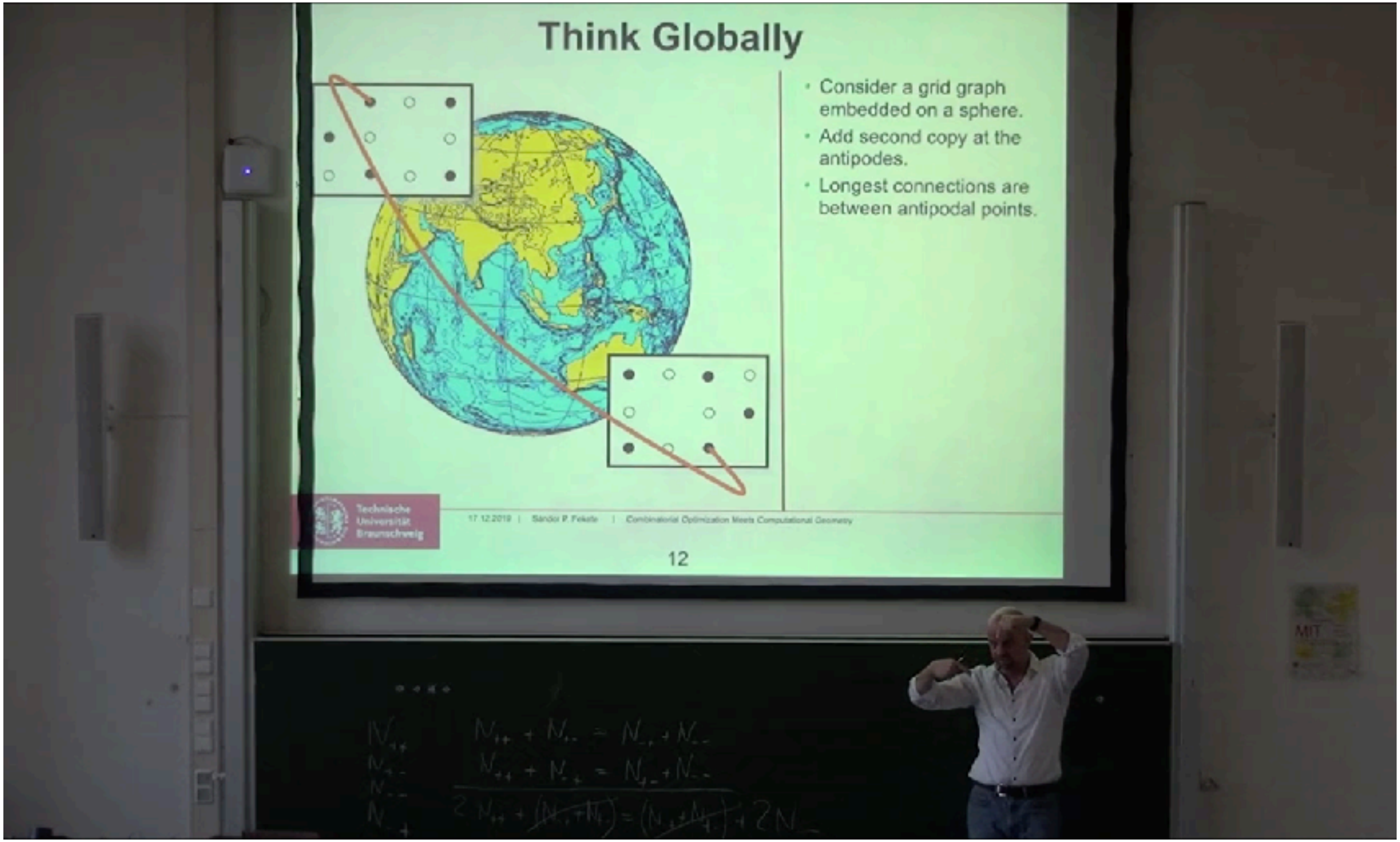
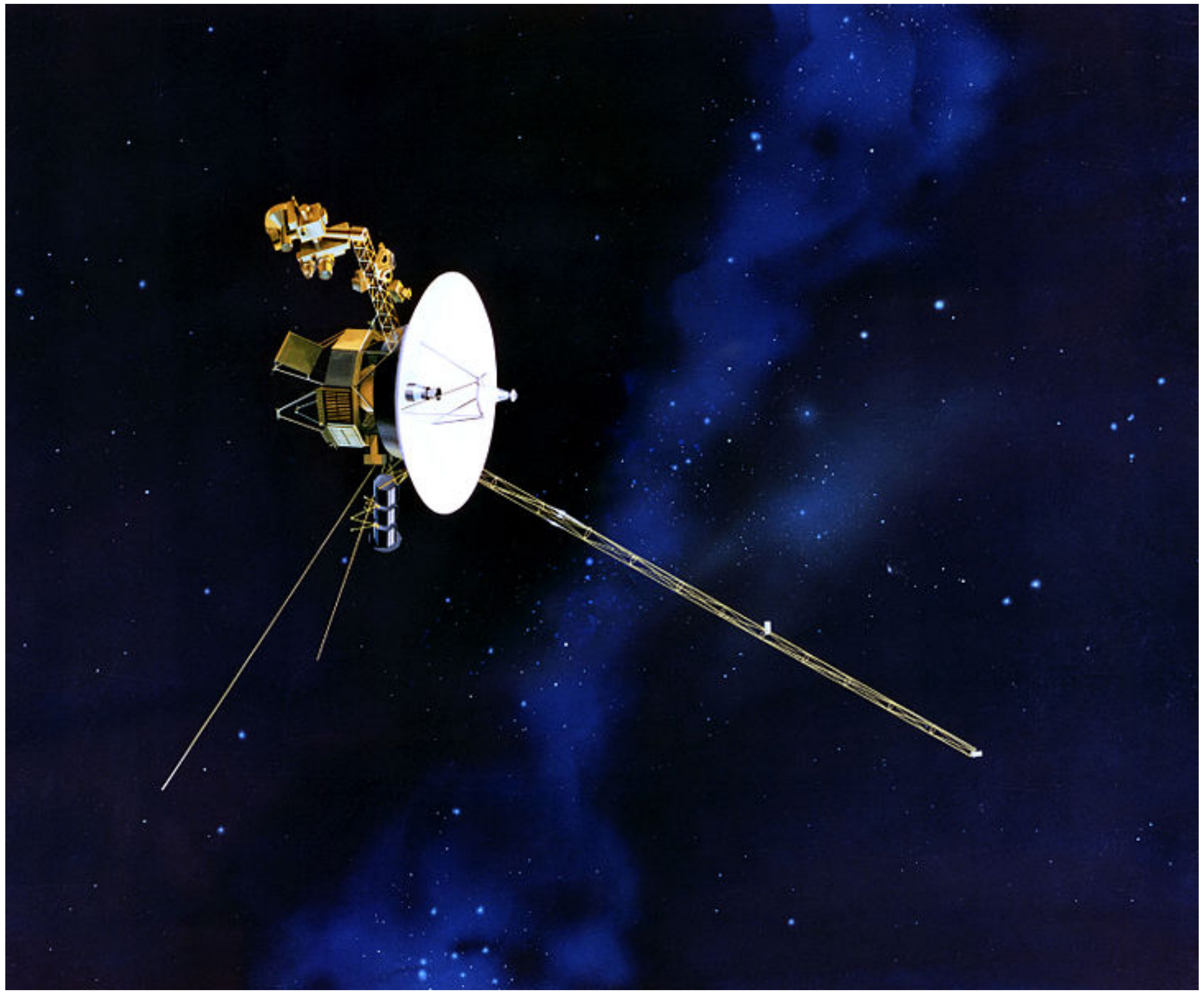


The Freeze-Tag Problem: How to Wake Up a Swarm of Robots

Estie Arkin[1]
Michael Bender[1]
Sándor Fekete[2]
Joe Mitchell[1]
Martin Skutella[3]

[1] University at Stony Brook
[2] TU Braunschweig
[3] TU Berlin

Motivation



- Highly focused antennas.
- Expensive rotations.

How can we quickly distribute information from one satellite to all others?

d.krupke@tu-bs.de
Institute of Operating Systems and Computer Networks

Technische Universität Braunschweig

Student Presentation

Minimum Scan Cover with Angular Transition Costs



Sándor Fekete

Linda Kleist

Dominik Krupke

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Dispersion (1)

Algorithmica (2001) 30: 451–470
DOI: 10.1007/s00453-001-0022-x

Algorithmica

Approximation of Geometric Dispersion Problems[1]

C. Baur[2] and S. P. Fekete[3]

Abstract. We consider problems of distributing a number of points within a polygonal region P, such that the points are "far away" from each other. Problems of this type have been considered before for the case where the possible locations form a discrete set. Dispersion problems are closely related to packing problems. While Hochbaum and Maass [20] have given a polynomial-time approximation scheme for packing, we show that geometric dispersion problems cannot be approximated arbitrarily well in polynomial time, unless P = NP. A special case of this observation solves an open problem by Rosenkrantz et al. [31]. We give a $\frac{2}{3}$ approximation algorithm for one version of the geometric dispersion problem. This algorithm is strongly polynomial in the size of the input, i.e., its running time does not depend on the area of P. We also discuss extensions and open problems.

Key Words. Packing, Dispersion, Location problems, Geometric optimization, Bounds on approximation factors.

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Dispersion (2)

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PACK(k, L)
Input: A polygonal region P with n vertices, a parameter k, a parameter L.
Question: Can k many L-squares be packed into P?

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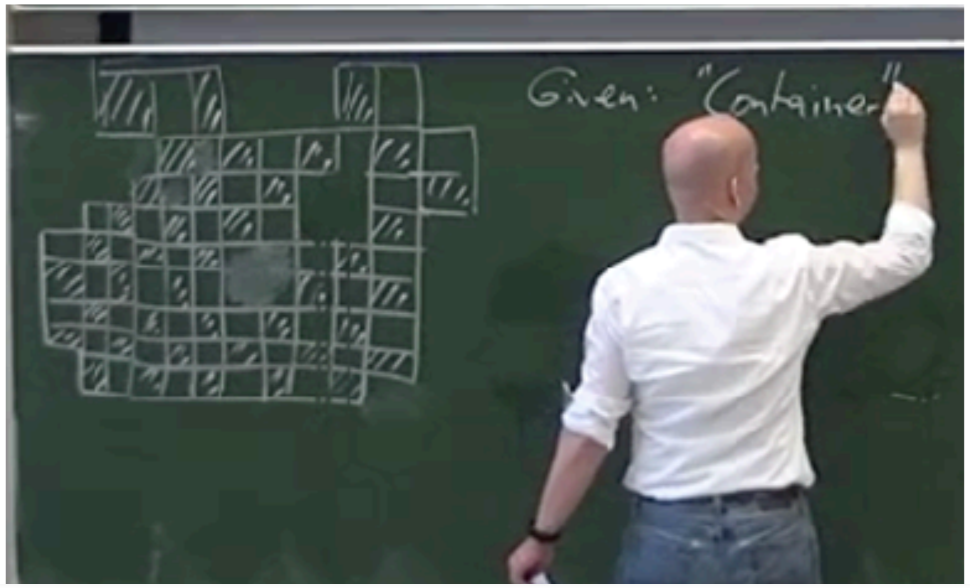

$\max_k$PACK(L)
Input: A polygonal region P with n vertices.
Task: Pack k many L-squares into P, such that k is as big as possible.

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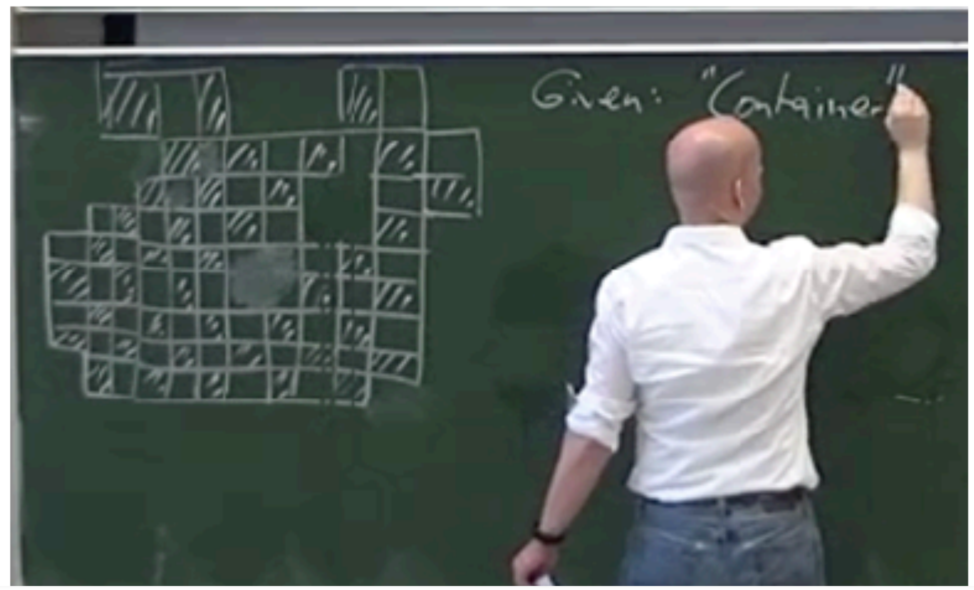

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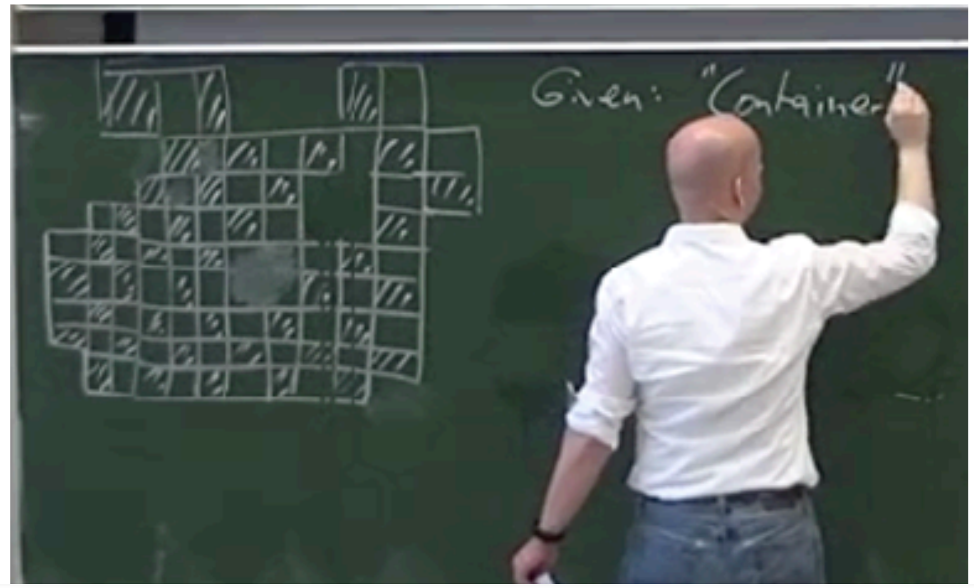

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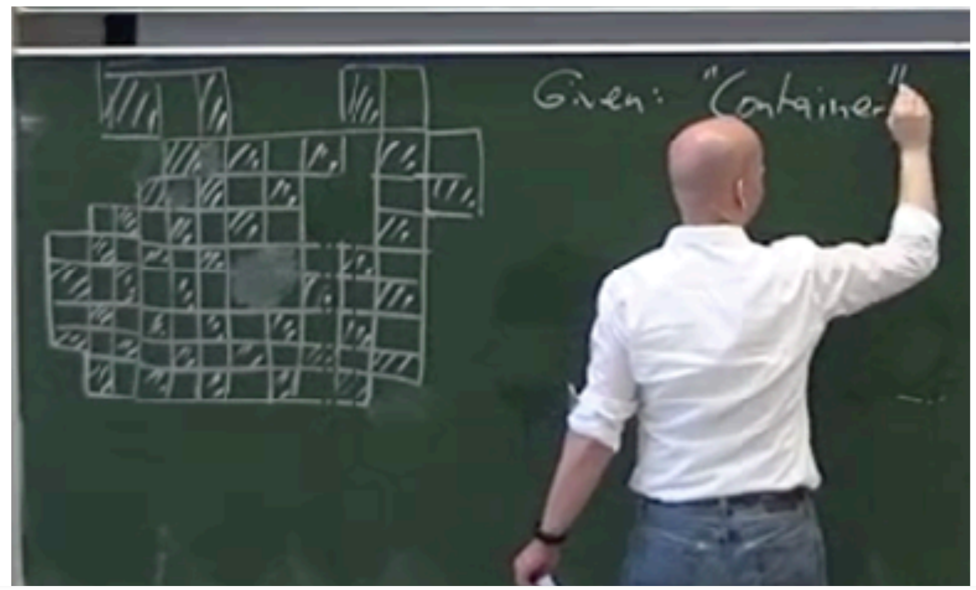

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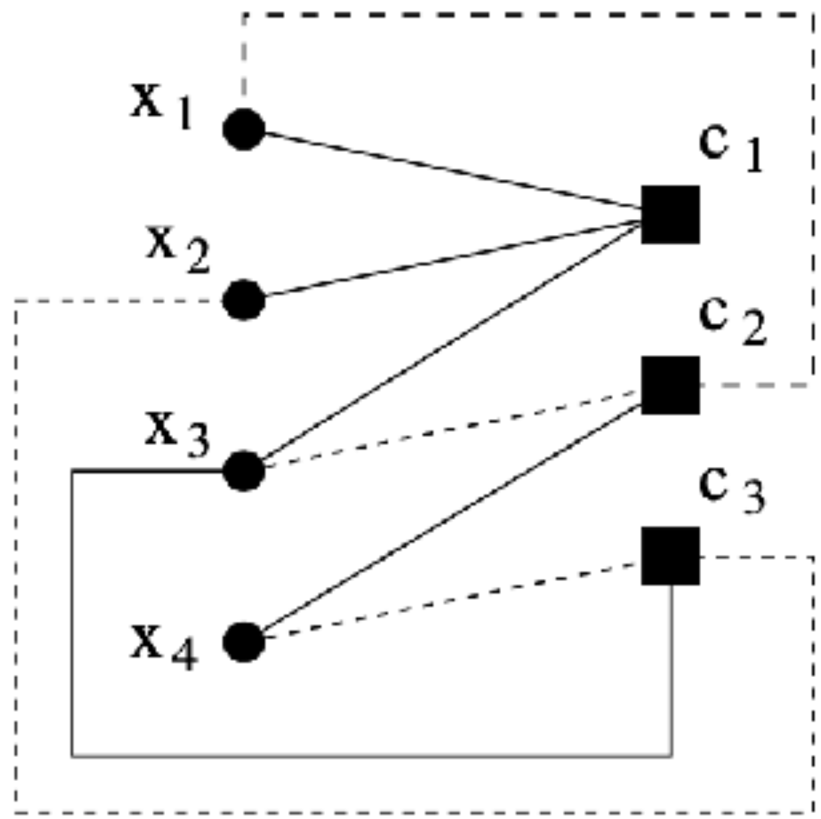

Fig. 1. The graph G_I representing the PLANAR 3SAT instance $(x_1 \vee x_2 \vee x_3) \wedge (\bar{x}_1 \vee \bar{x}_3 \vee x_4) \wedge (\bar{x}_2 \vee x_3 \vee \bar{x}_4)$. Edges are distinguished according to the logical parity of the corresponding literals.

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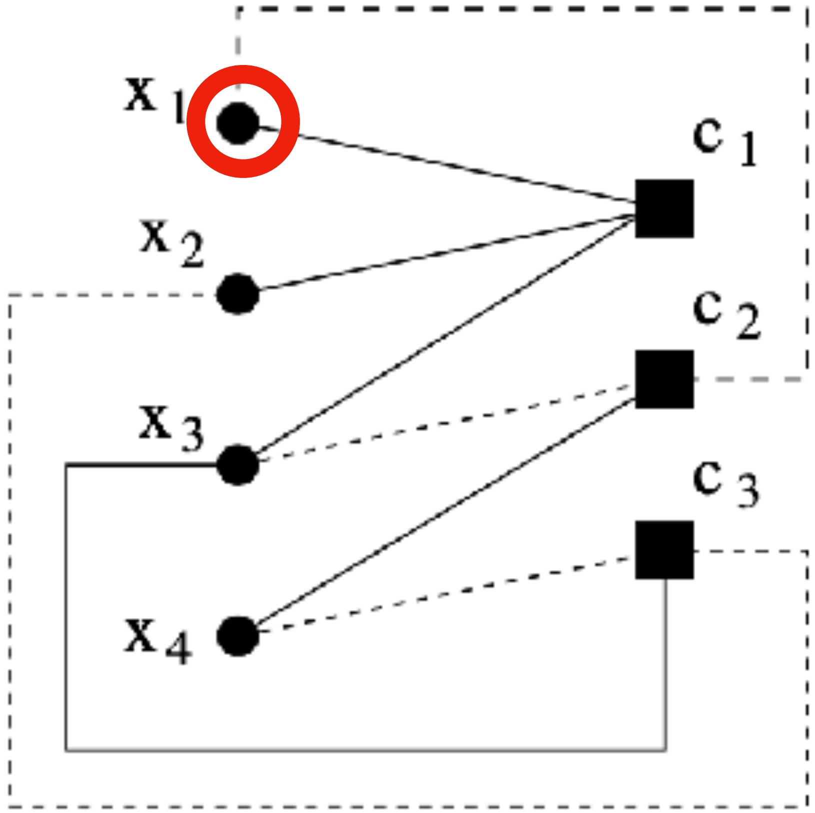

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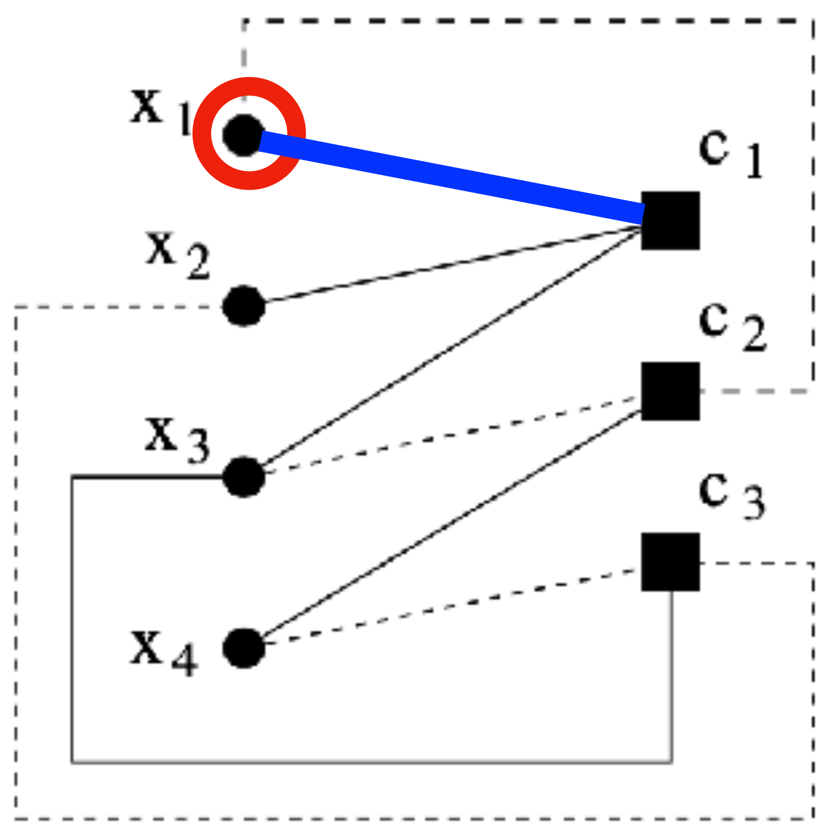

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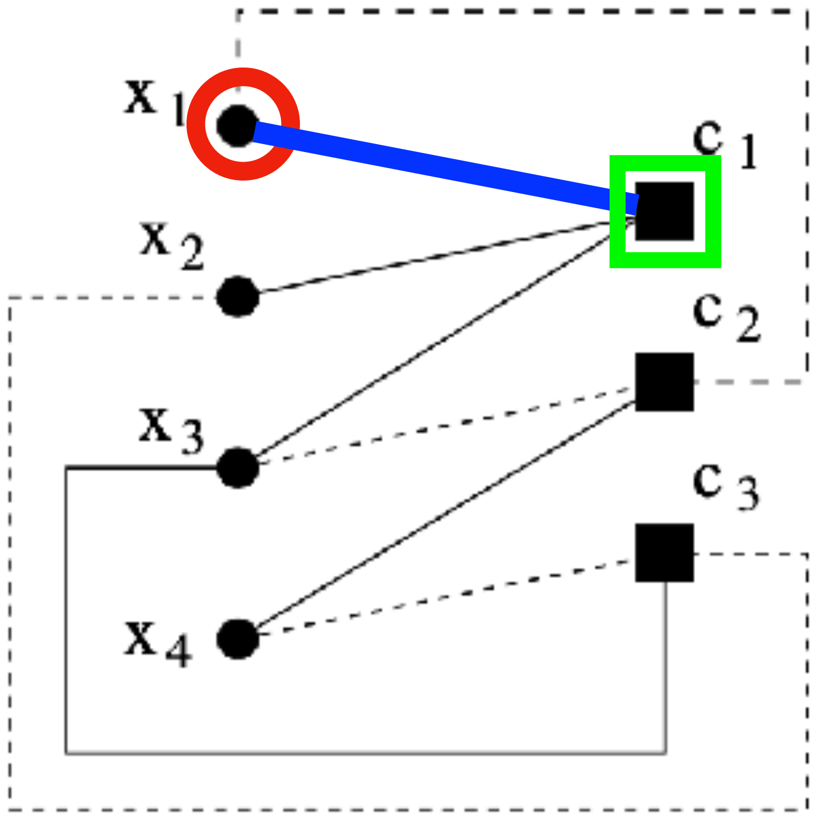

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Variables (1)

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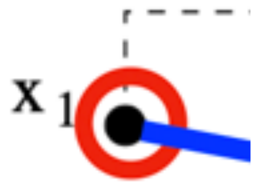

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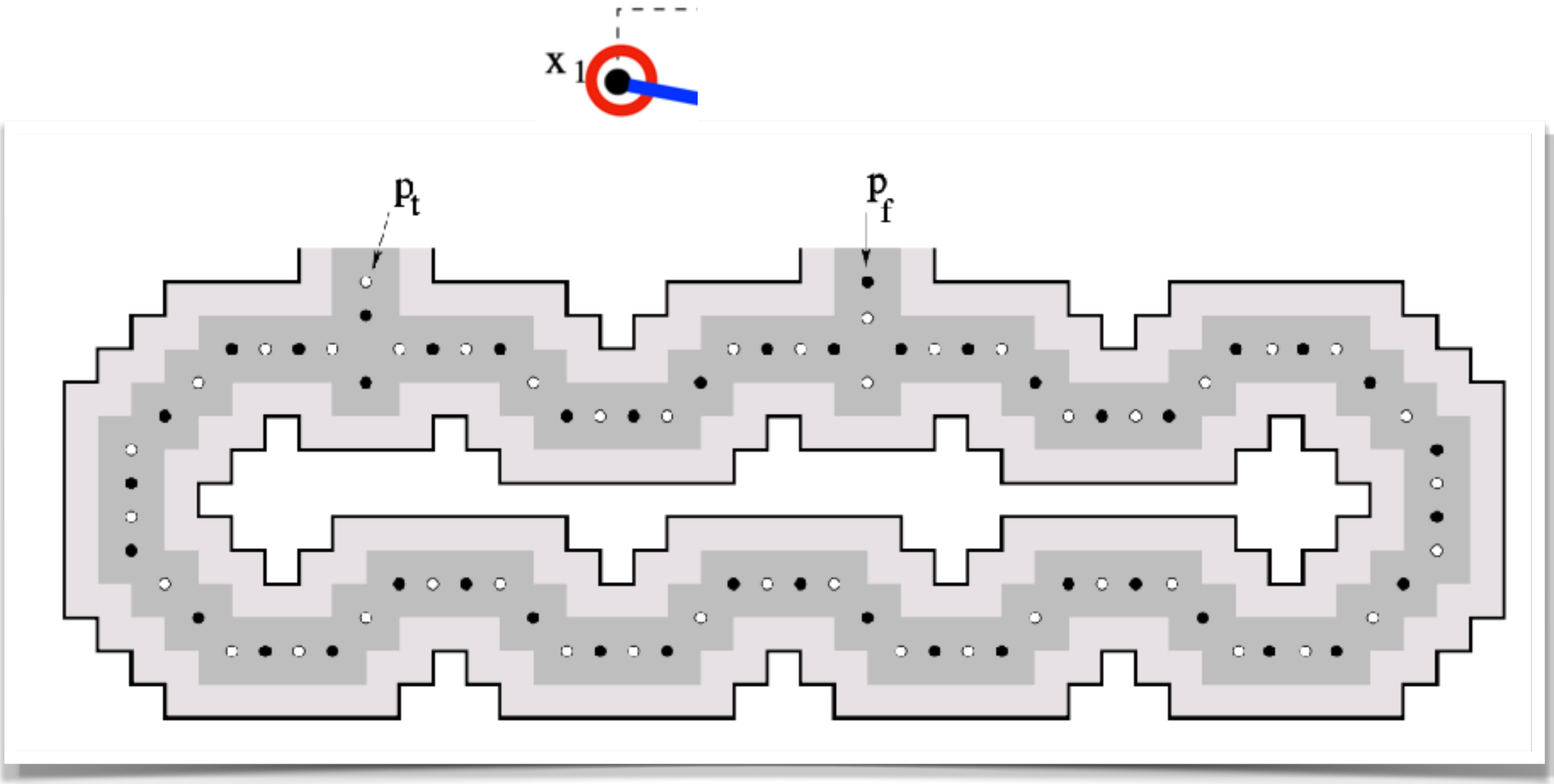
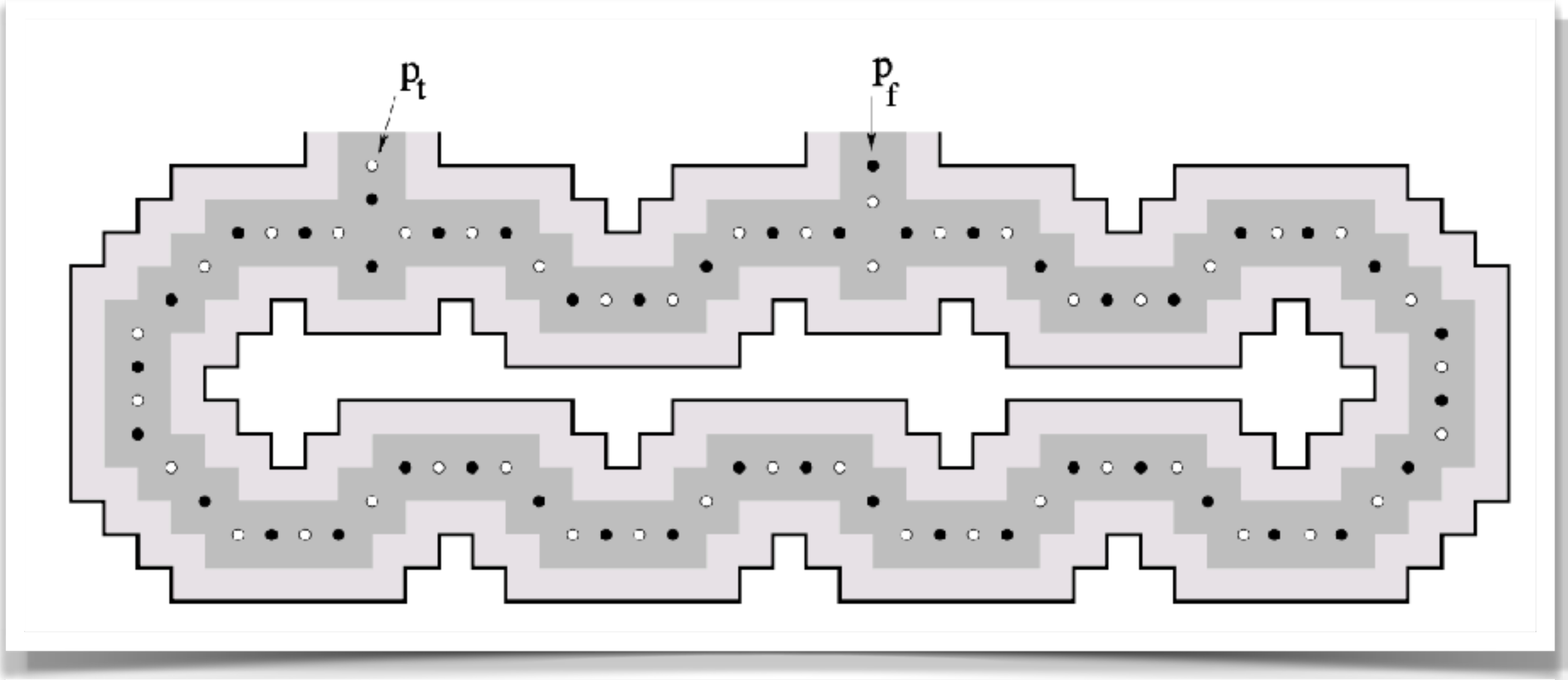
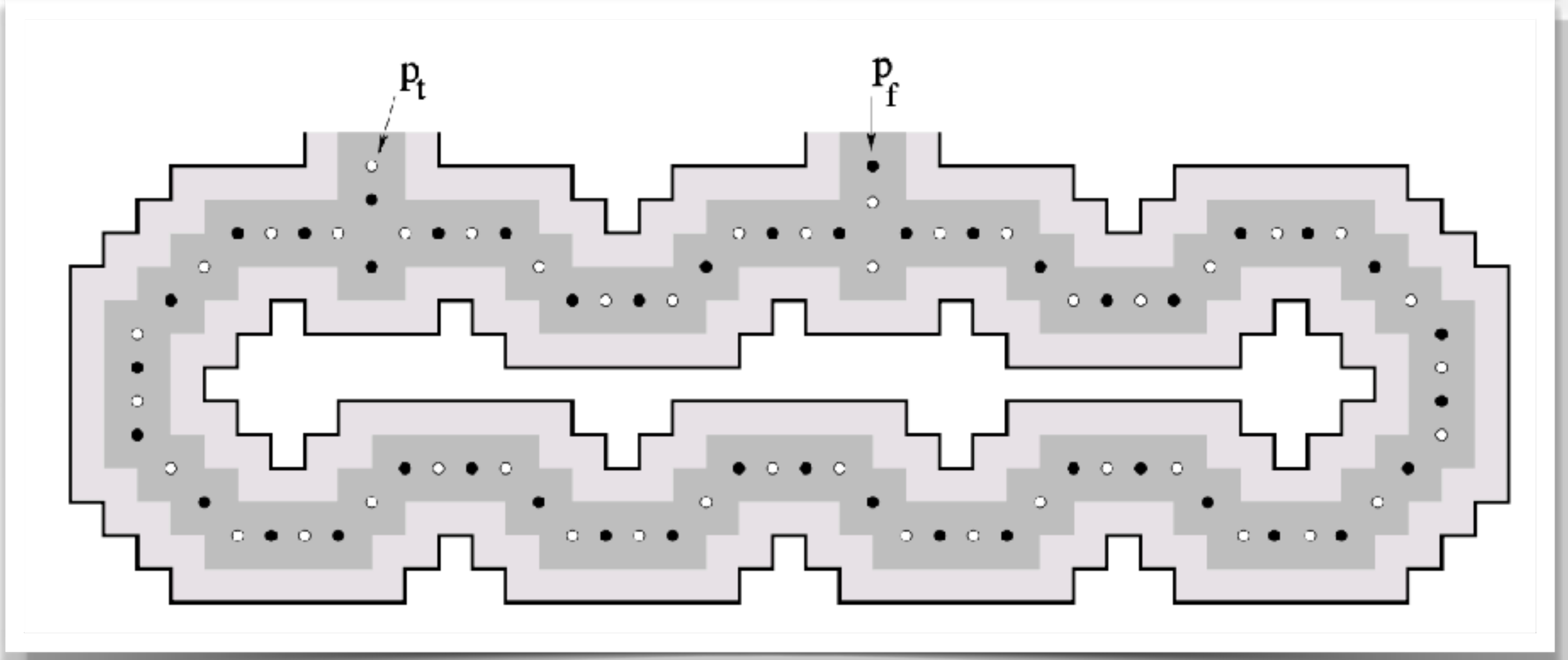


Variables (2)





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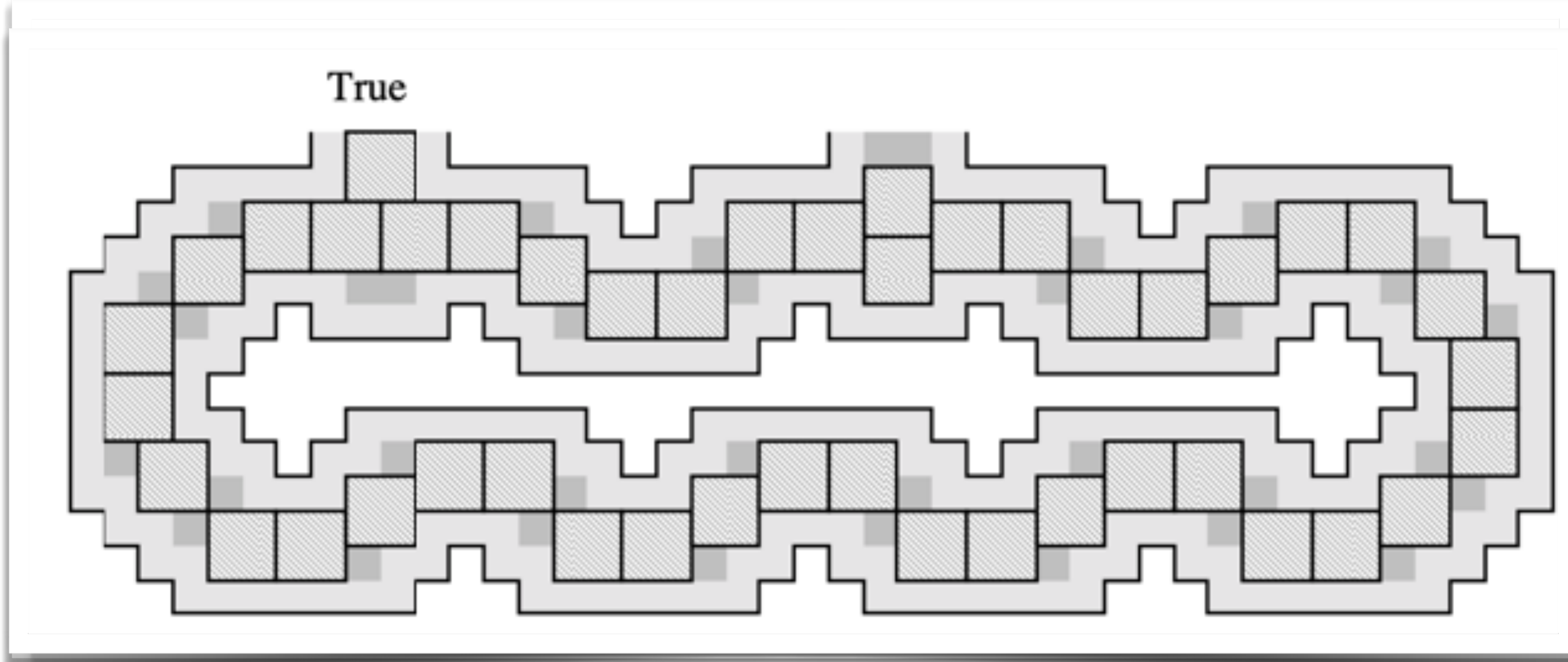

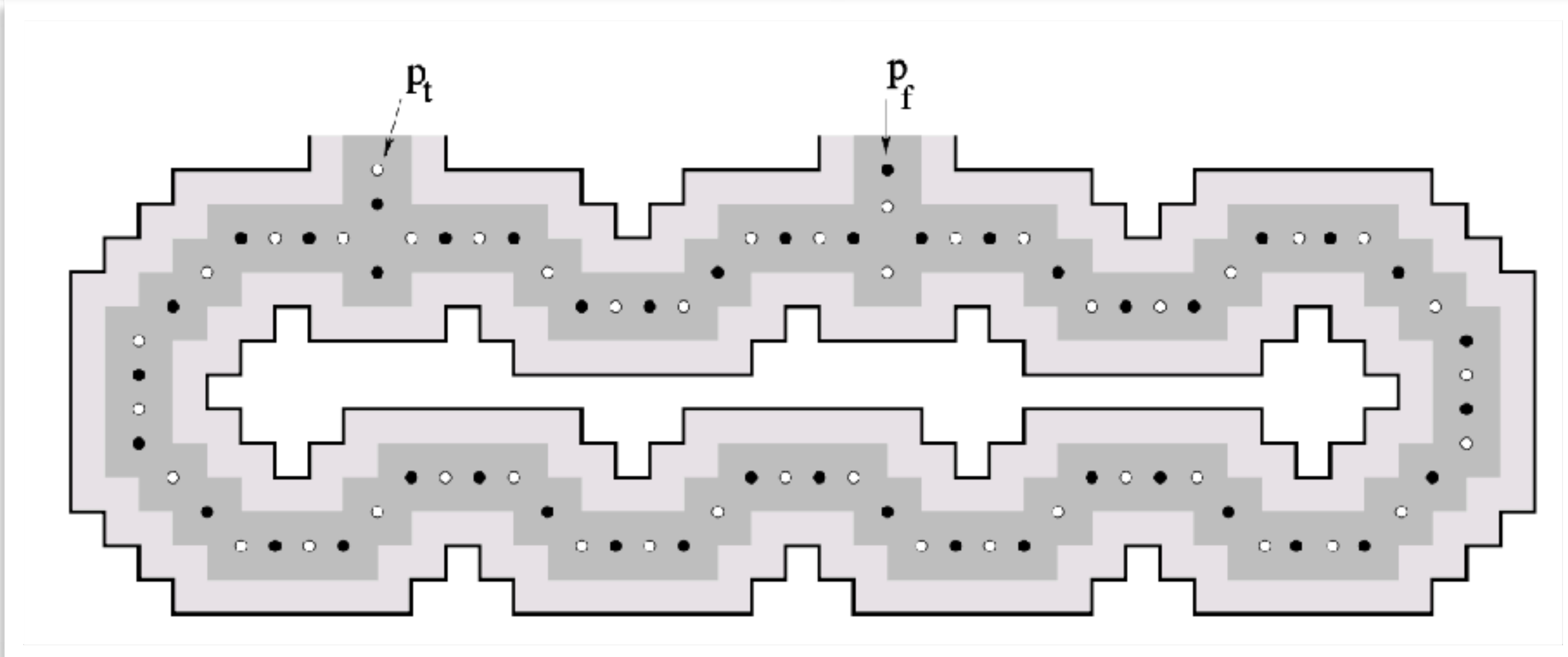

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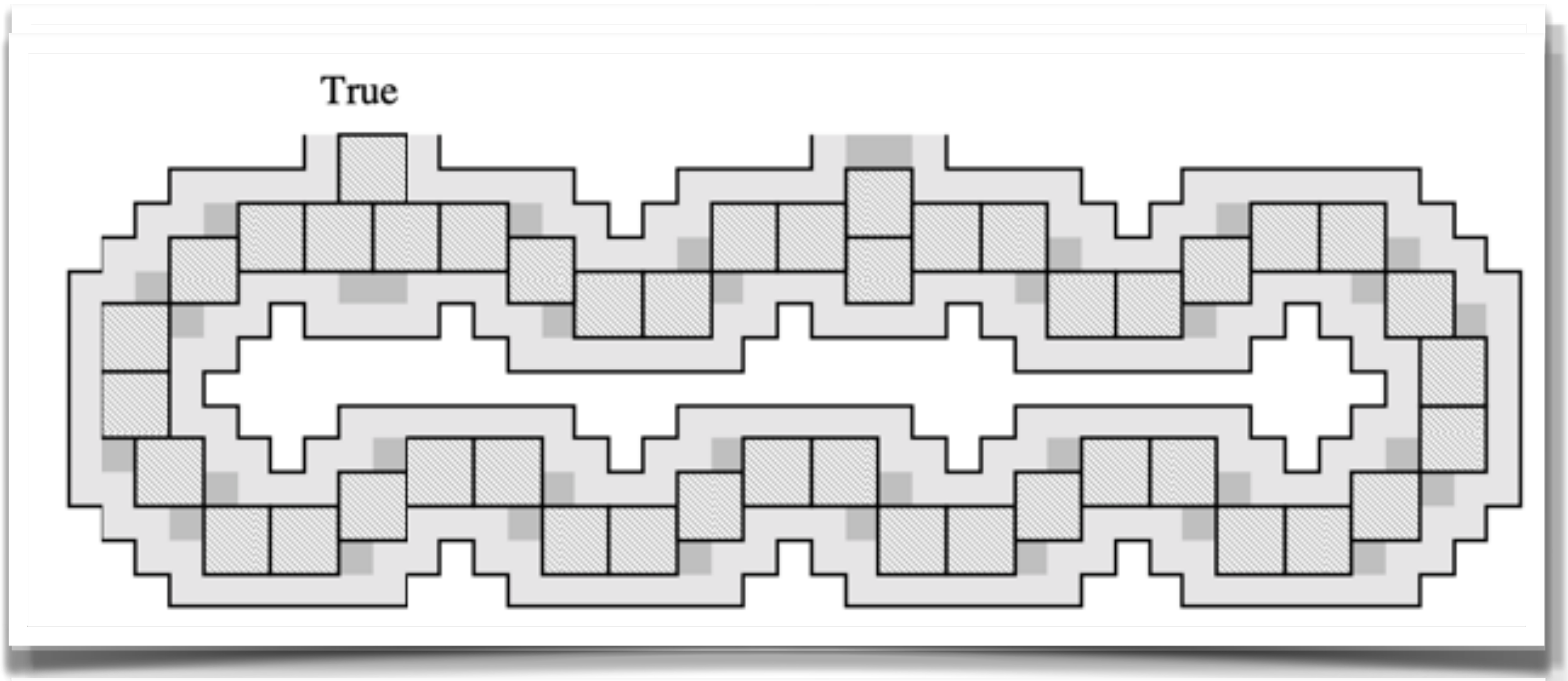

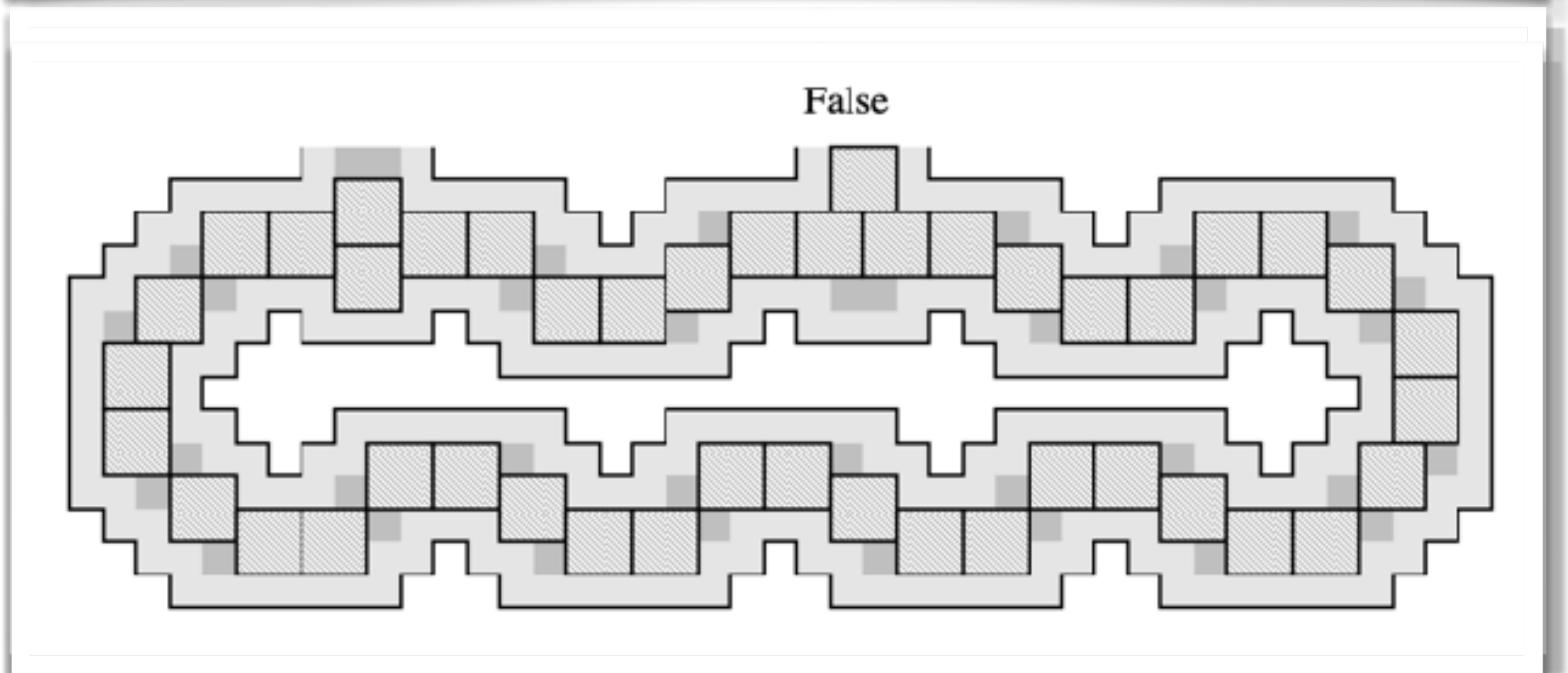

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Connectors

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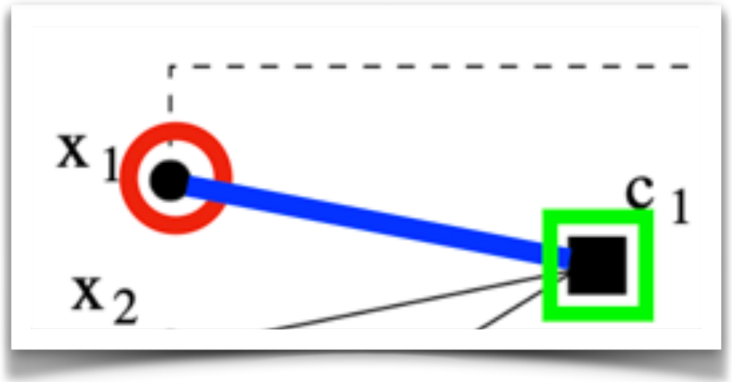

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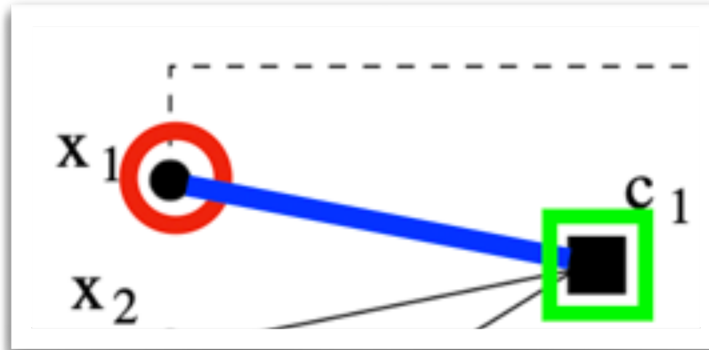


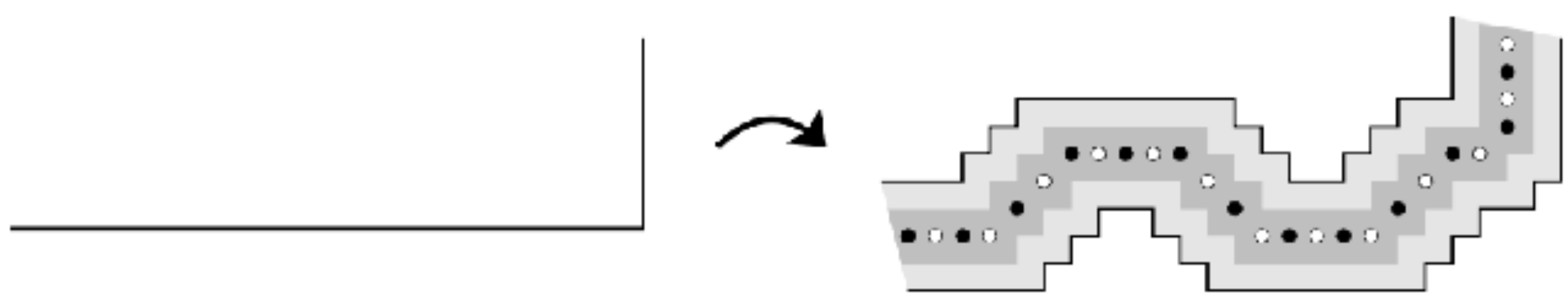
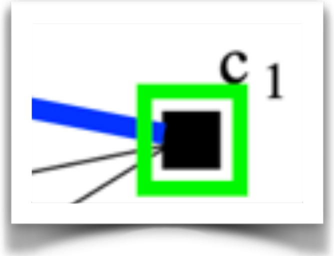


Fig. 3. Part of a connector component (right) for representing an edge between variable nodes and clause nodes (left).

Clauses

☐ Clauses



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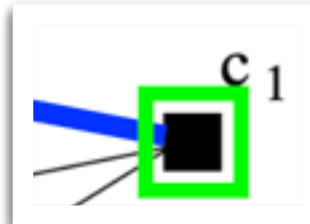

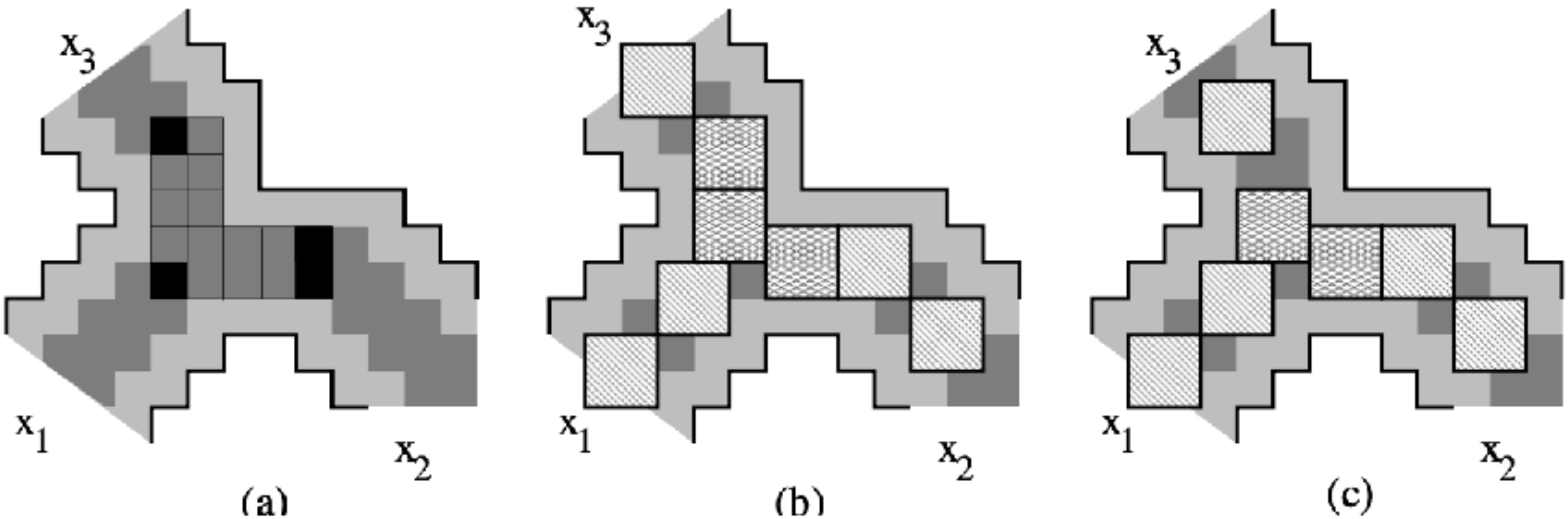

Fig. 4. A clause component for dispersion with boundaries and its receptor region (a); a satisfying placement (b); and an unsatisfying placement (c).

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Smaller Squares (1)

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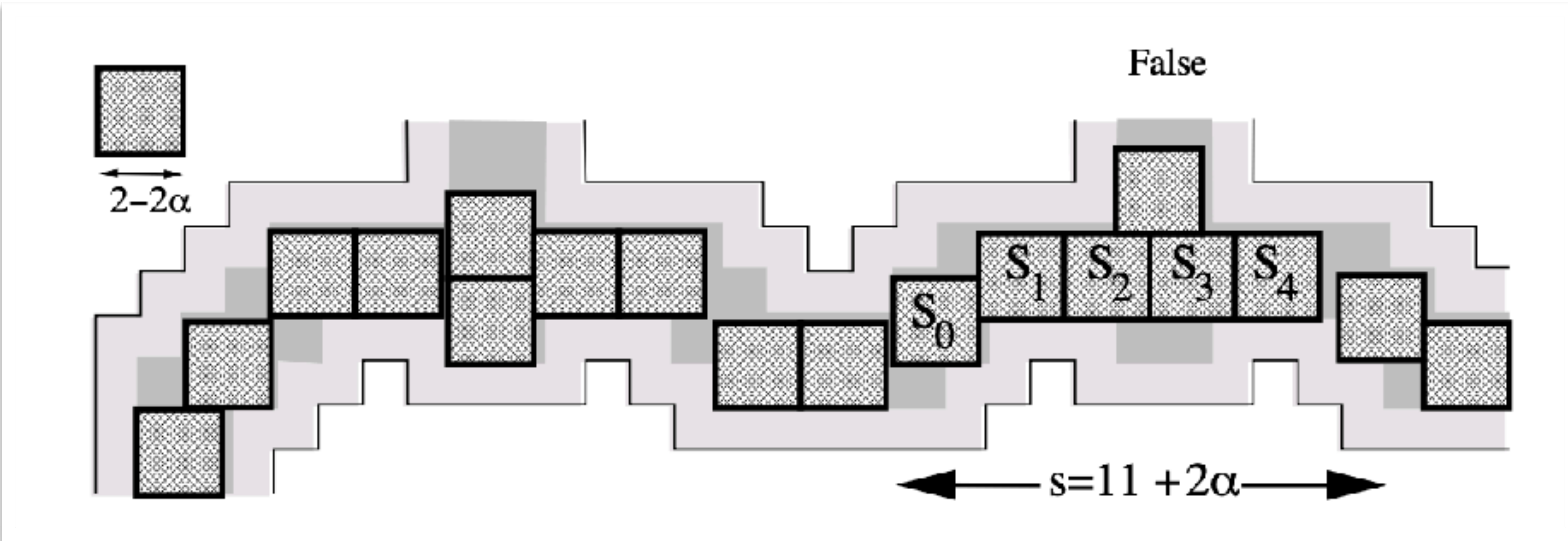

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Smaller Squares (2)

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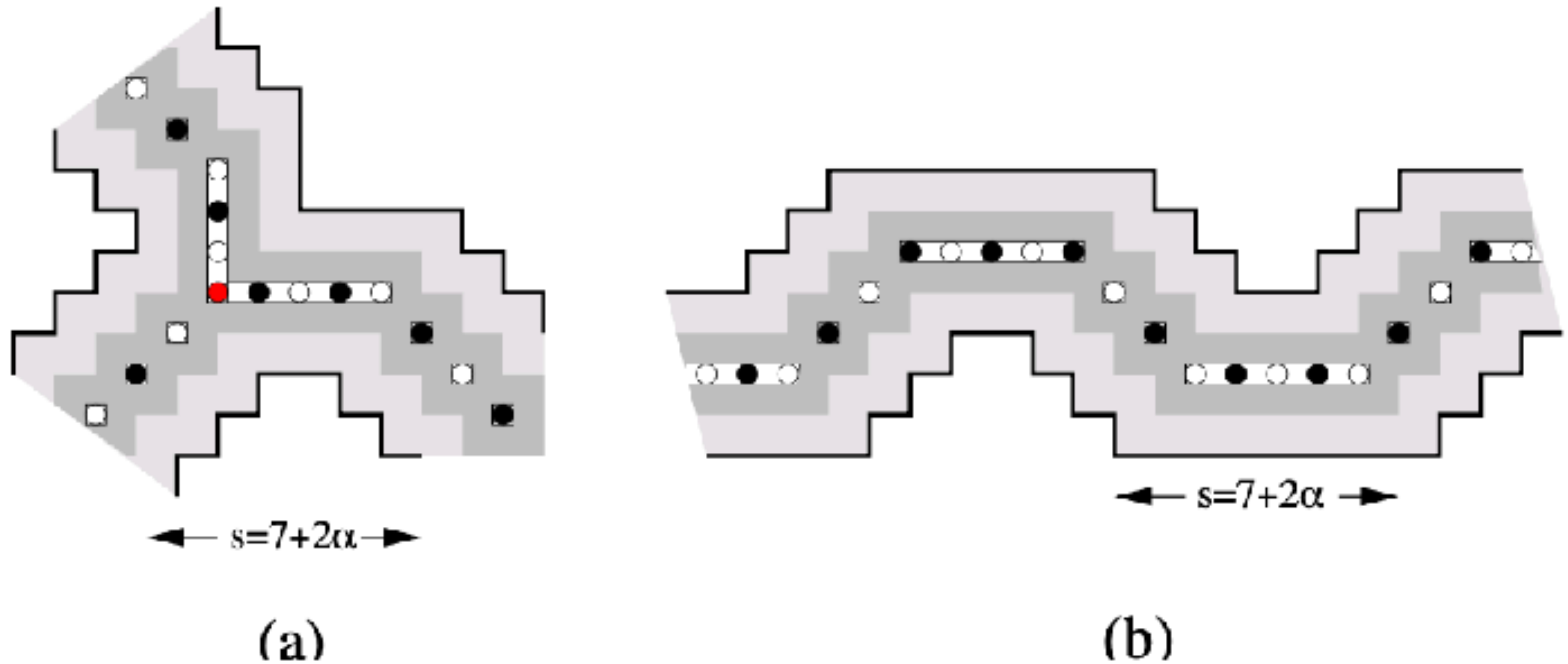

Fig. 7. An upper bound on the approximation factor: clause components (a) and connector components (b).

Other Metrics

Other Metrics

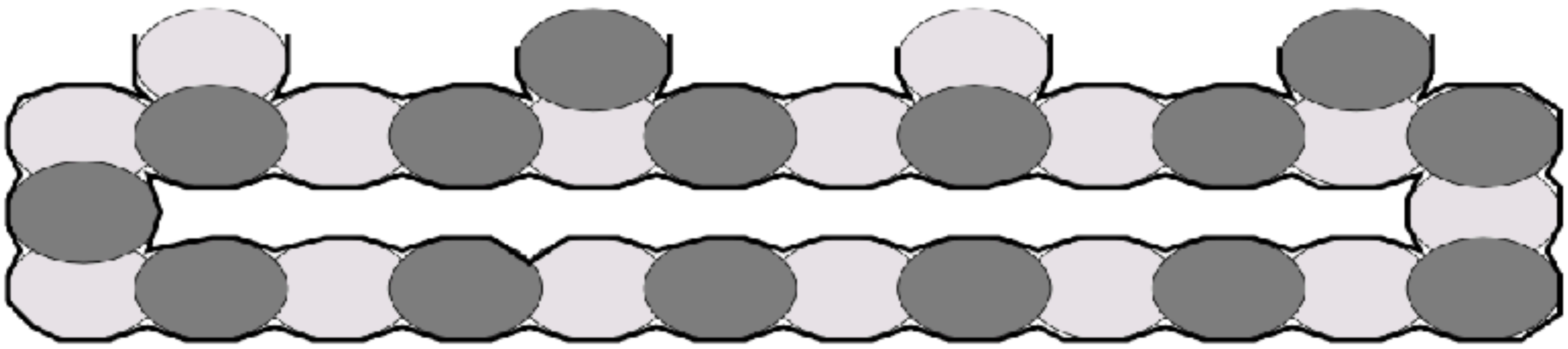

Fig. 9. A variable gadget for the case where the unit ball is an ellipse.

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Approximation

Approximation

THEOREM 9. *For rectilinear geometric dispersion with boundaries of k locations in a rectilinear polygon P with n vertices, Algorithm 8 computes a solution $A_{Dis}(P,k)$, such that*

$$A_{Dis}(P,k) \geq \frac{2}{3} OPT(P,k).$$

The running time is strongly polynomial.

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ALGORITHM 8.
Input: rectilinear polygon P, positive integer k.
Output: a set of k locations, such that $A_{Dis}(P,k) := d$ is the minimum L_∞ distance between a location and the boundary, or between two locations.

1. **For all** $(e_i, e_j) \in Par(P)$ **do**
   (a) Perform binary search for the smallest integer m, $2 \leq m \leq k+1$, with the following property:
      - For $d_{ijm} := Dist(e_i, e_j)/m$, $AS(P - d_{ijm}/2, d_{ijm}, 6)$ returns a feasible solution for at least k locations at distance d_{ijm}.

   (b) Let d_{ij} be the distance d_{ijm} for the critical value m.
2. Let d be the maximum d_{ij} for any (e_i, e_j).

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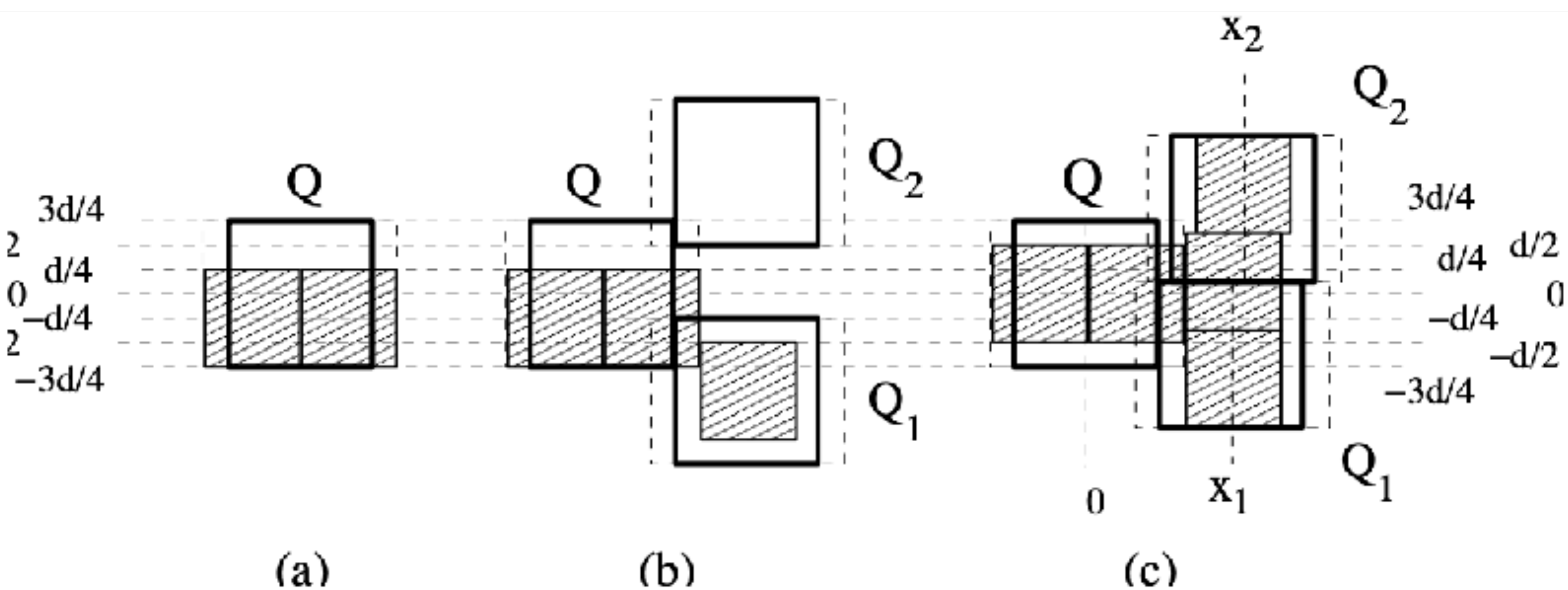

Fig. 12. Constructing a packing of *d*-squares.

Average Distance

Average Distance

Algorithmica (2004) 38: 501–511
DOI: 10.1007/s00453-003-1074-x

Algorithmica

Maximum Dispersion and Geometric Maximum Weight Cliques[1]

Sándor P. Fekete[2] and Henk Meijer[3]

Abstract. We consider a facility location problem, where the objective is to "disperse" a number of facilities, i.e., select a given number k of locations from a discrete set of n candidates, such that the average distance between selected locations is maximized. In particular, we present algorithmic results for the case where vertices are represented by points in d-dimensional space, and edge weights correspond to rectilinear distances. Problems of this type have been considered before, with the best result being an approximation algorithm with performance ratio 2. For the case where k is fixed, we establish a linear-time algorithm that finds an optimal solution. For the case where k is part of the input, we present a polynomial-time approximation scheme.

Key Words. Dispersion, Facility location, Maximum weight cliques, Remote clique, Heaviest subgraph, Geometric optimization, Approximation, PTAS.

Average Distance

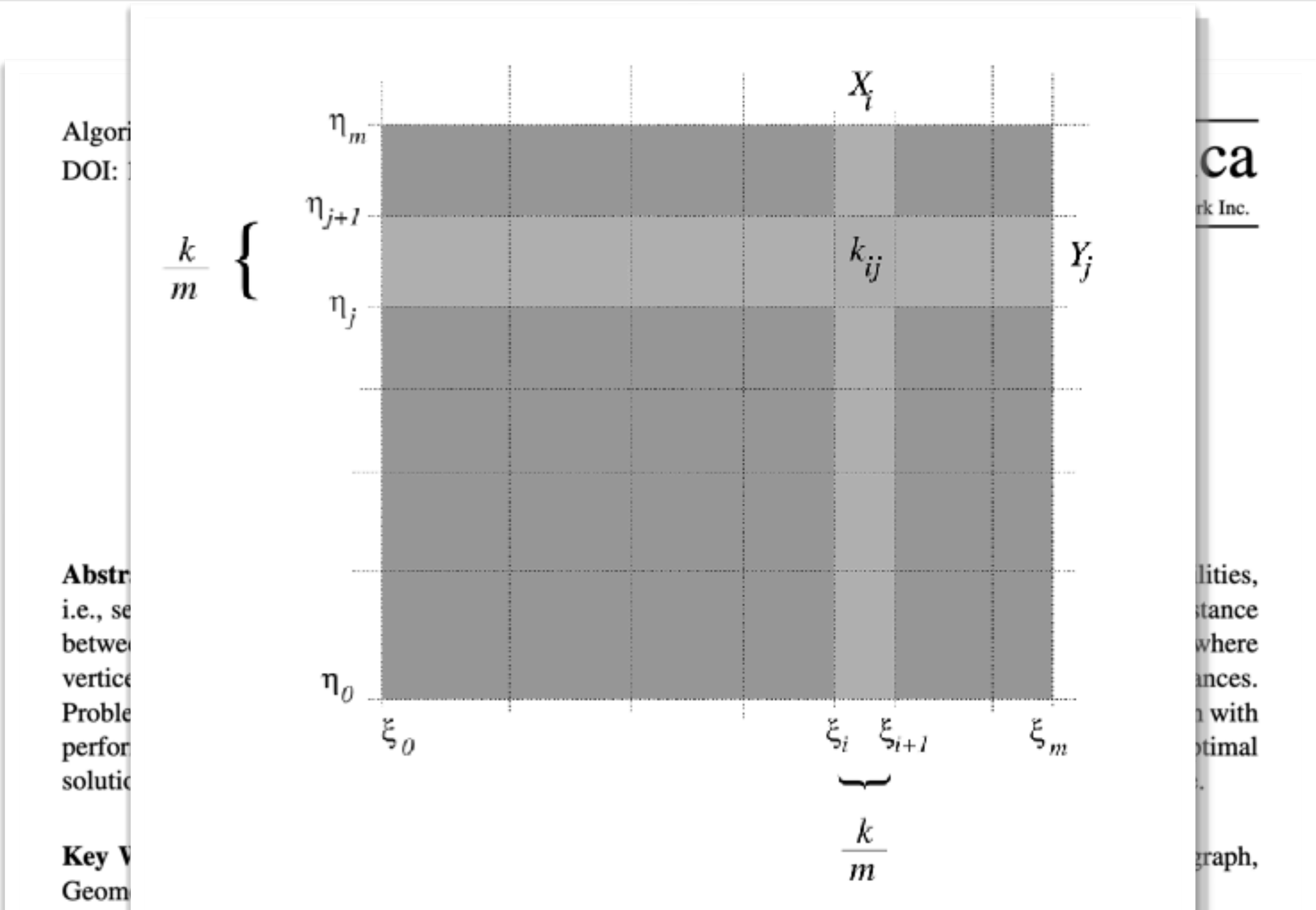

Fig. 1. Subdividing the plane into cells.

Average Distance

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Algorithmica

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Lawn Mowing (1)

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Lawn Mowing (1)

ELSEVIER

Computational Geometry 17 (2000) 25–50

Computational Geometry
Theory and Applications

www.elsevier.nl/locate/comgeo

Approximation algorithms for lawn mowing and milling ☆

Esther M. Arkin [a,1], Sándor P. Fekete [b,*], Joseph S.B. Mitchell [a,2]

[a] *Department of Applied Mathematics and Statistics, SUNY Stony Brook, NY 11794-3600, USA*
[b] *Department of Mathematics, TU Berlin, 10623 Berlin, Germany*

Communicated by K. Mehlhorn; received 18 March 1997; accepted 12 August 1999

Abstract

We study the problem of finding shortest tours/paths for "lawn mowing" and "milling" problems: Given a region in the plane, and given the shape of a "cutter" (typically, a circle or a square), find a shortest tour/path for the cutter such that every point within the region is covered by the cutter at some position along the tour/path. In the milling version of the problem, the cutter is constrained to stay within the region. The milling problem arises naturally in the area of automatic tool path generation for NC pocket machining. The lawn mowing problem arises in optical inspection, spray painting, and optimal search planning.

Lawn Mowing (1)

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Computational Geometry 17 (2000) 25–50

Computational Geometry
Theory and Applications
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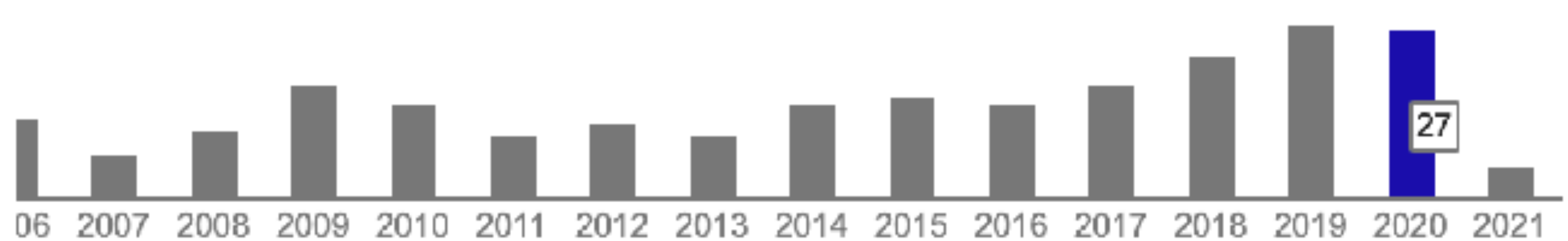
Total citations Cited by 283



Lawn Mowing (2)

Lawn Mowing (2)

E.M. Arkin et al. / Computational Geometry 17 (2000) 25–50

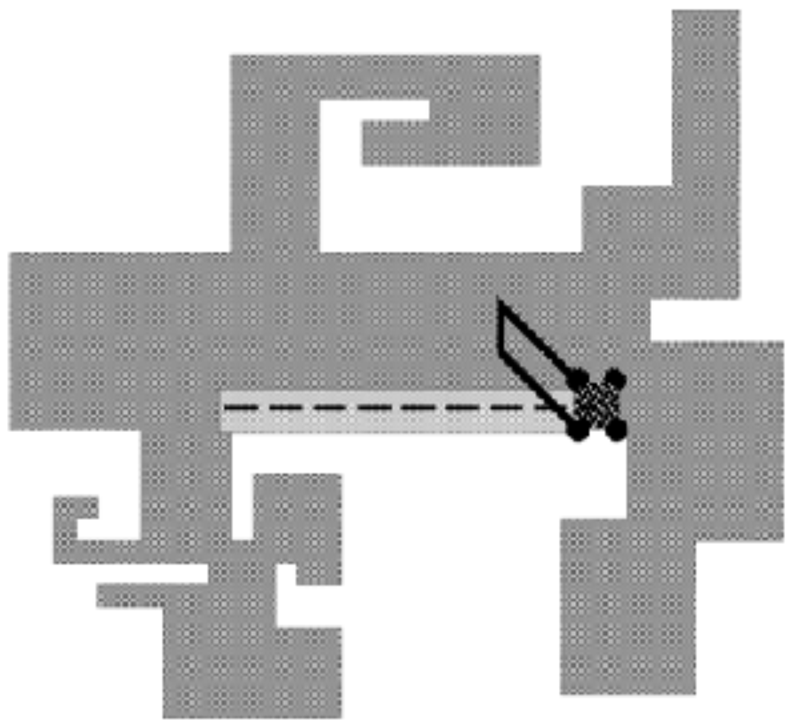


Fig. 1. The lawn mowing problem.

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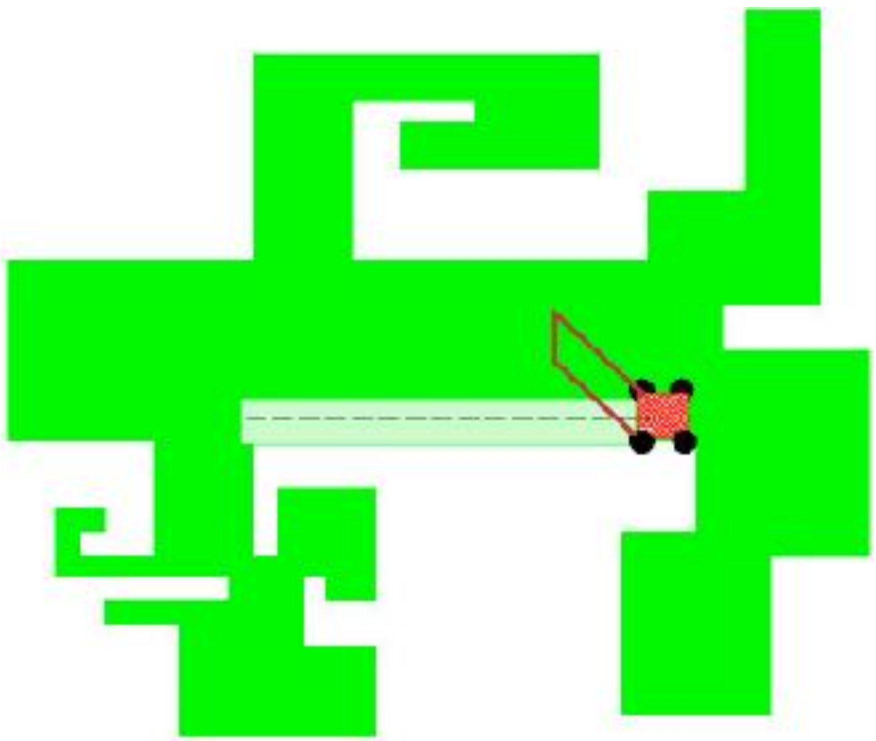
E.M. Arkin et al. / Computational Geometry 17 (2000) 25–50



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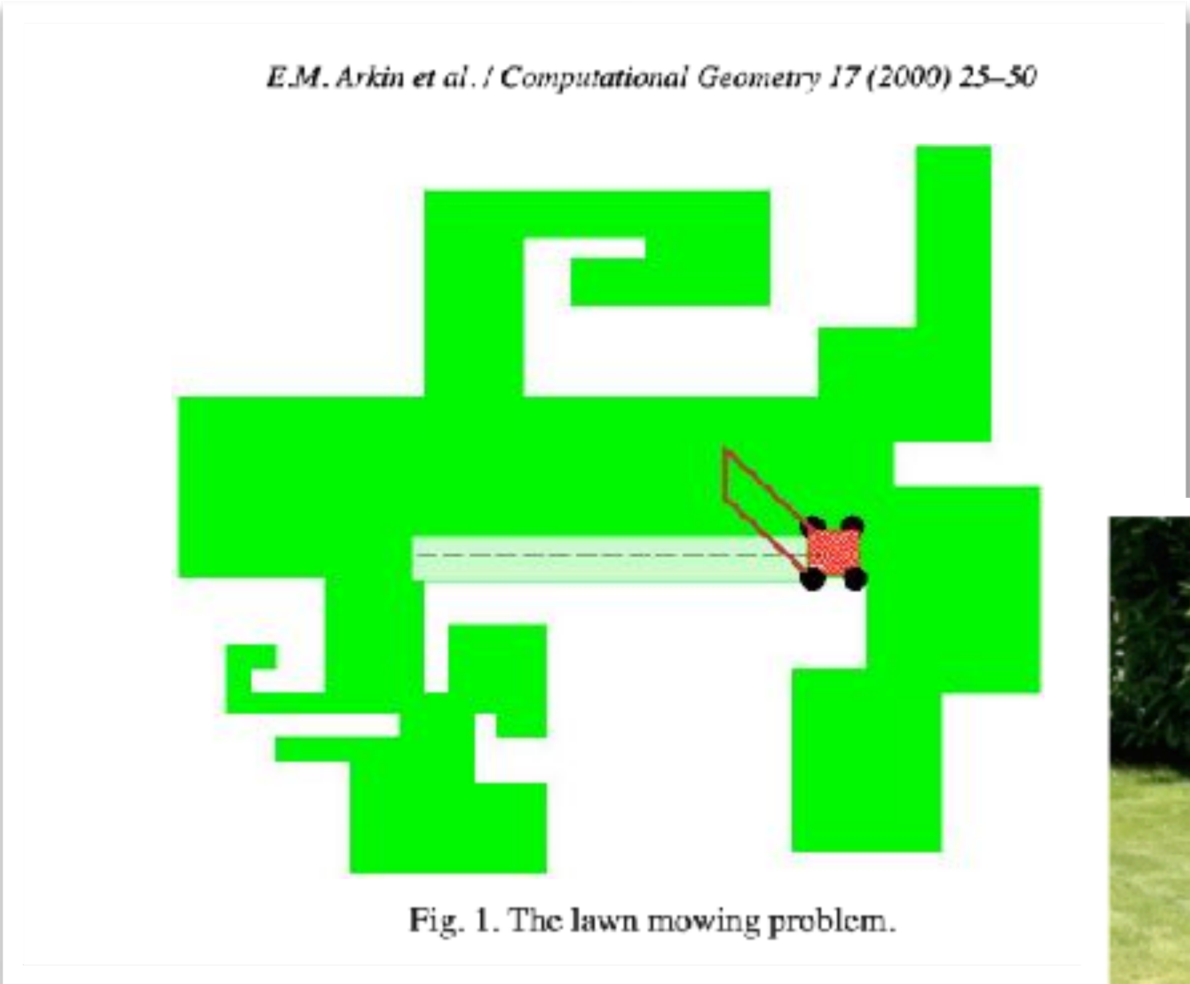
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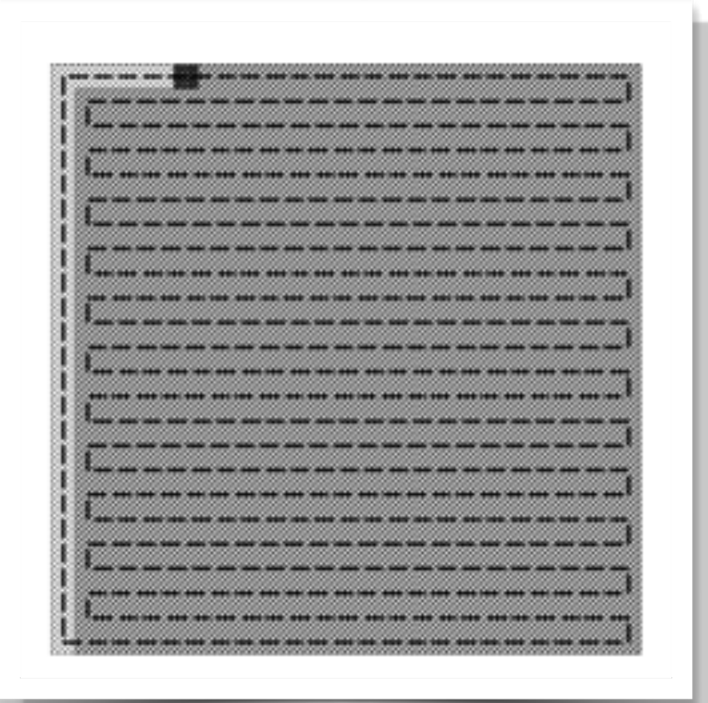
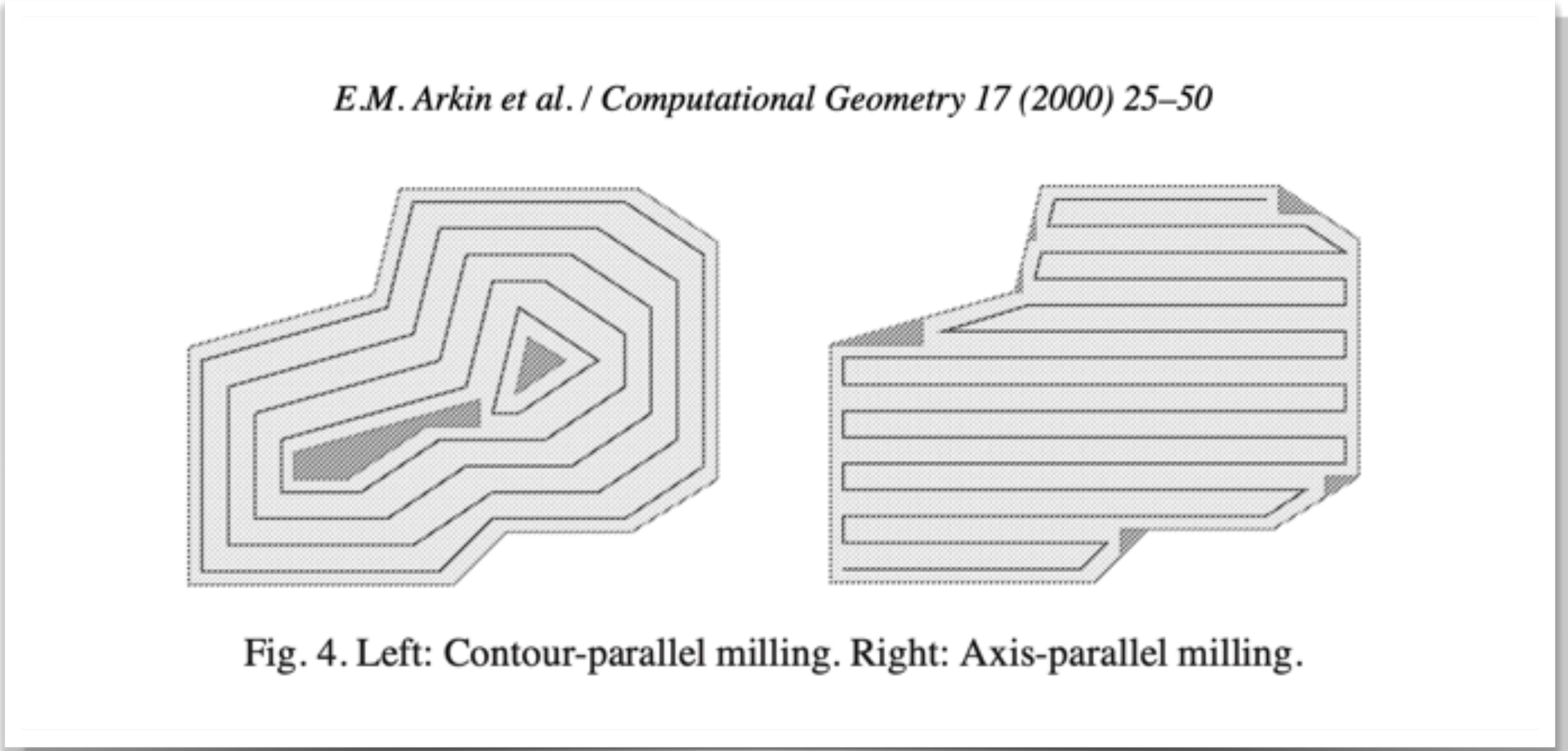
Lawn Mowing (3)





Lawn Mowing (4)

Theorem 1. *The lawn mowing problem for a connected polygonal region is NP-hard for the case of an aligned unit square cutter* χ.

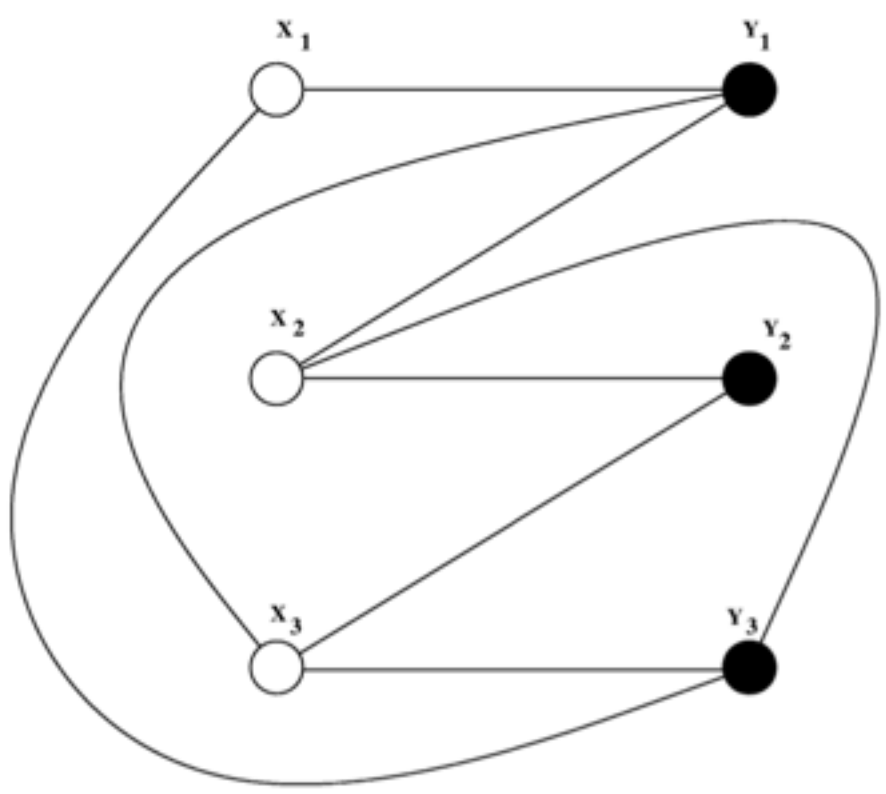
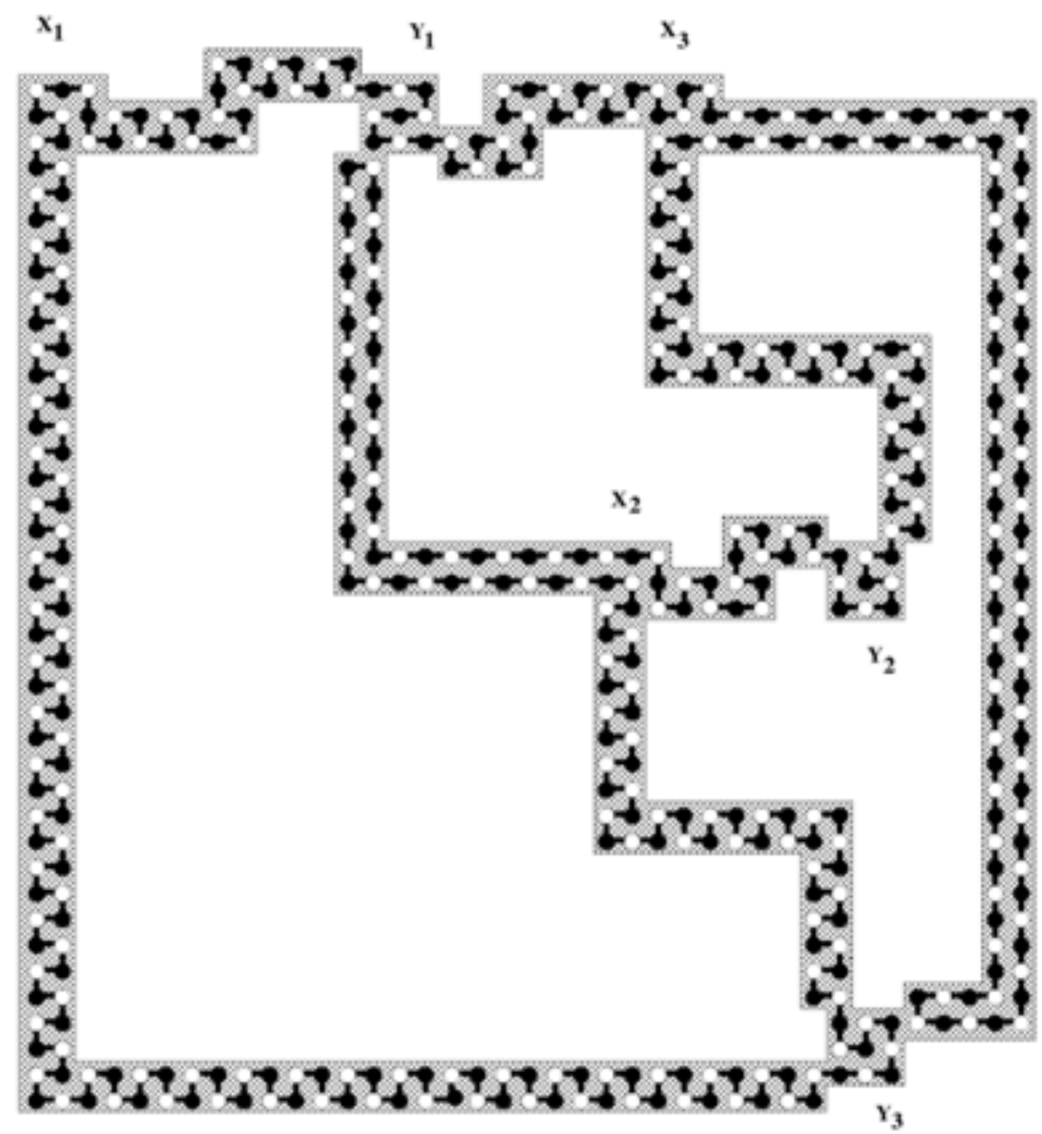


Fig. 5. A planar bipartite graph G, with maximum vertex degree 3.



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Corollary 1. *The lawn mowing problem is NP-hard even for simple polygonal regions R.*

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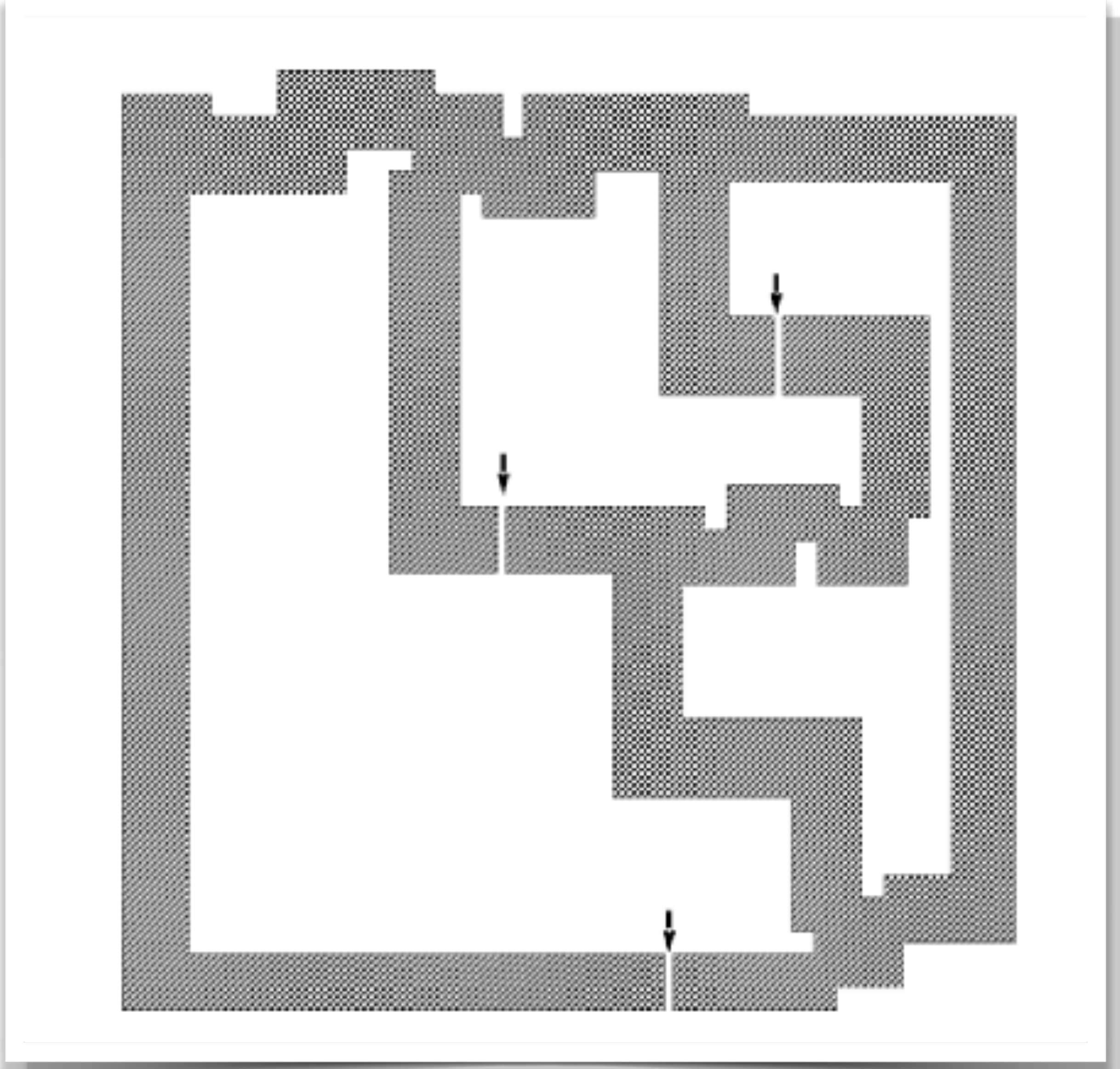

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Lawn Mowing (6)

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Theorem 2. *Finding a rectilinear TSP approximation (with factor α_{TSP}) on the set of centerpoints S yields a lawn mowing tour of length at most $\alpha_{\mathrm{TSP}}(3\ell^* + 6)$, where ℓ^* is the length of an optimal lawn mowing tour.*

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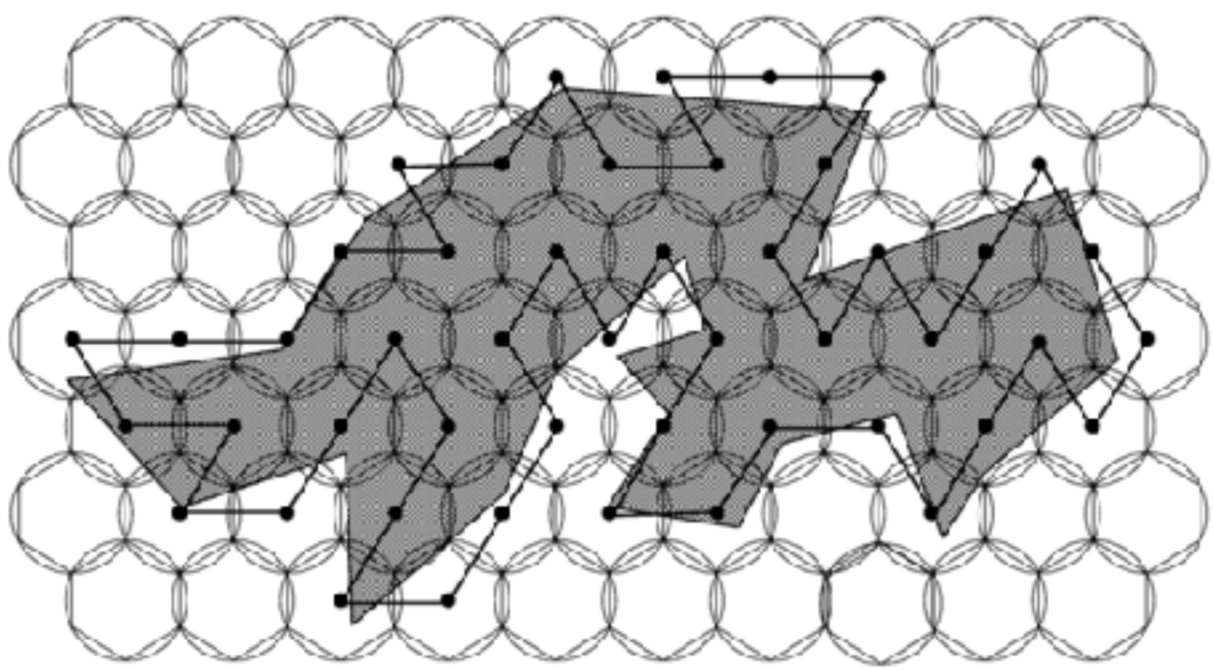

Fig. 10. Approximation for the case of a circular cutter χ.

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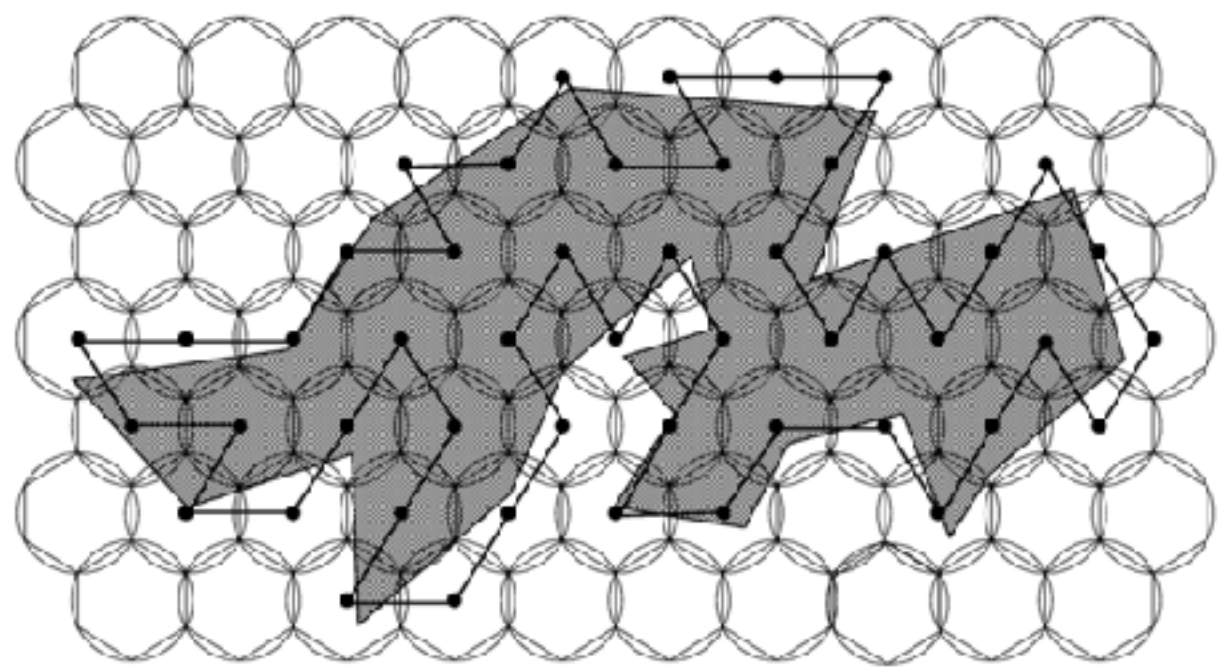

Theorem 3. *The lawn mowing problem has a constant-factor approximation algorithm that runs in polynomial time (dependent on the TSP heuristic employed). For the case of an aligned unit square cutter, the approximation factor is $3\alpha_{\mathrm{TSP}}$ for rectilinear motion, and is $3\beta\alpha_{\mathrm{TSP}}$ for arbitrary translational motion. For the case of a unit circular cutter, with arbitrary motion, the approximation factor is $3\gamma\alpha_{\mathrm{TSP}}$. Here, $\beta = \frac{2}{\sqrt{2+\sqrt{2}}} \approx 1.08$ and $\gamma = \frac{2\sqrt{3}}{3} \approx 1.15$.*

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Lawn Mowing (7)

Theorem 4. *In time* $O(n \log n)$, *one can decide whether a (multiply) connected region with n sides (straight or circular arc) can be milled by a unit disk or unit square, and, within the same time bound, one can construct a tour of length at most* $2\frac{1}{2}$ *times the length of an optimal milling tour.*

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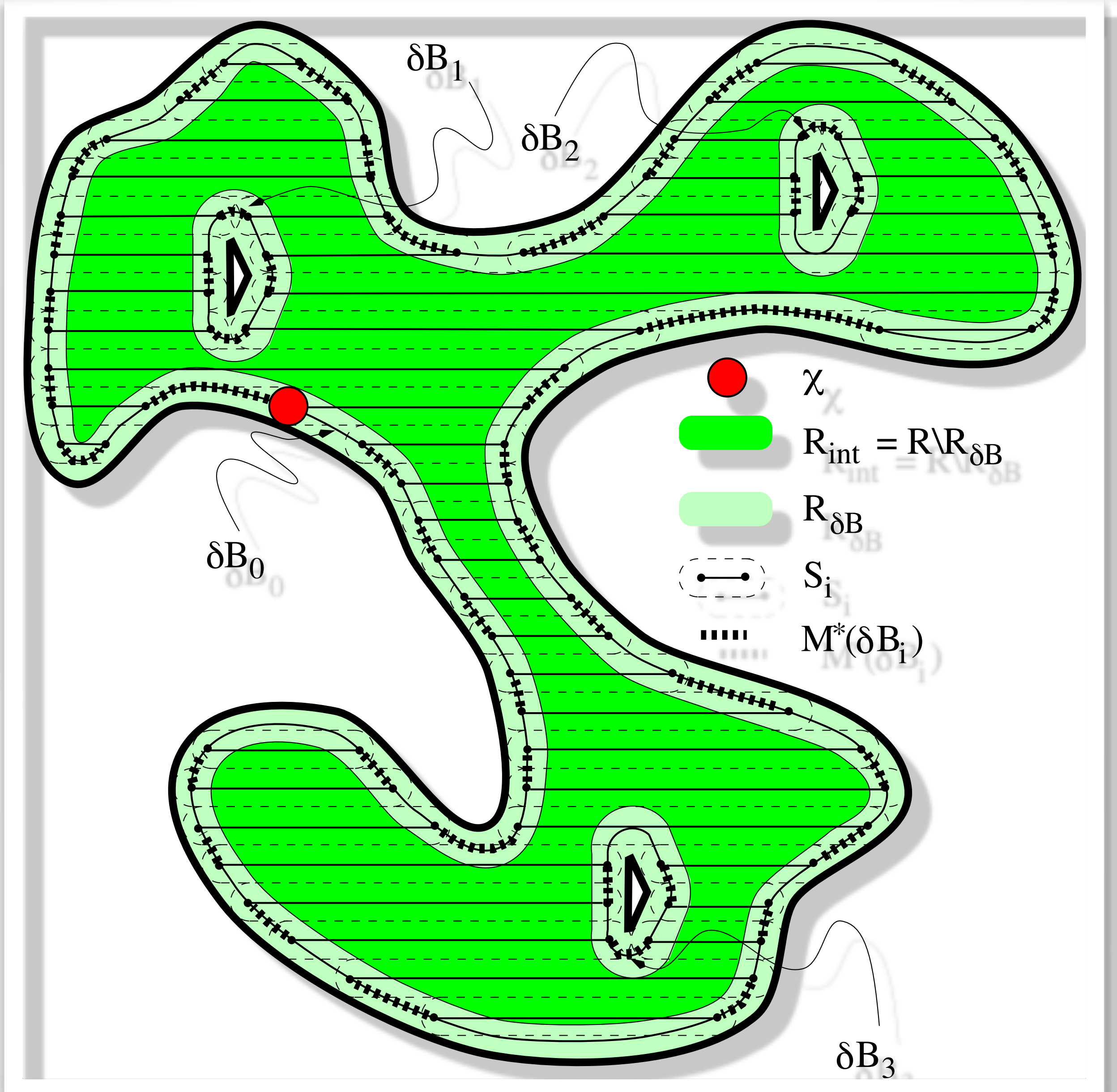

Lawn Mowing (8)

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Theorem 5. *Let G be a simple grid graph, having N nodes at the centerpoints, V, of pixels within a simple rectilinear polygon, R, having n (integer-coordinate) sides. Assume that G has no cut vertices. Then, in time $O(n)$, one can find a representation of a tour, T, that visits all N nodes of G, of length at most $\frac{6N-4}{5}$.*

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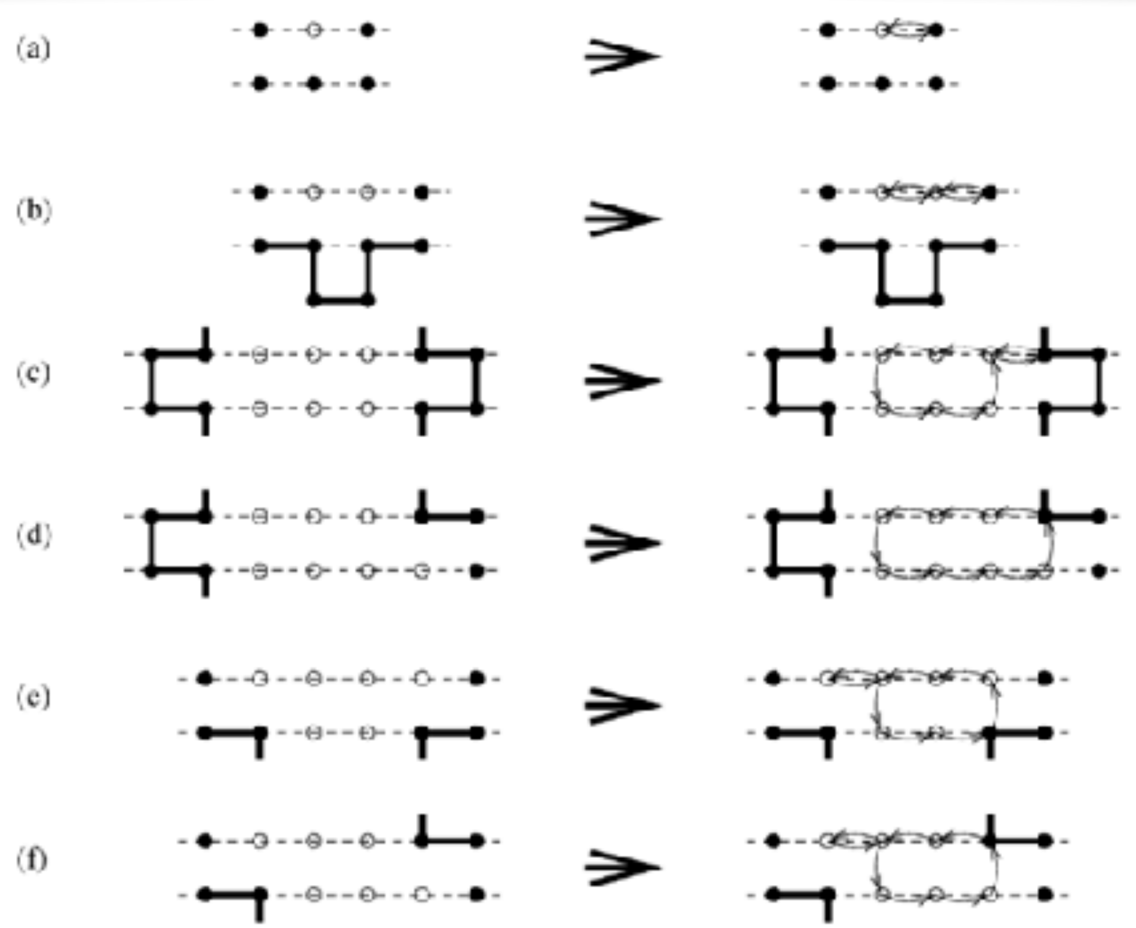
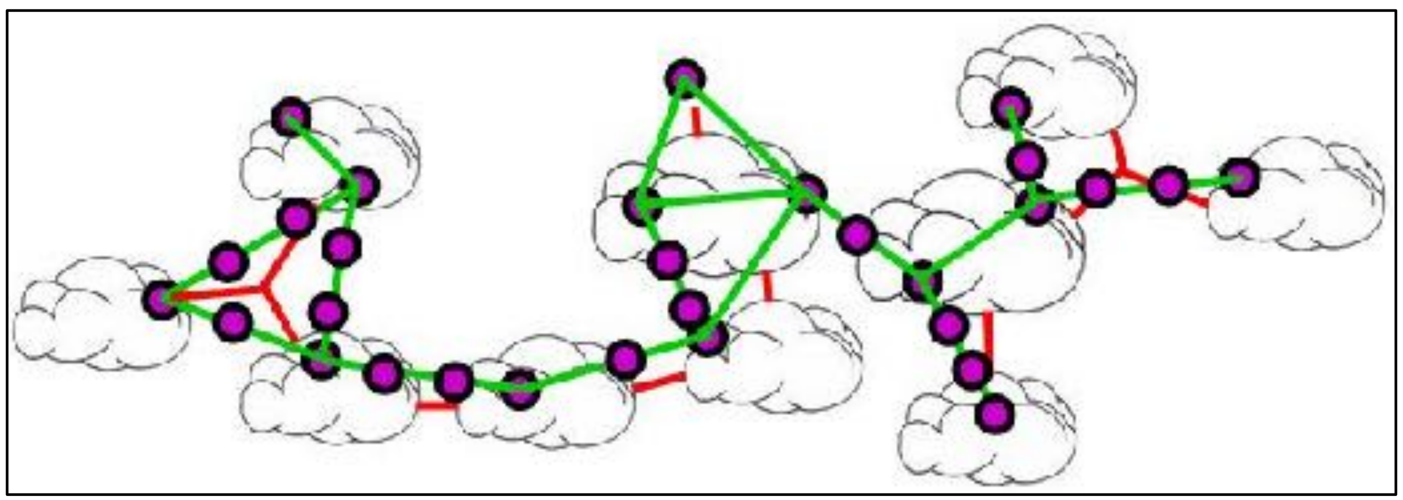


Fig. 15. Six cases for incorporating internal nodes into the modified contour tour. Hollow circles denote internal nodes, V_I', and solid circles denote nodes of C'. Solid edges are drawn where there *must* be edges of C'.

1. Introduction
2. Review
3. Extra Packing: Dispersion
4. Extra Tours: Lawn Mowing
5. Relay Placement
6. Coordinated Motion Planning

Approximation Algorithms for Relay Placement in the Plane



Alon Efrat, Sándor P. Fekete, Joe Mitchell, Valentin Polishchuk,
G. R. Poornananda, and Jukka Suomela

Relay Placement

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Input: V = set of n sensors (points in R^2)

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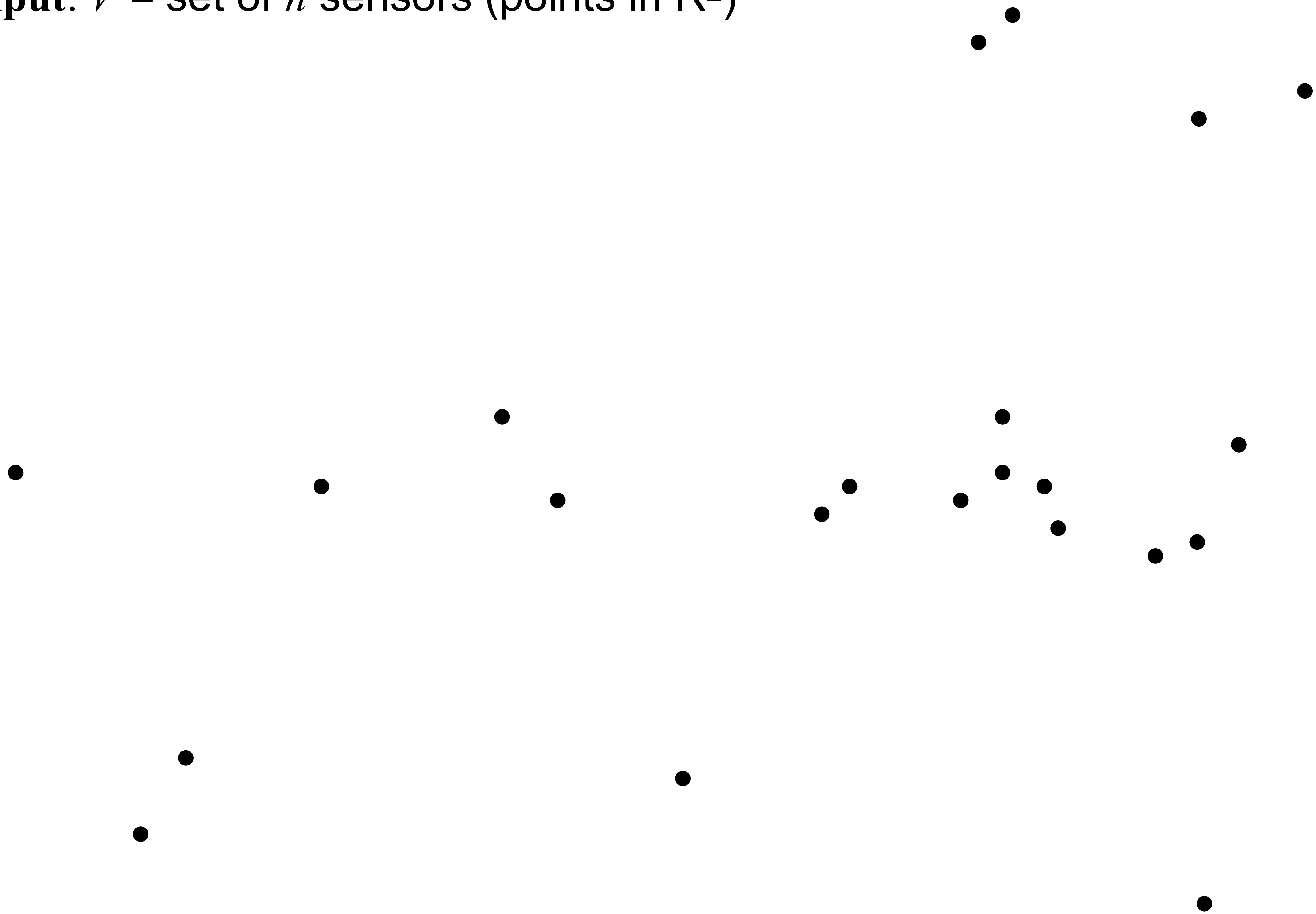

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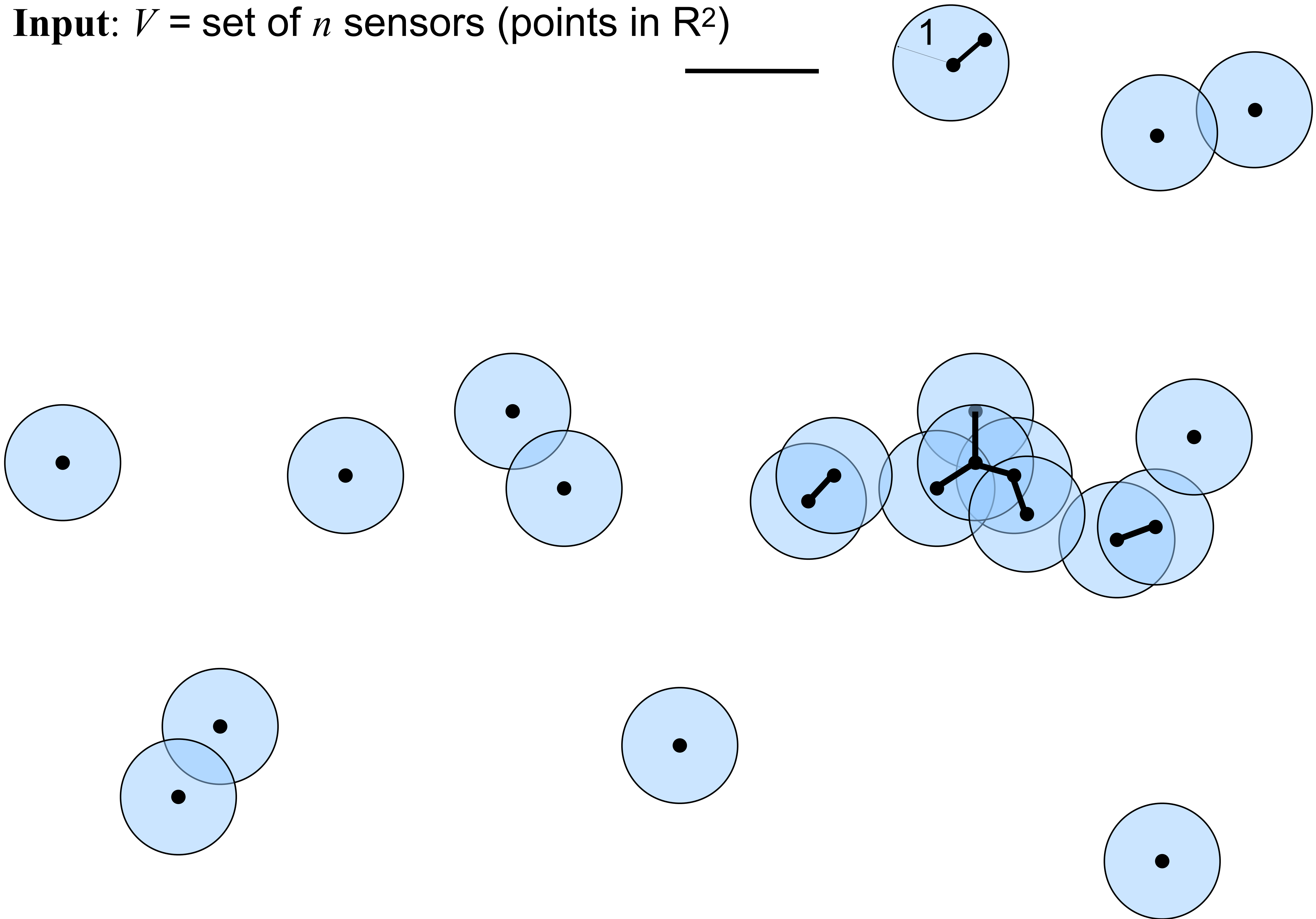

Relay Placement

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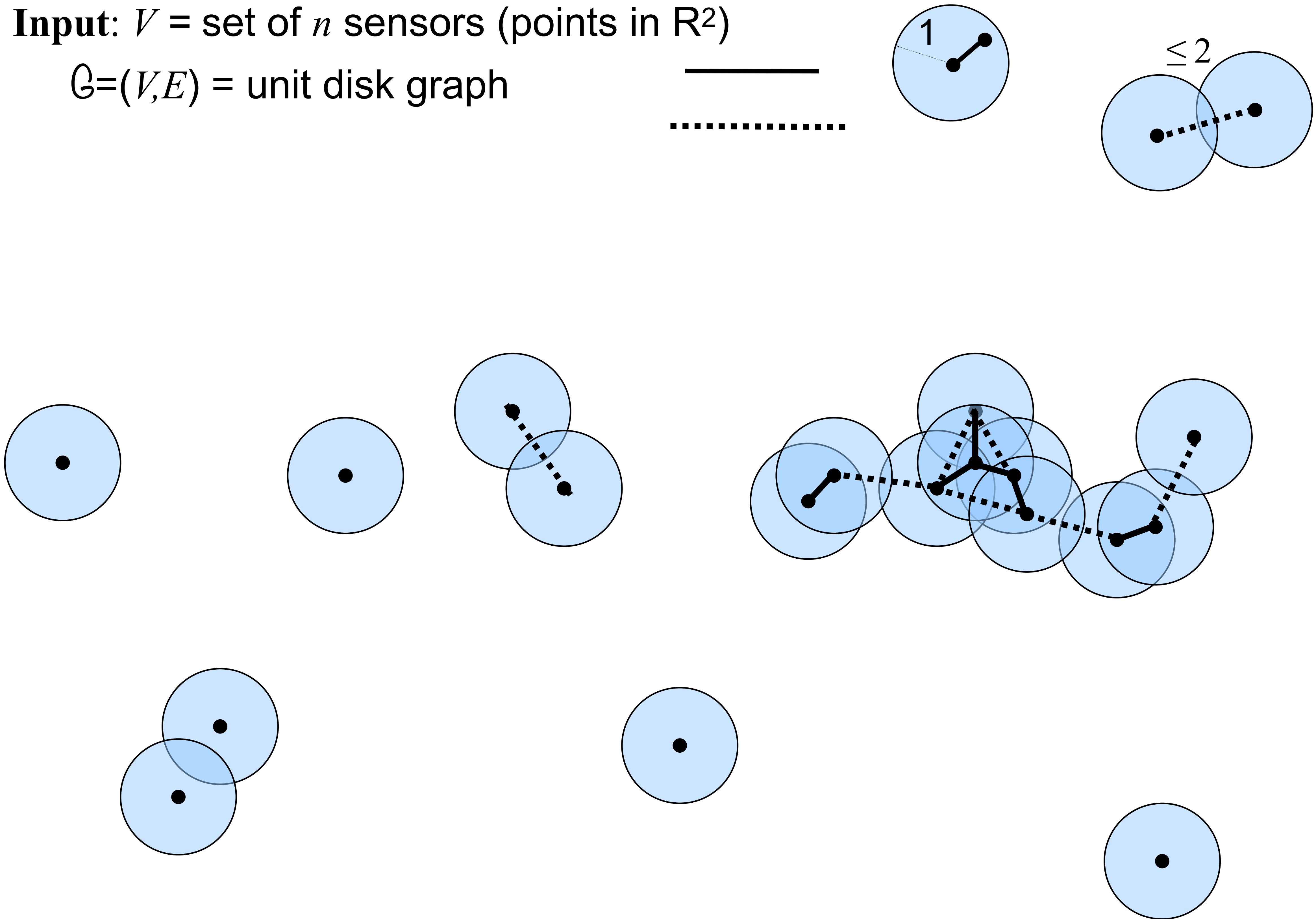
$G=(V,E)$ = unit disk graph



Relay Placement

Input: V = set of n sensors (points in R^2)

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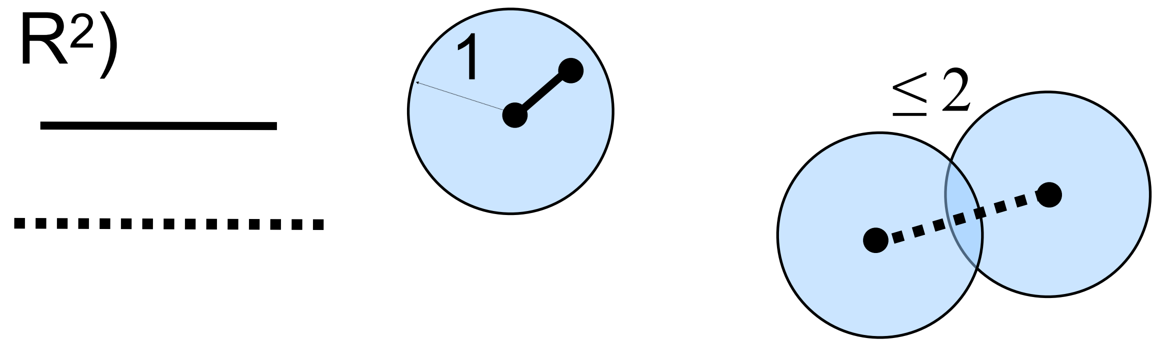
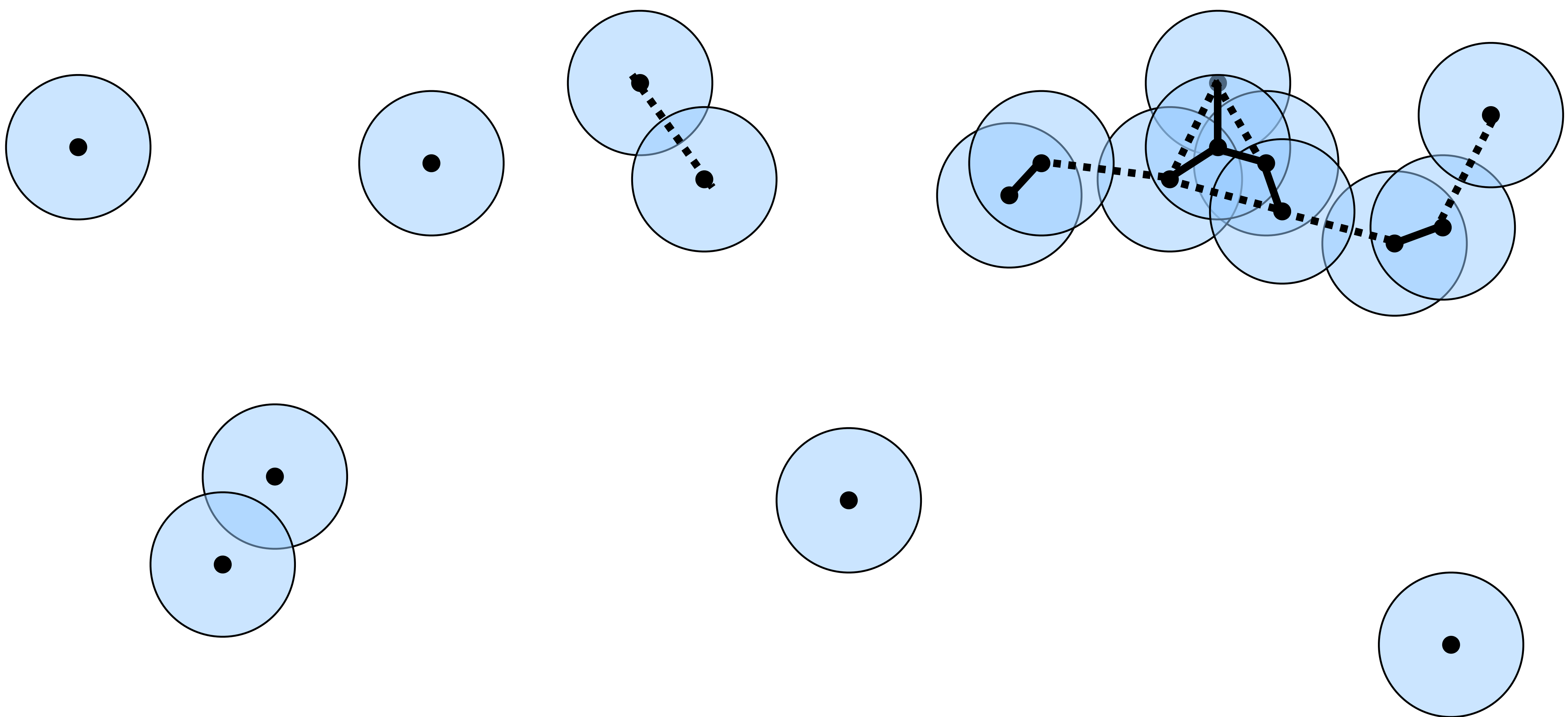
$\mathcal{F}=(V,F)$ = radius-2 disk graph





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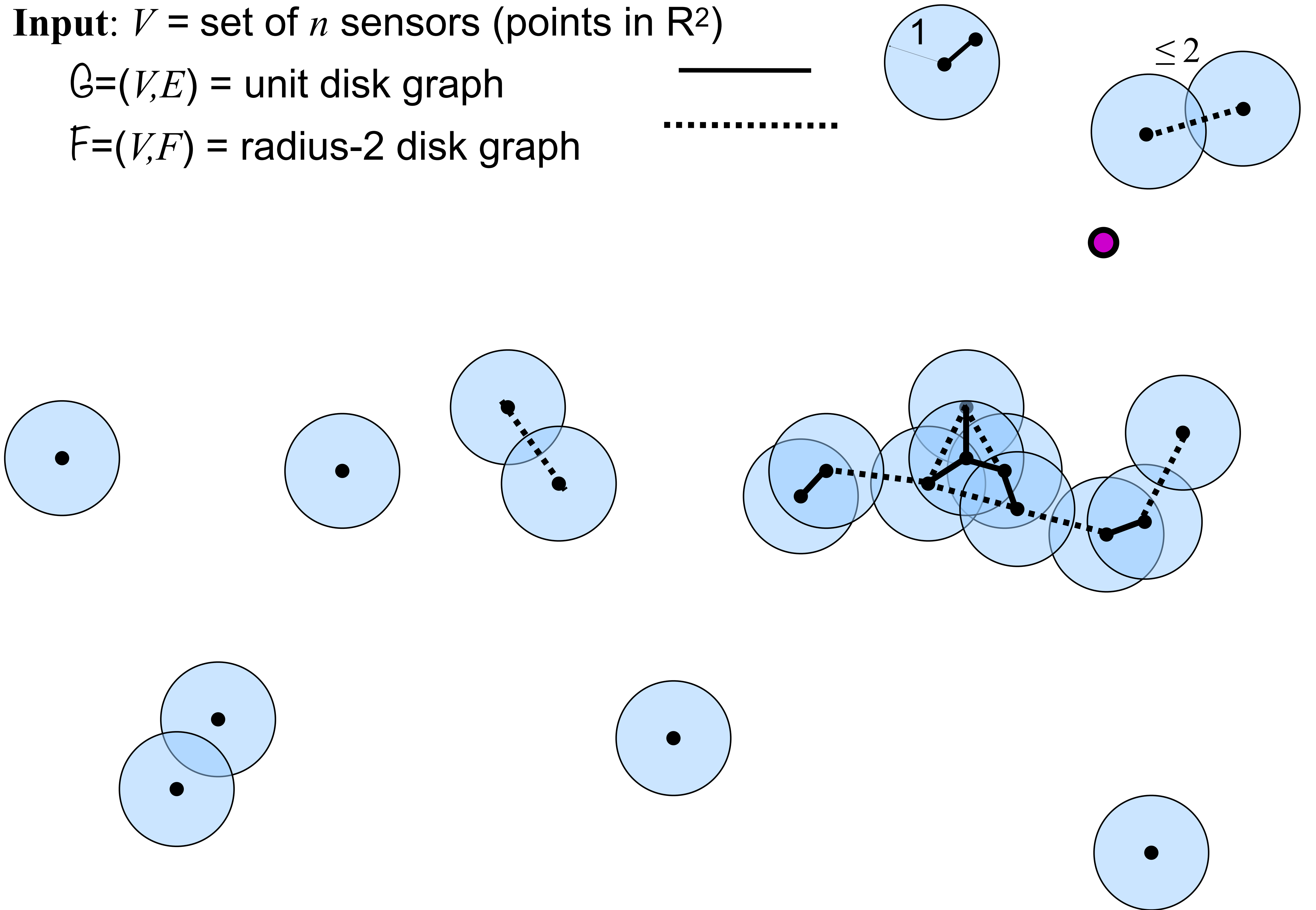

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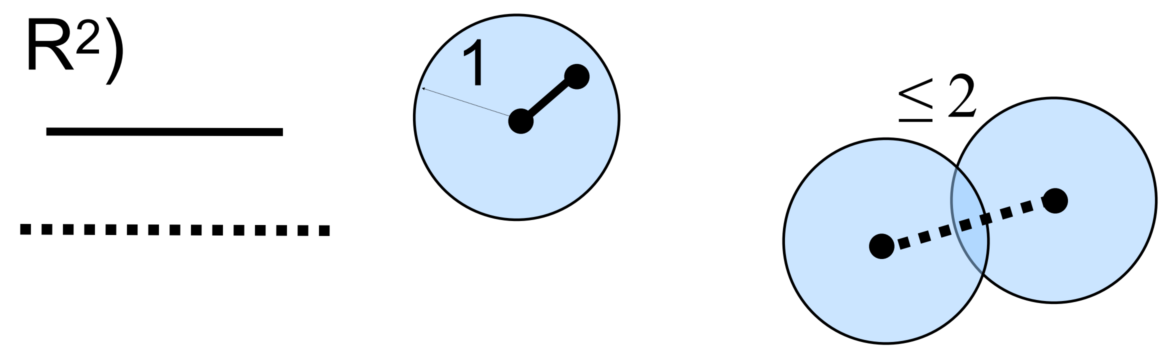

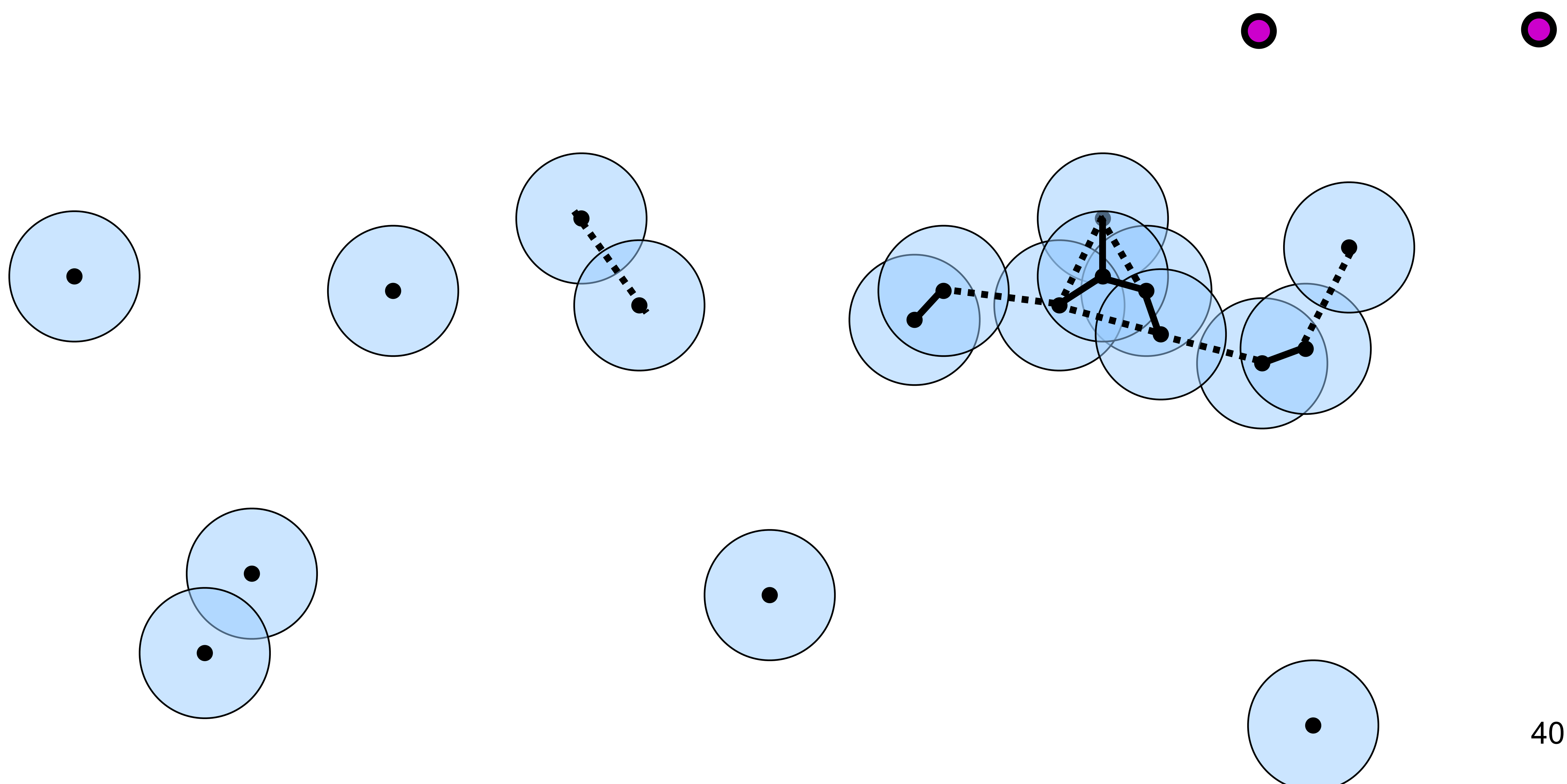

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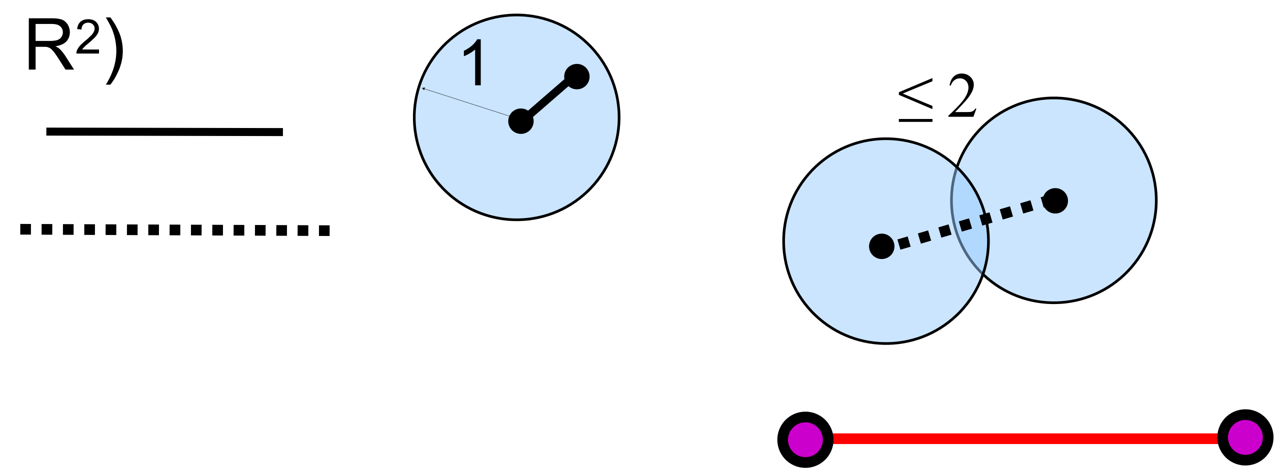

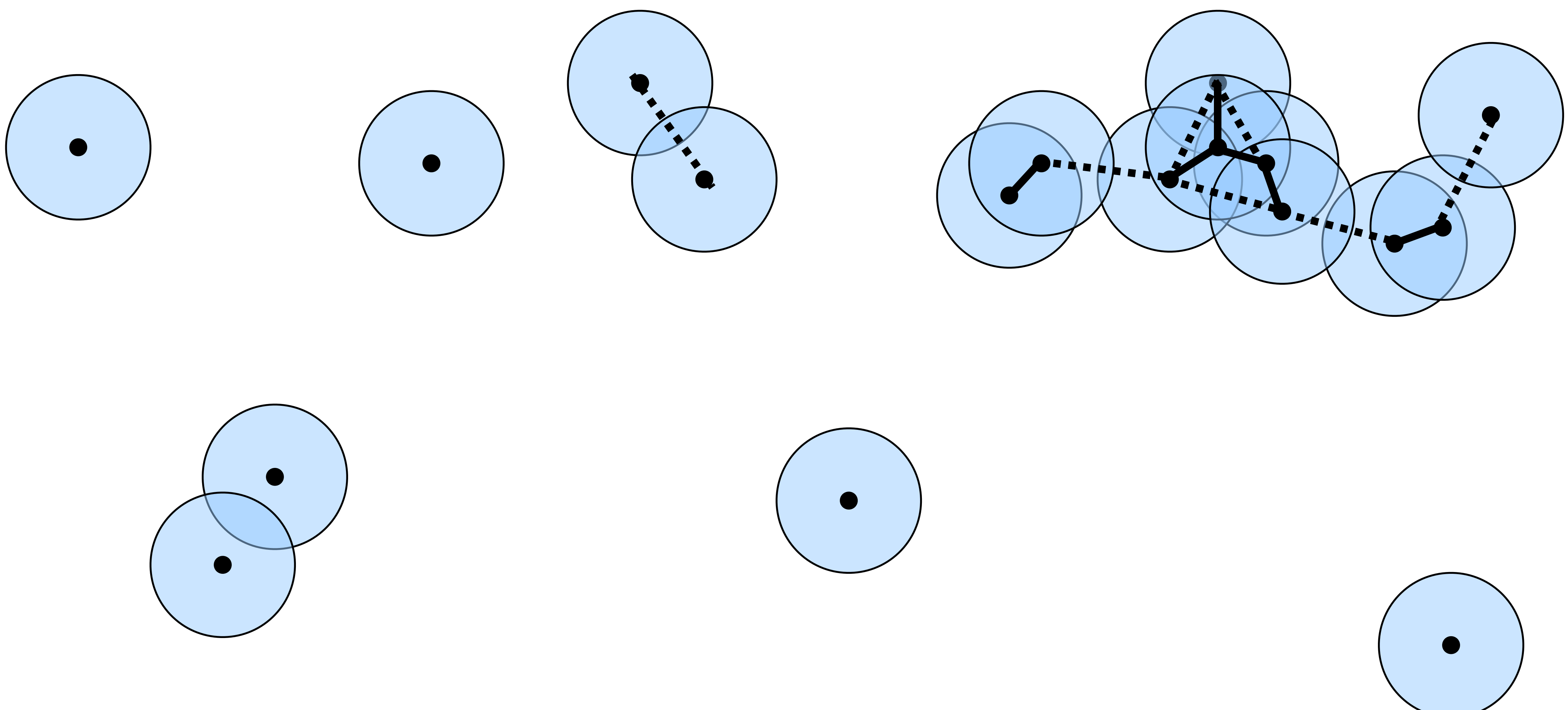

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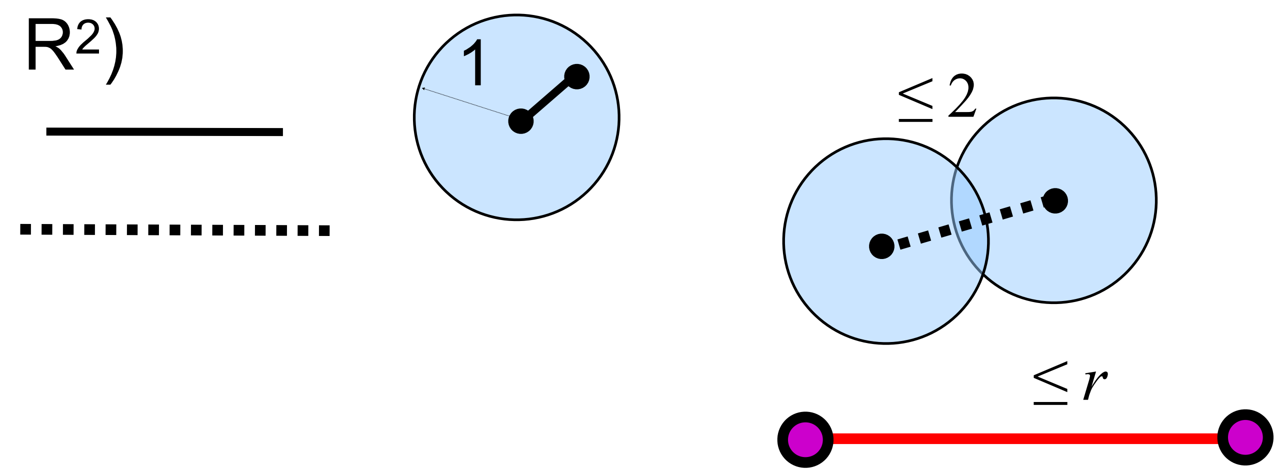

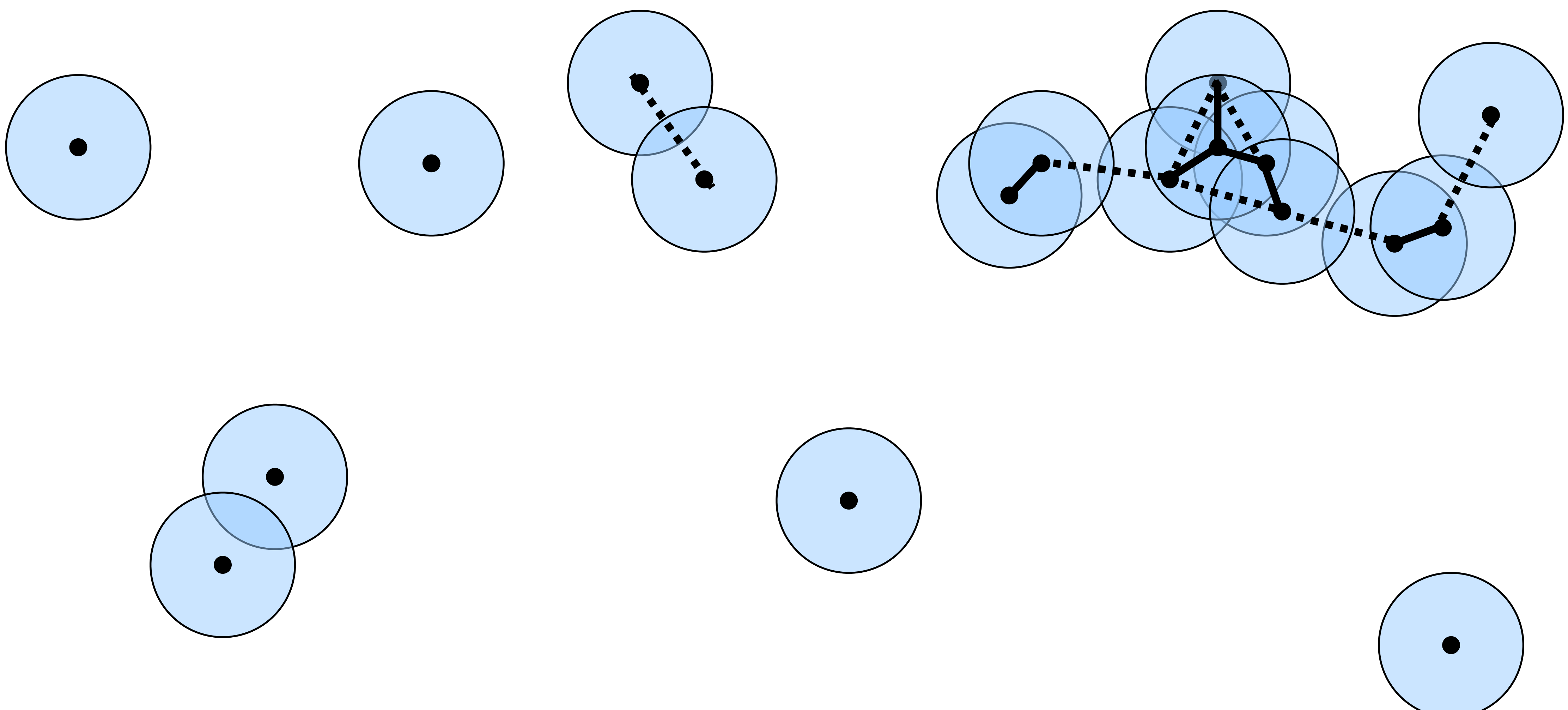

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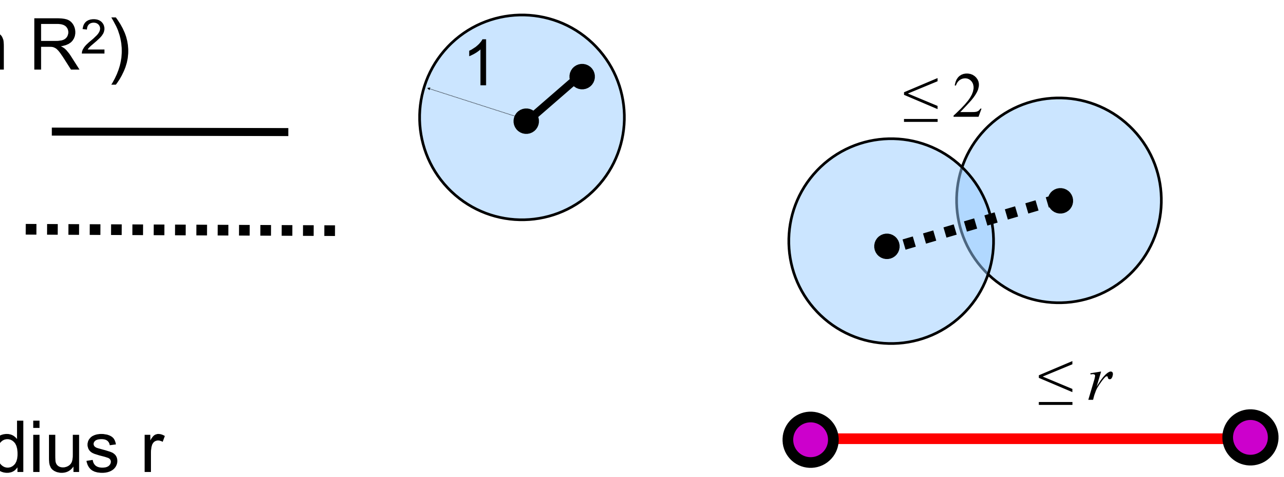
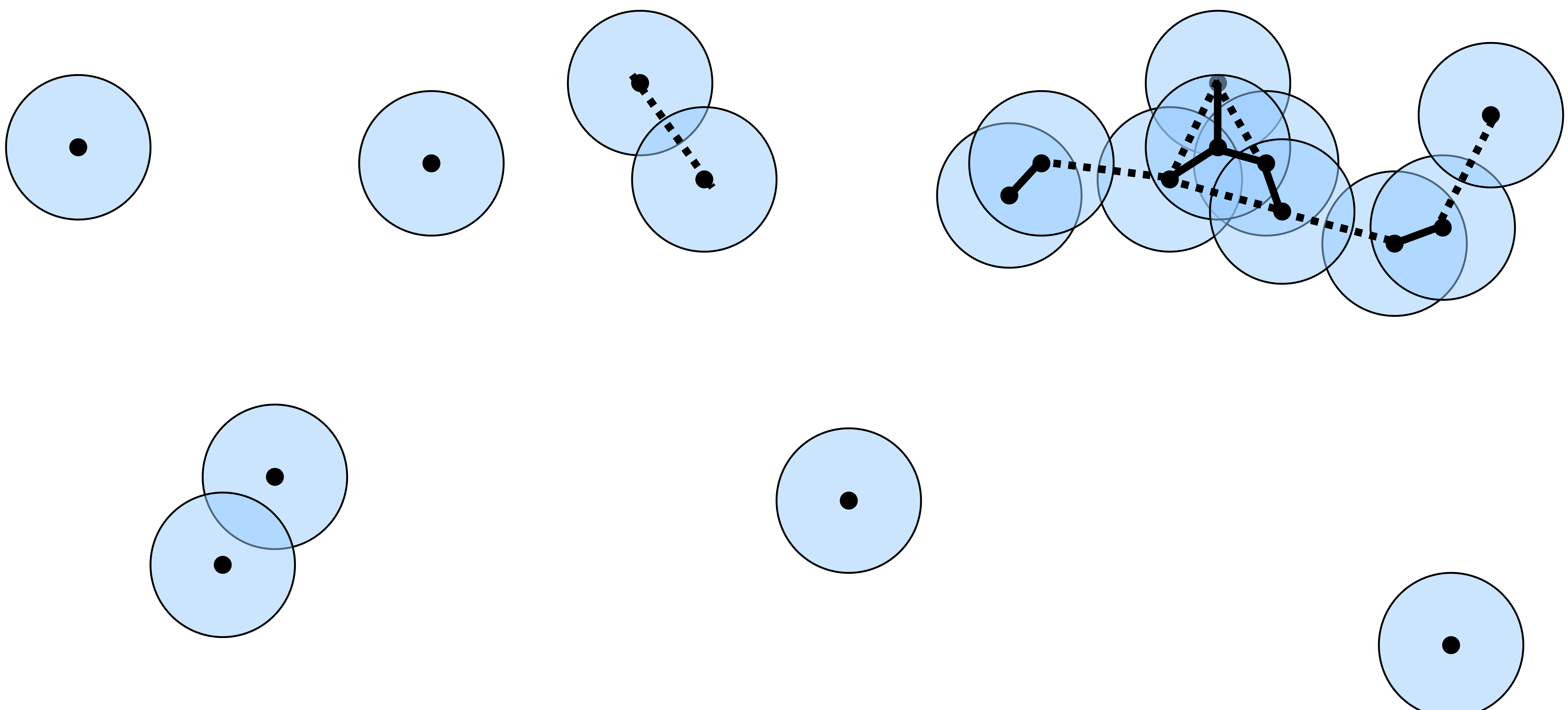


Add: relays with communication radius r



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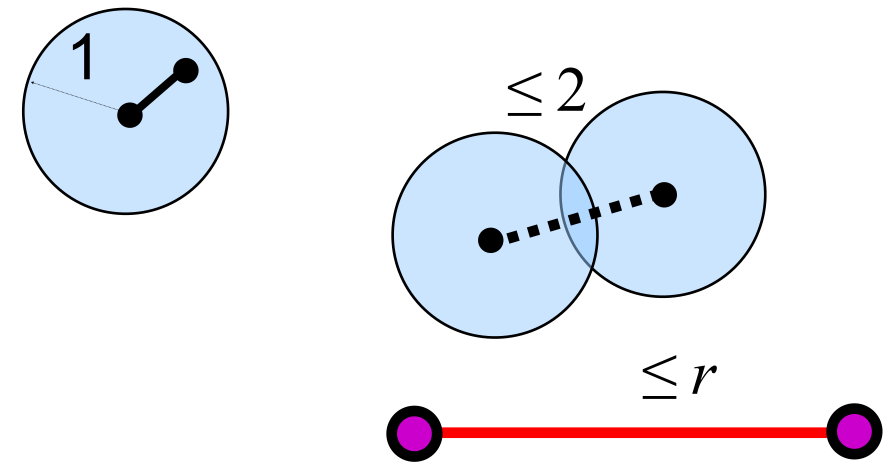

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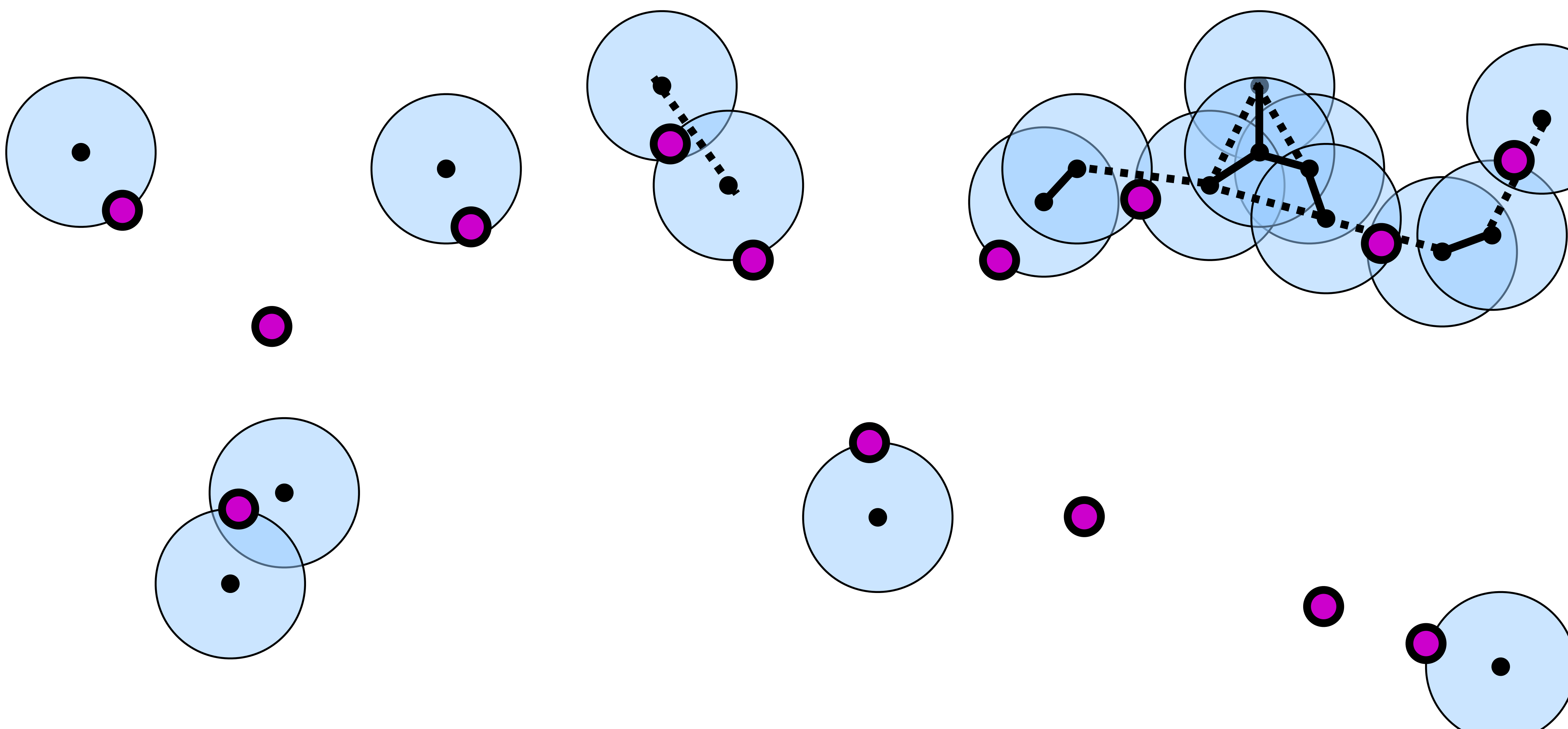

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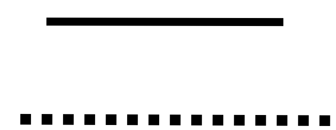
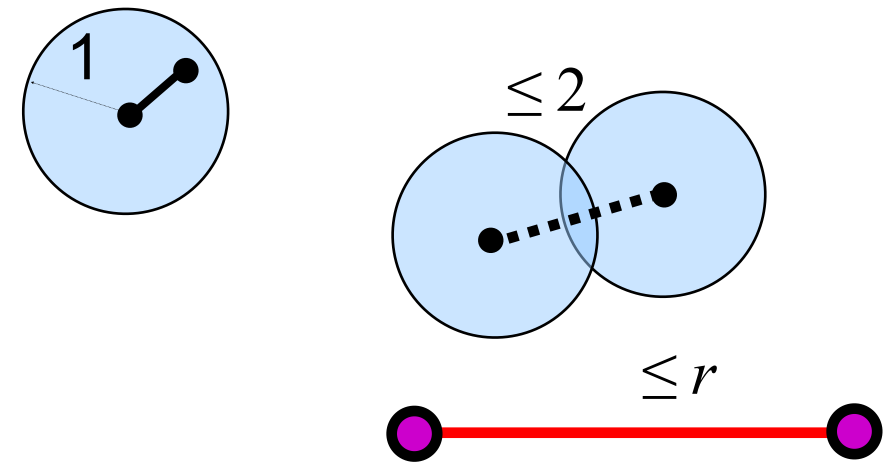
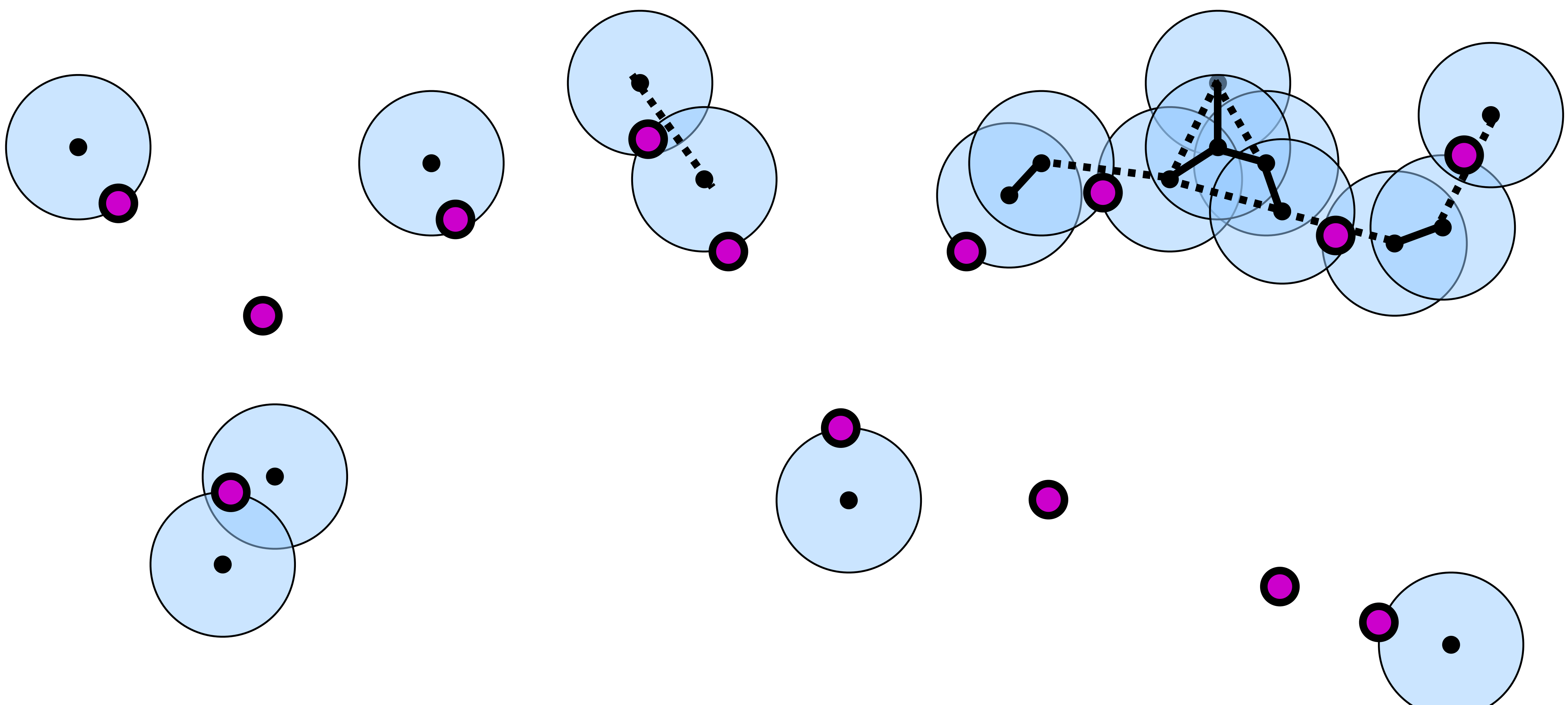




Add: relays with communication radius r

Goal: min # relays (communication radius *r*)



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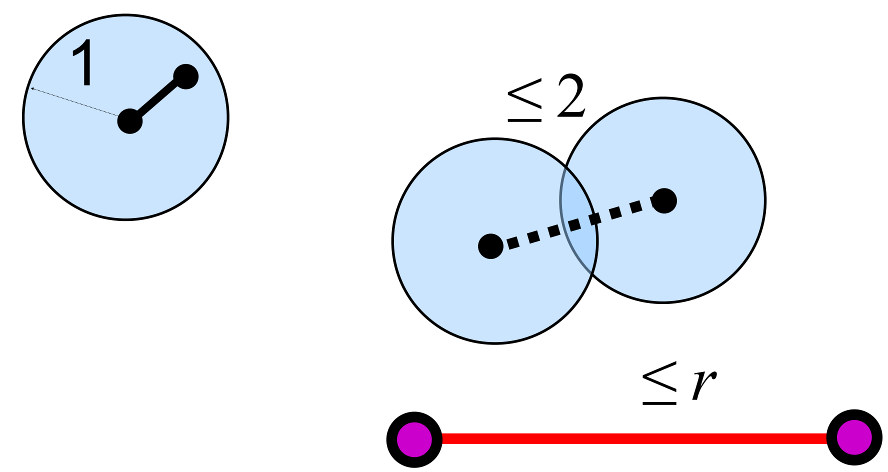

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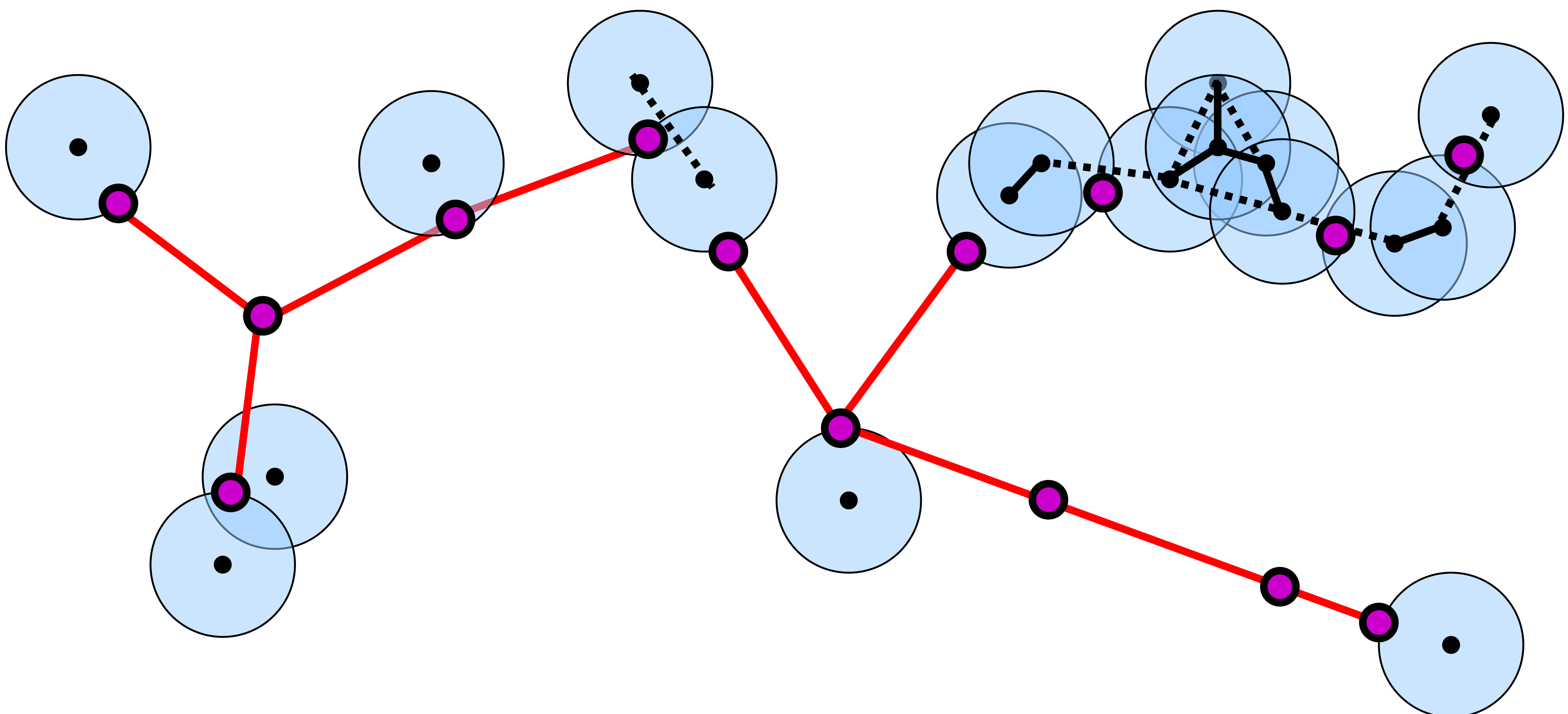

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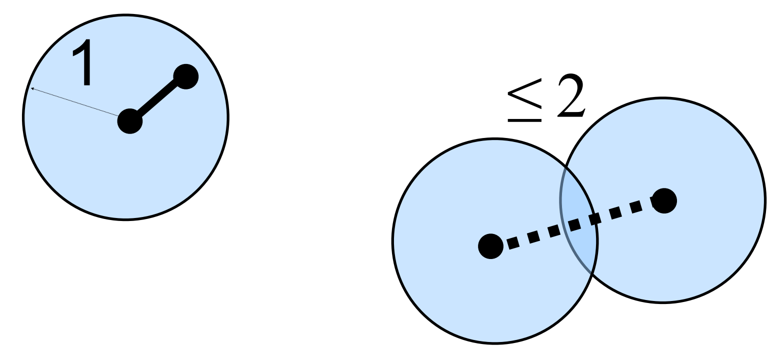
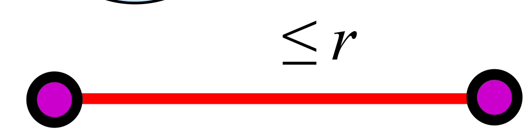
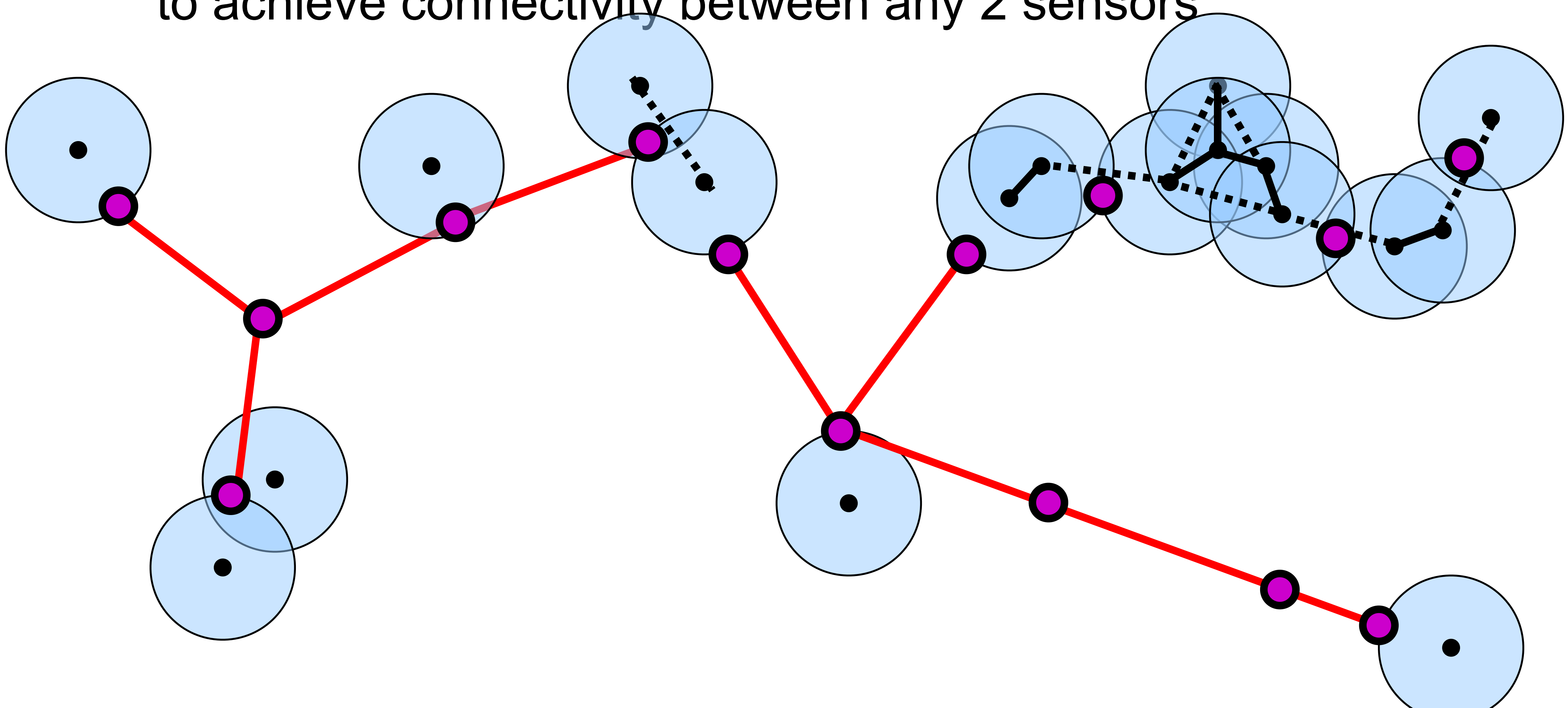




Add: relays with communication radius r

Goal: min # relays (communication radius r)
to achieve connectivity between any 2 sensors



Model of Communication

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- Sensor and relay: communicate if dist ≤ 1

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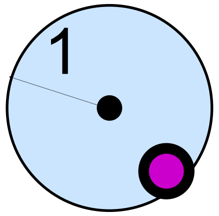

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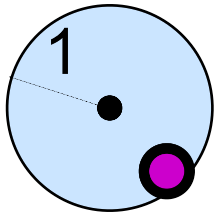

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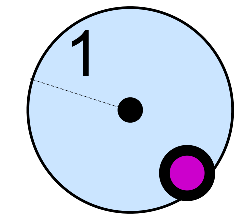
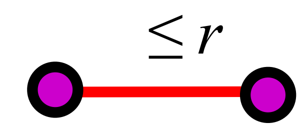


- Relay and relay: communicate if dist ≤ r



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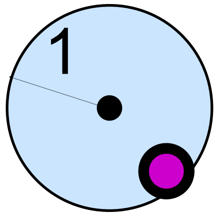

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- Two sensors:
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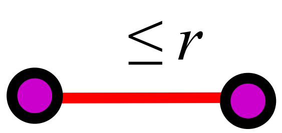
  - Two-tier: no direct communication (only via a *path of relays*)



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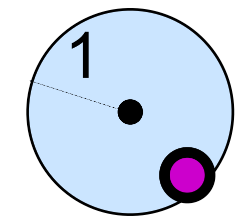

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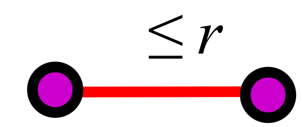

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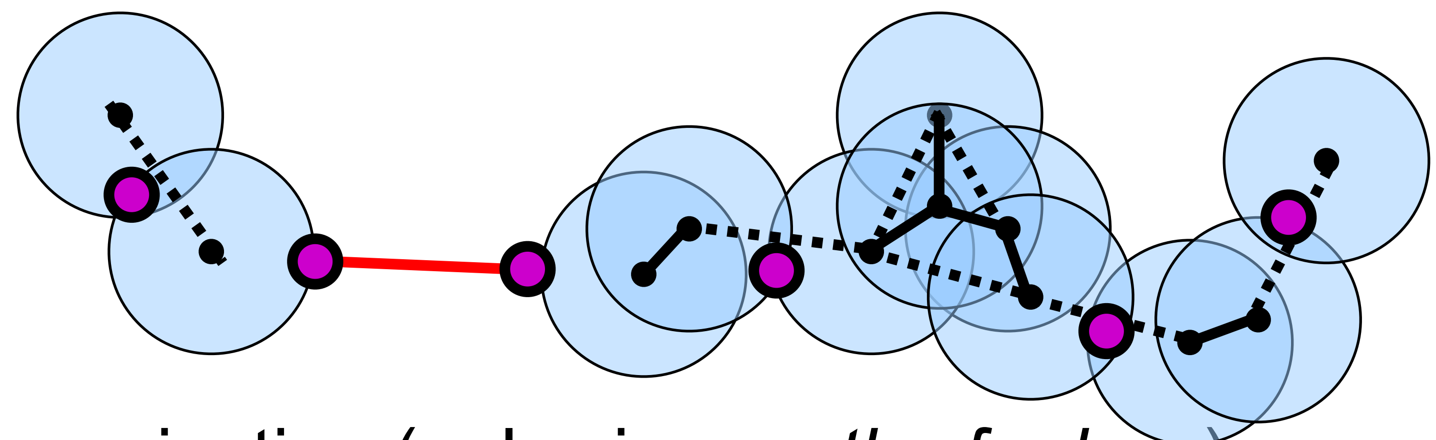

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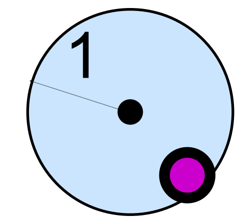

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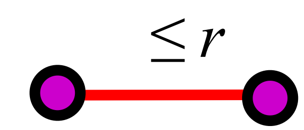

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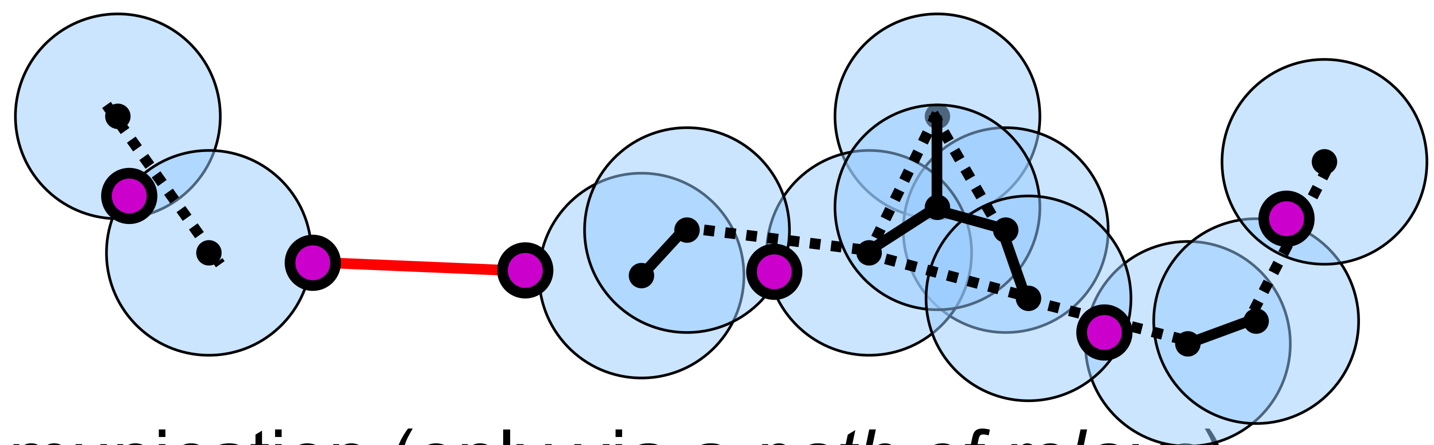

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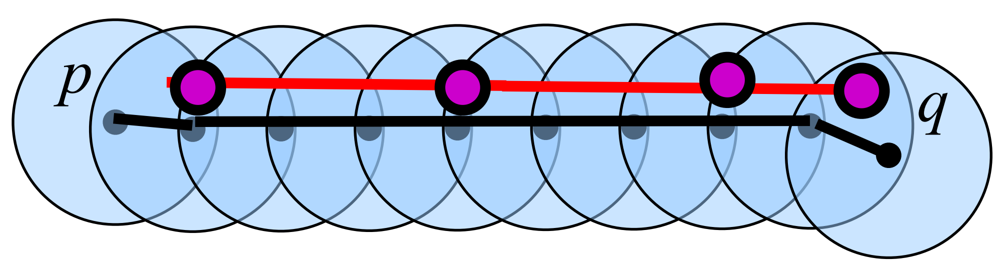

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  [Srinivas, Zussman, Modiano '06]

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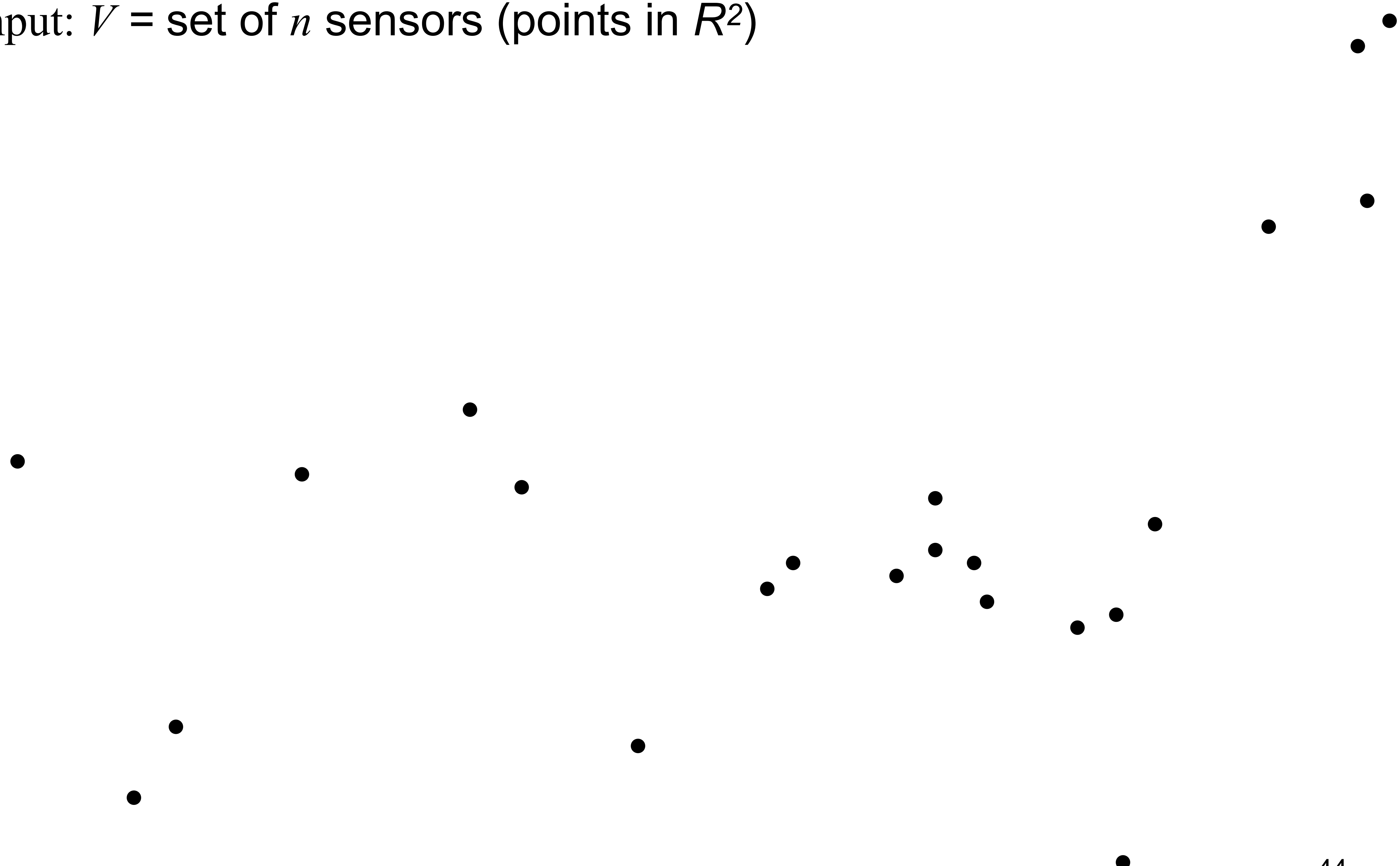

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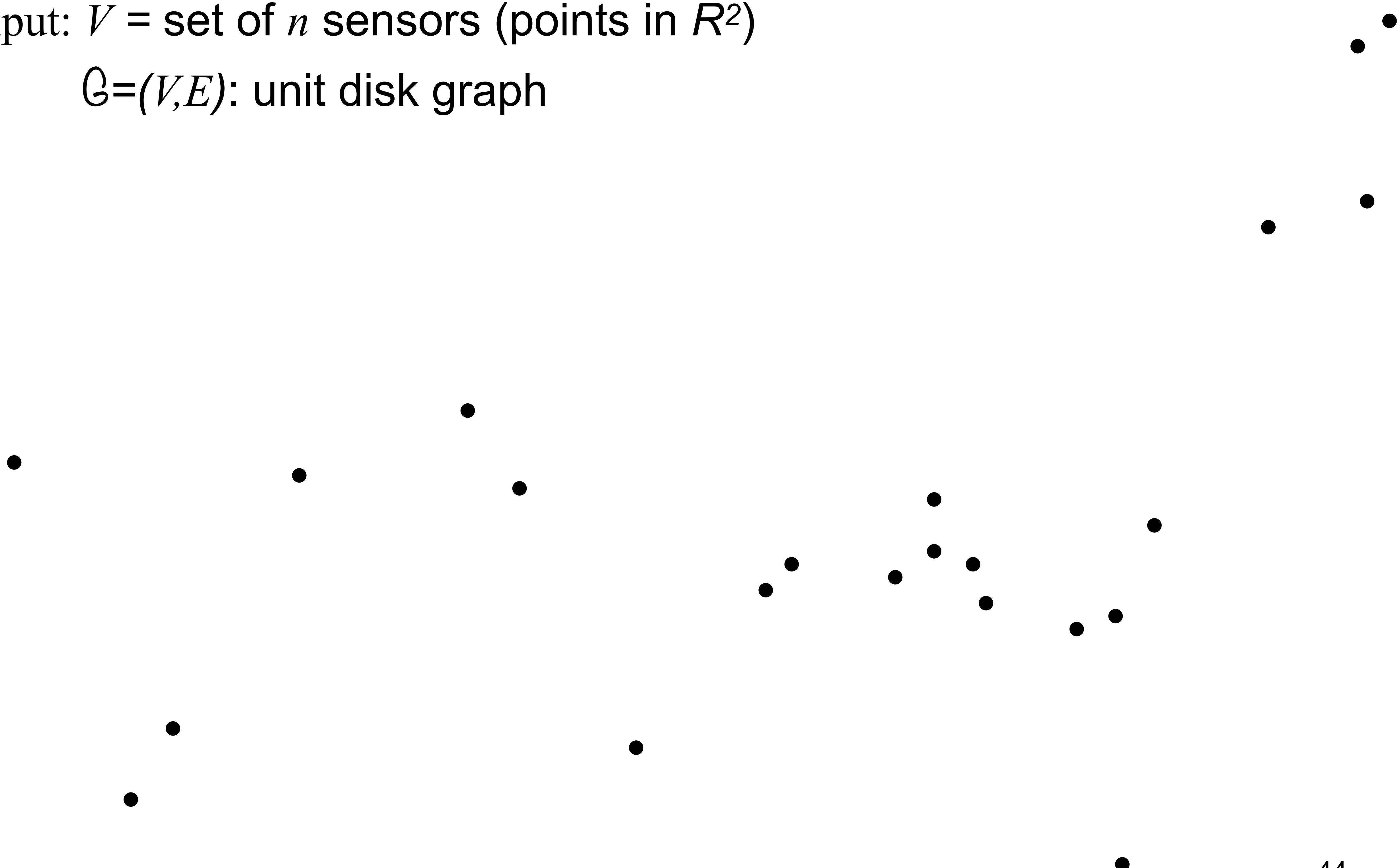

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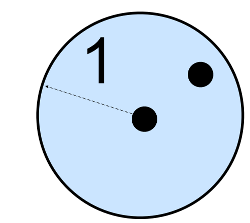

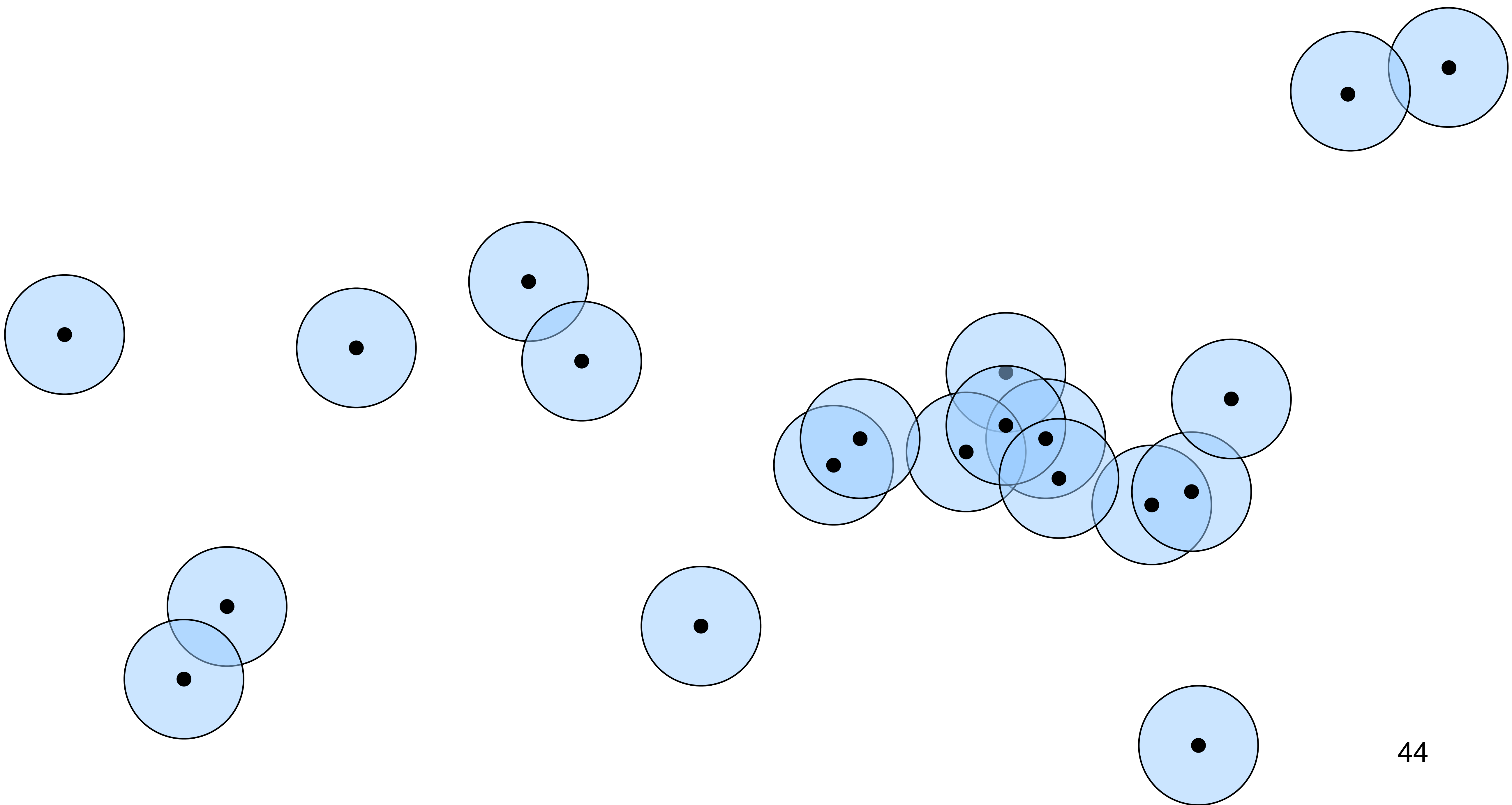

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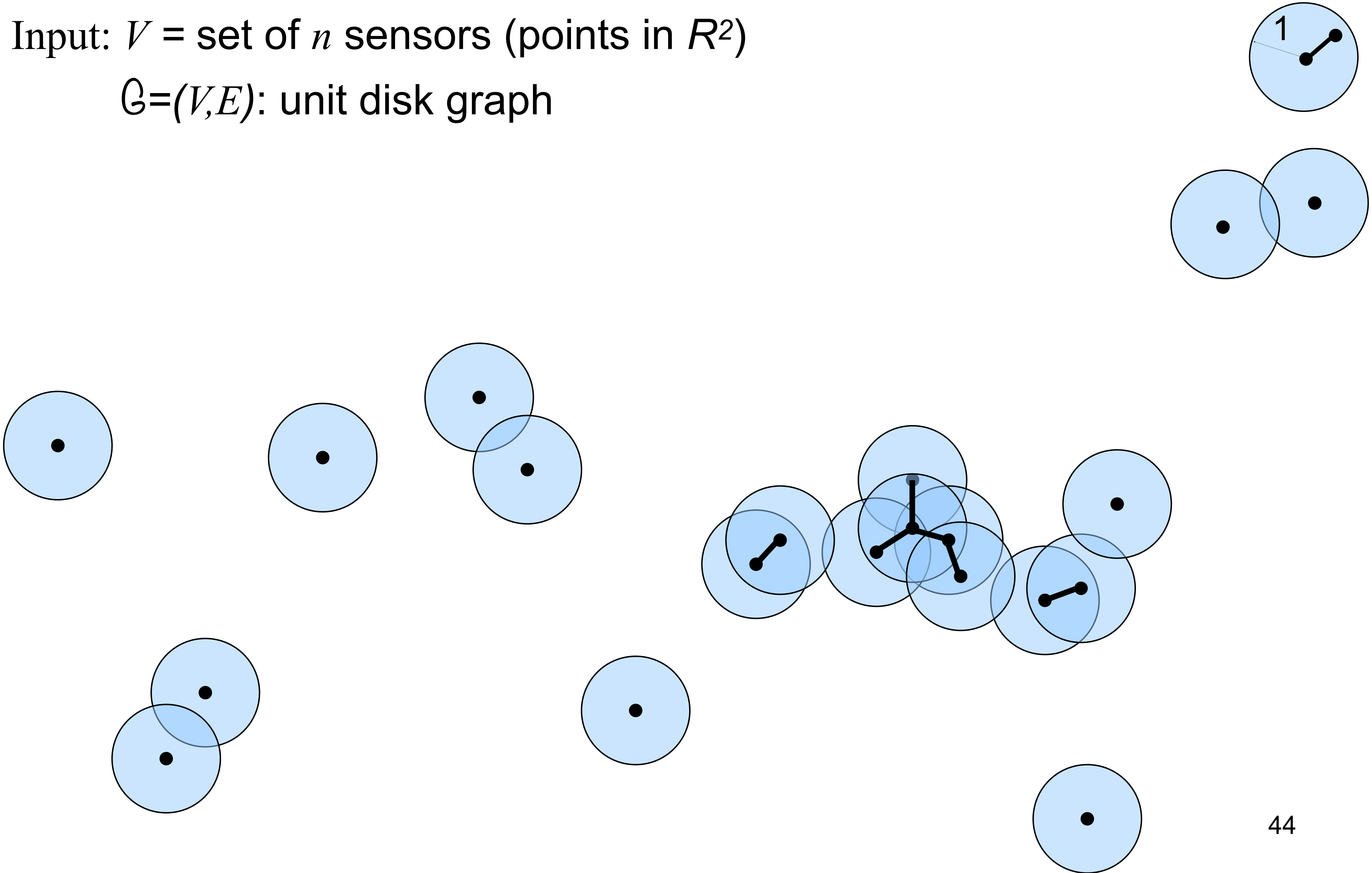

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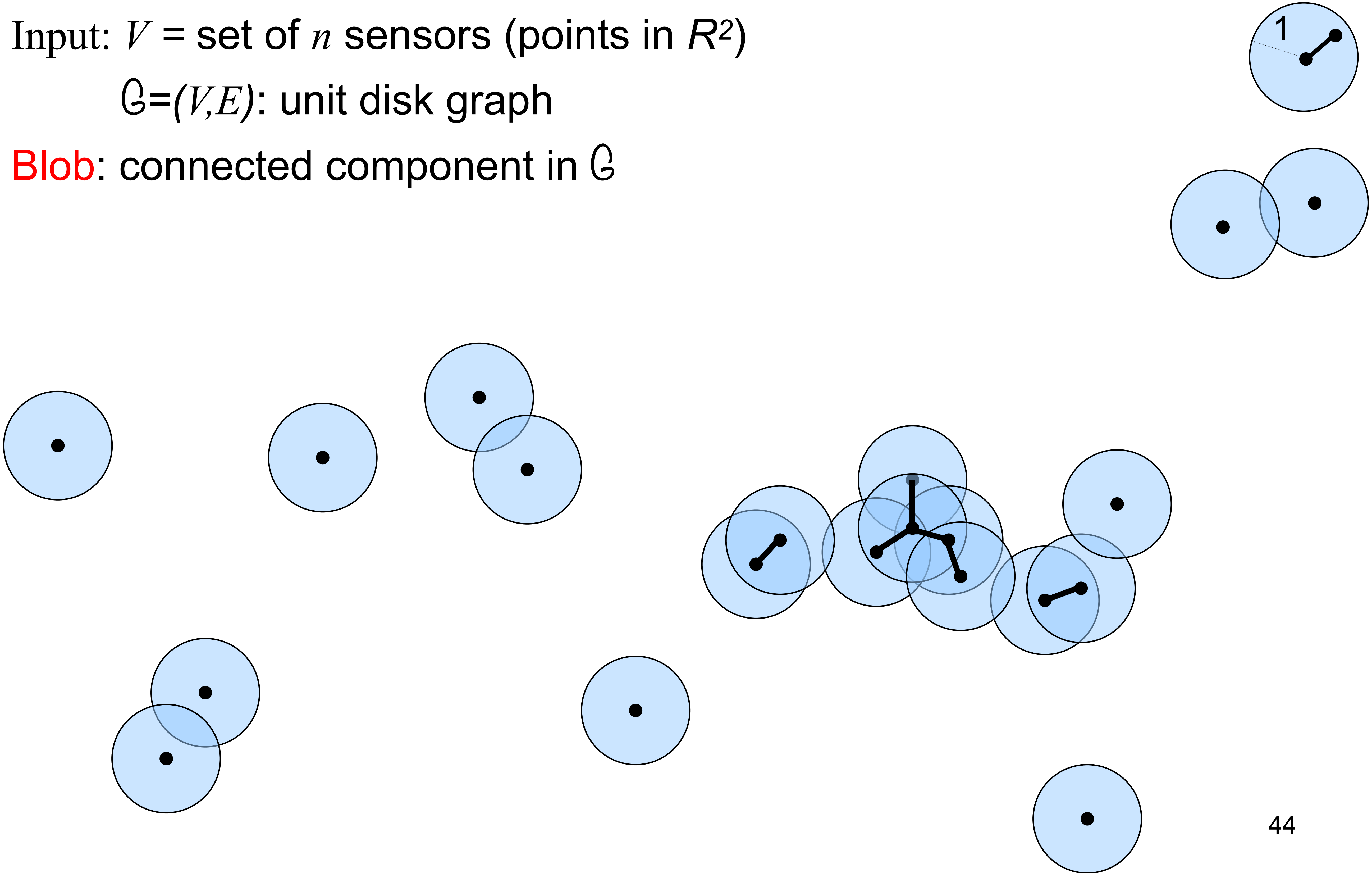

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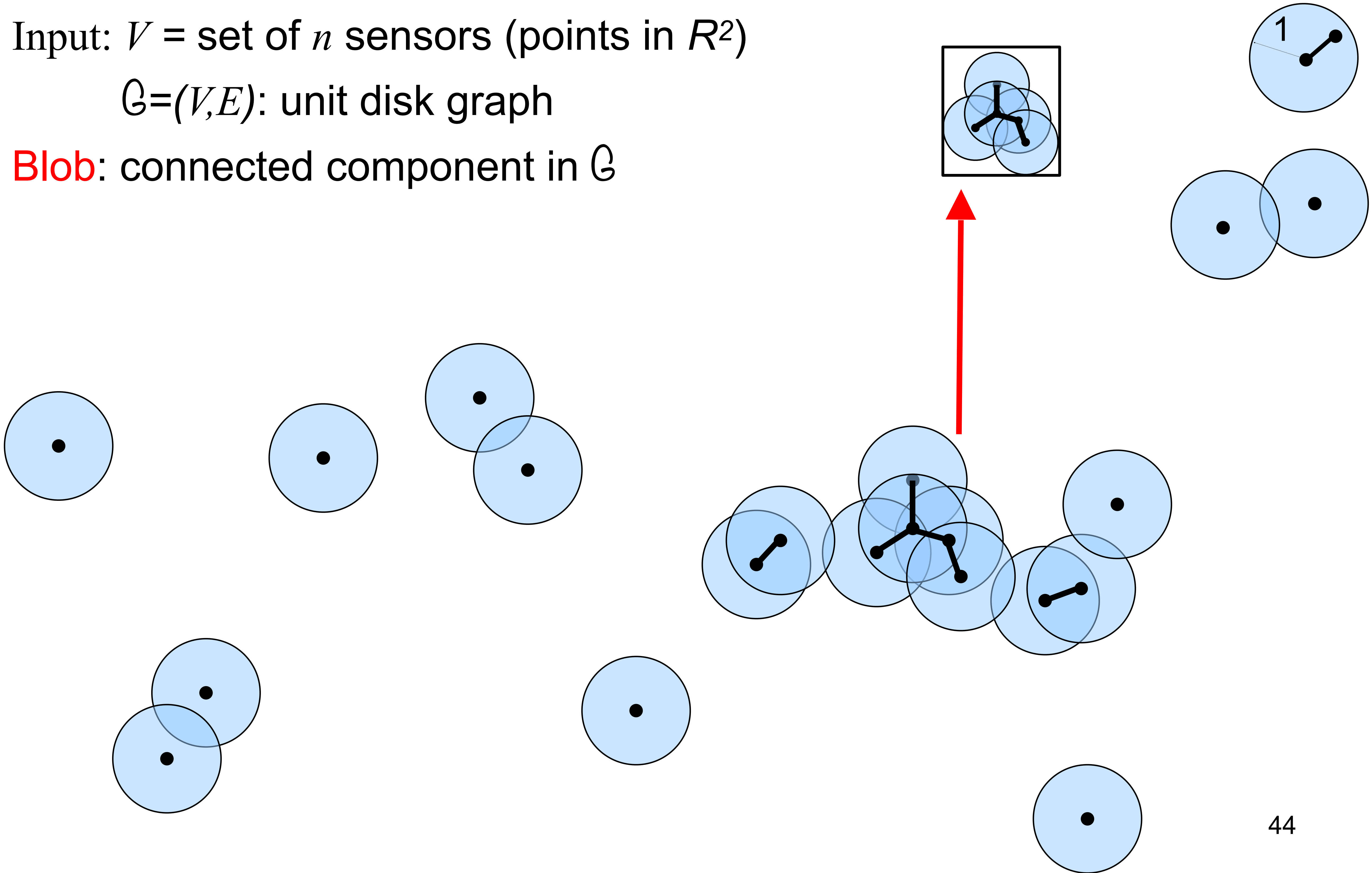

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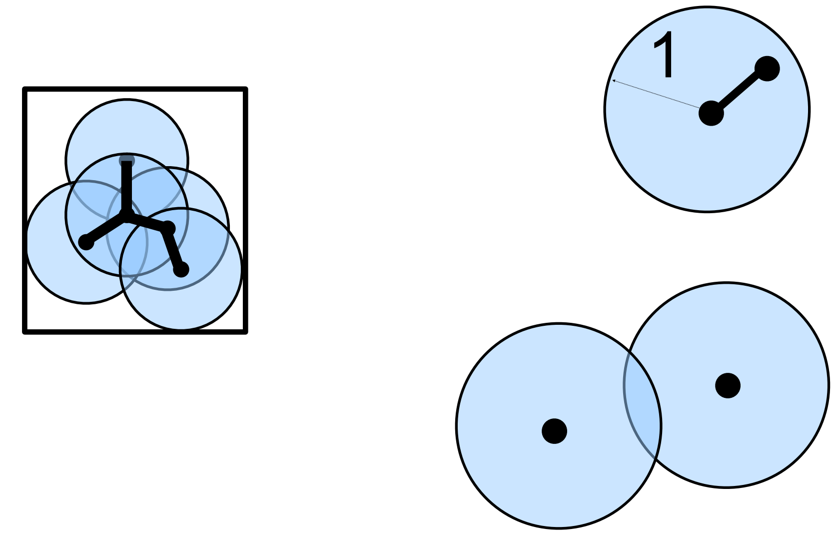

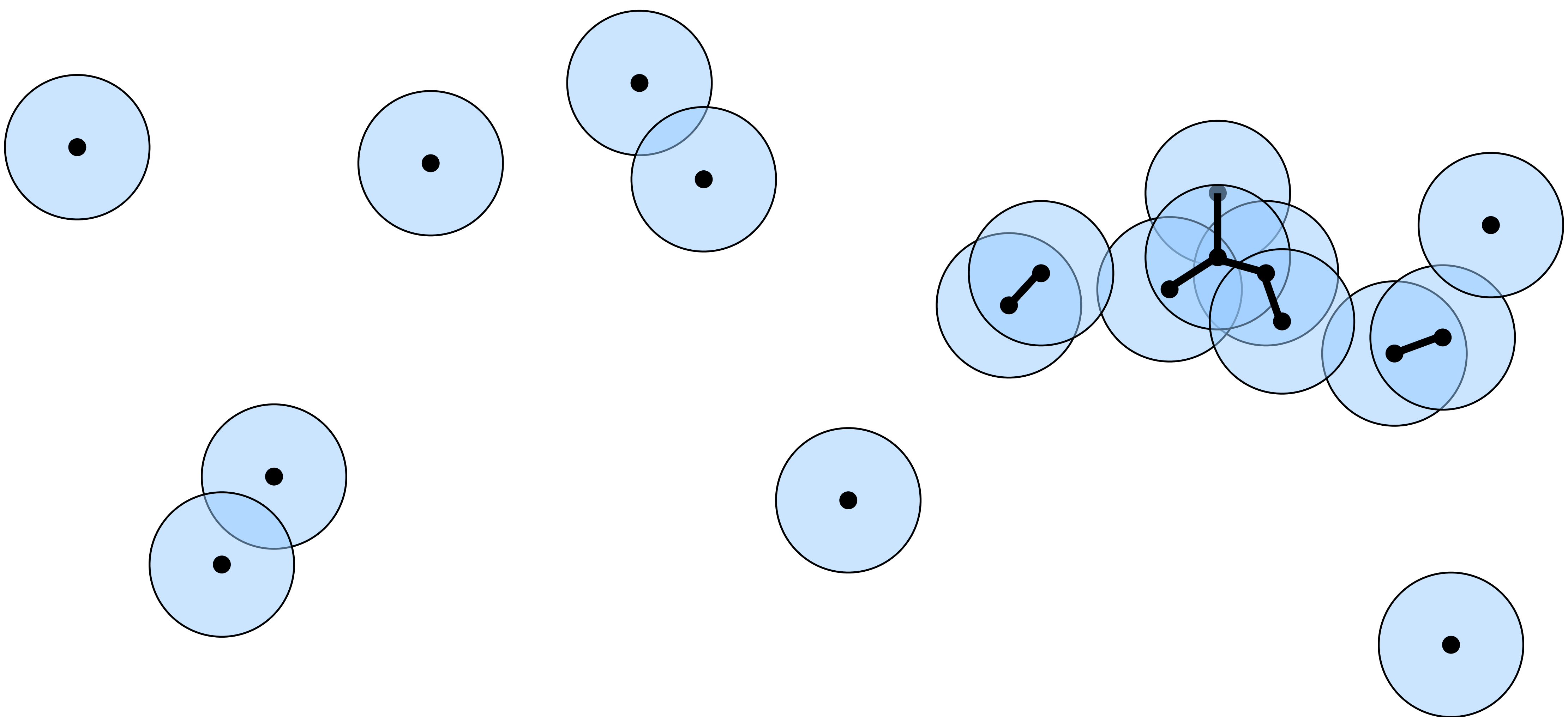

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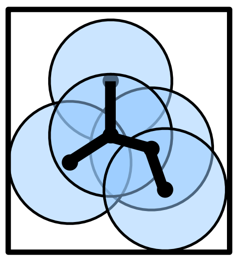

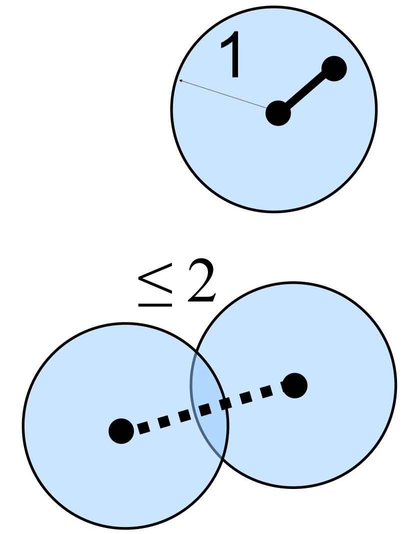

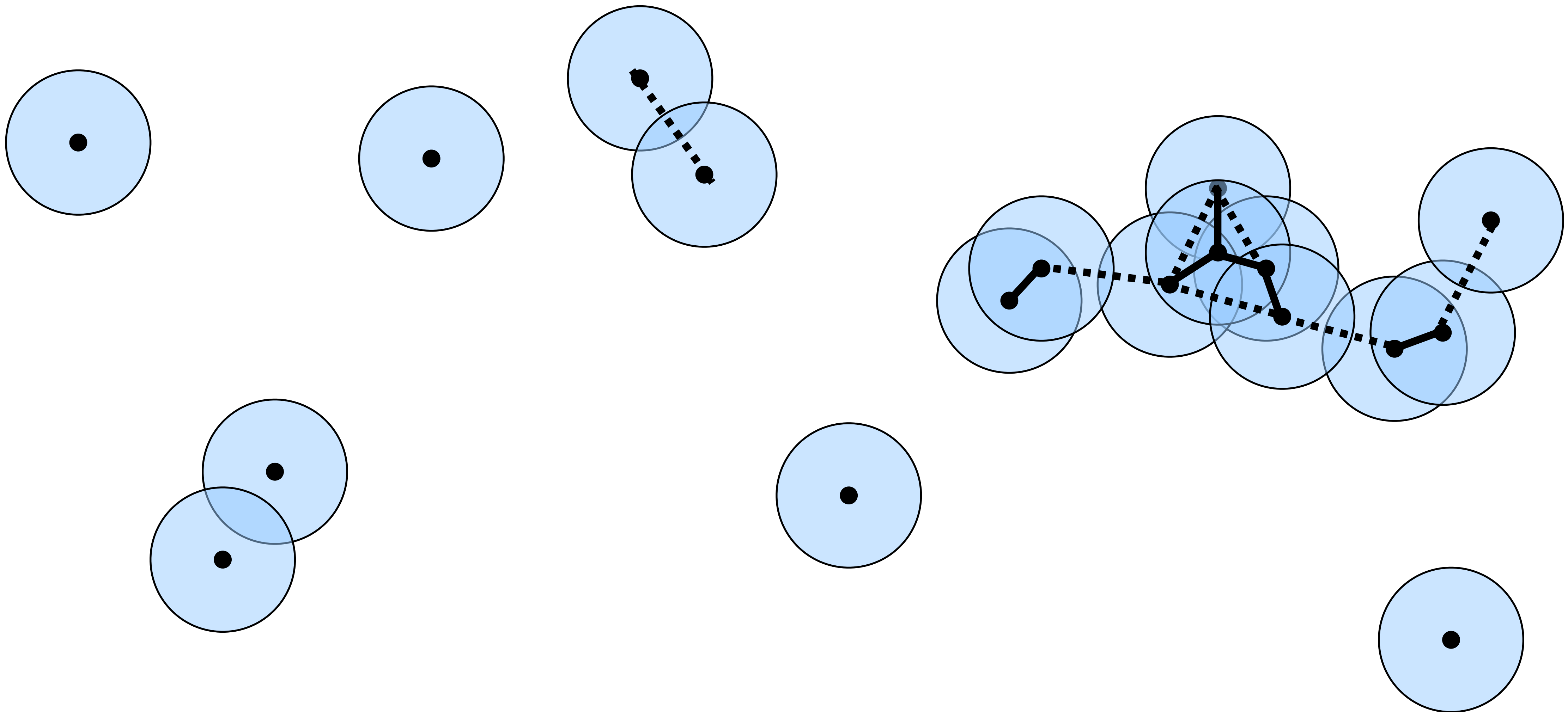

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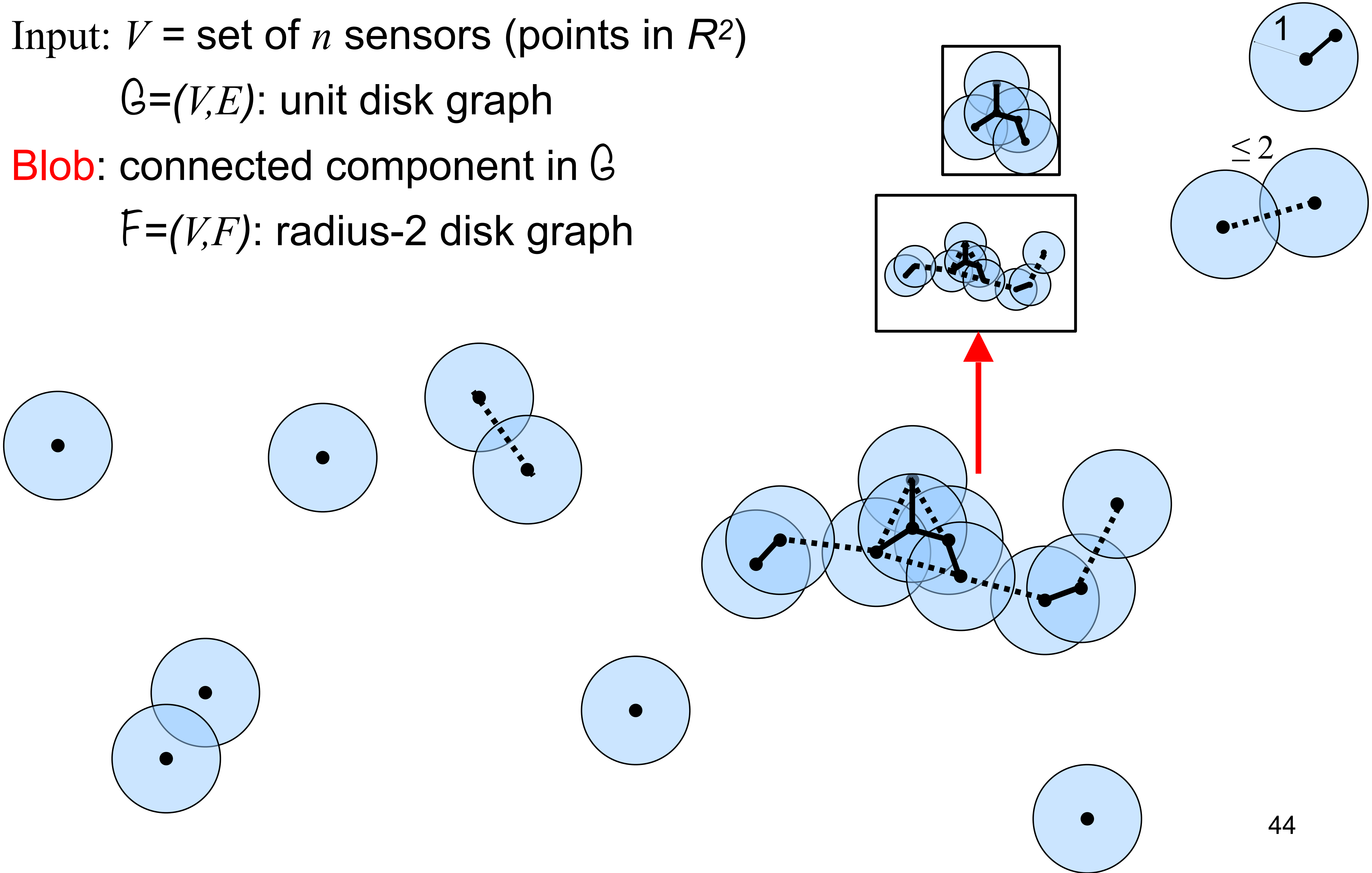

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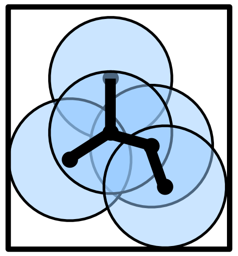

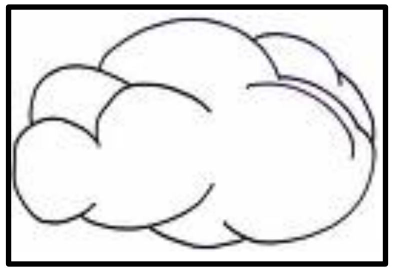

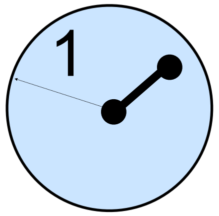

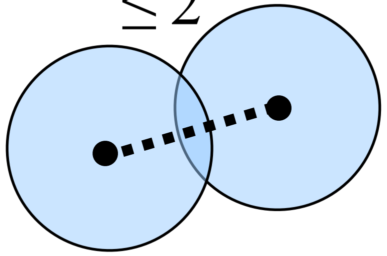

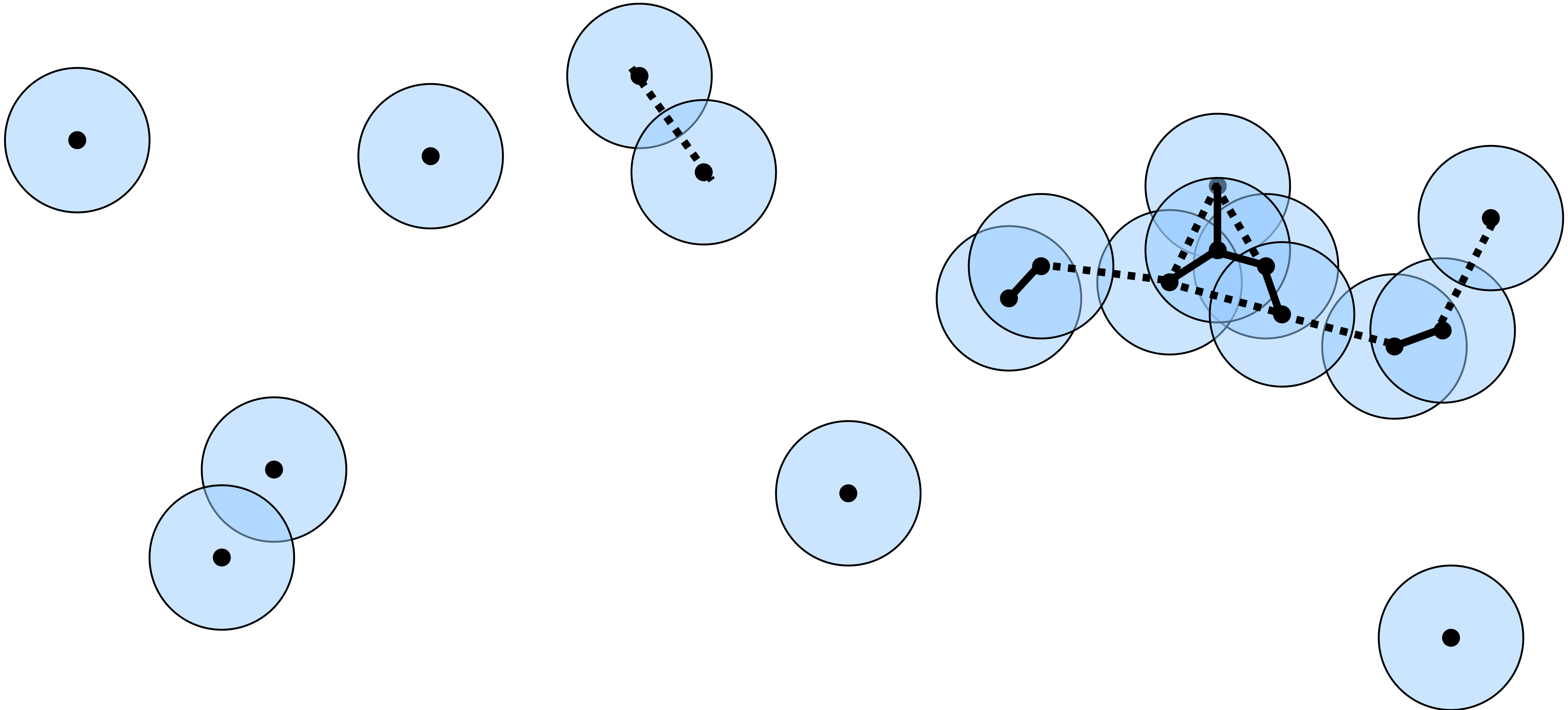

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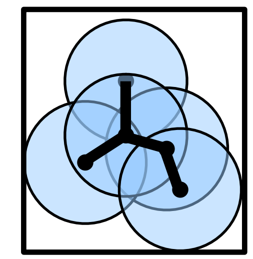

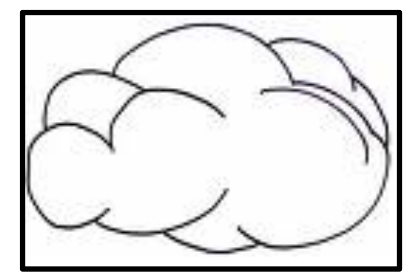

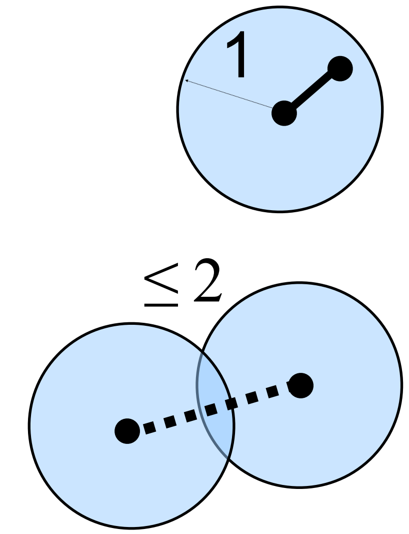

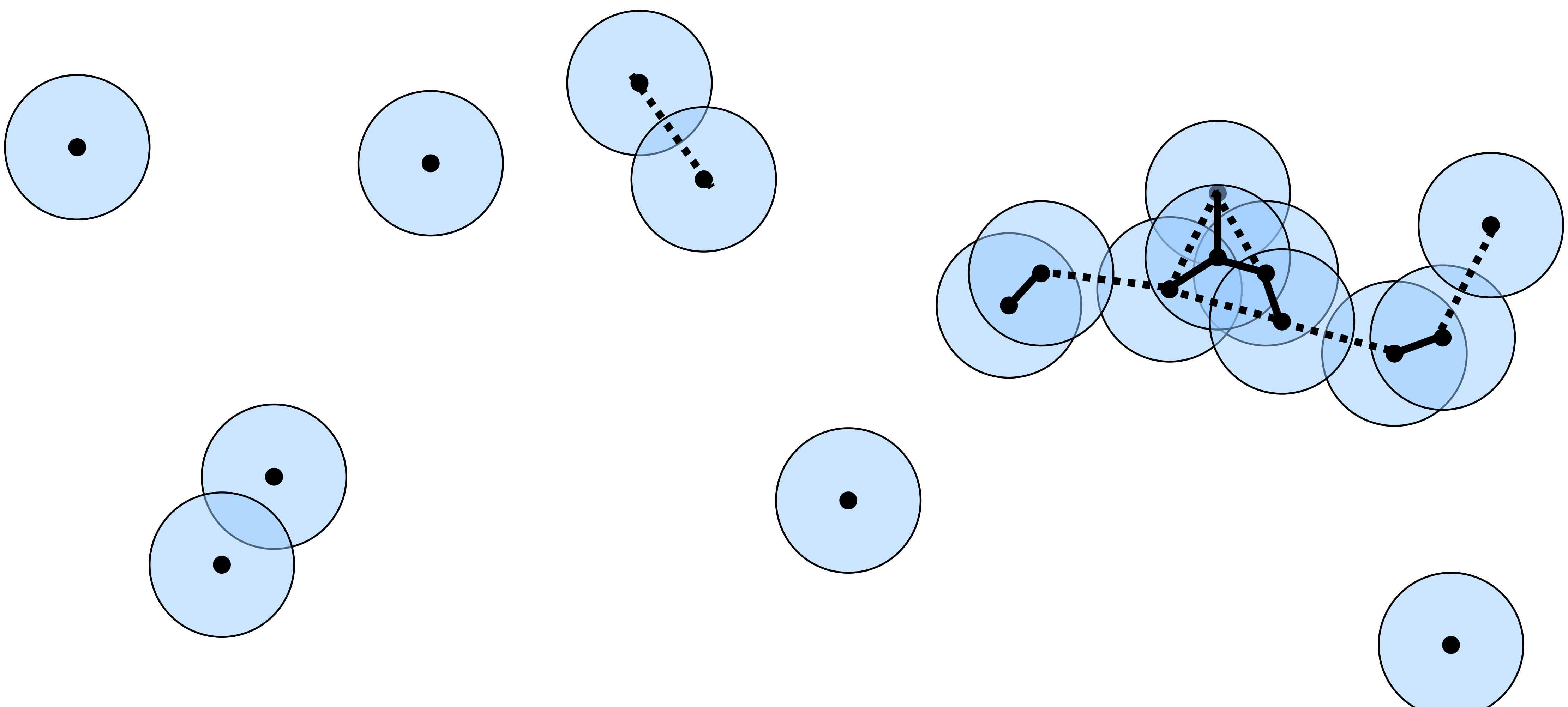

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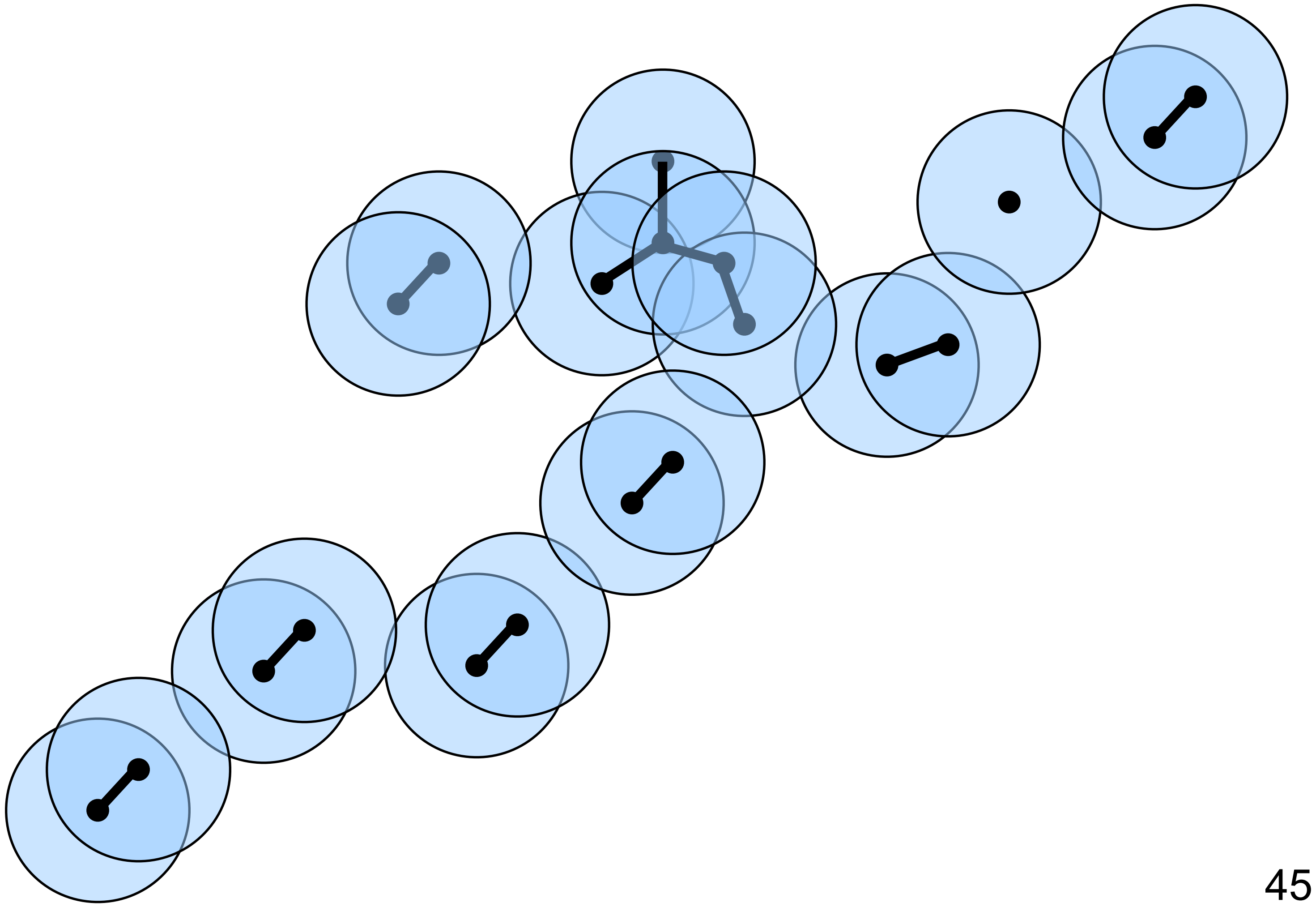

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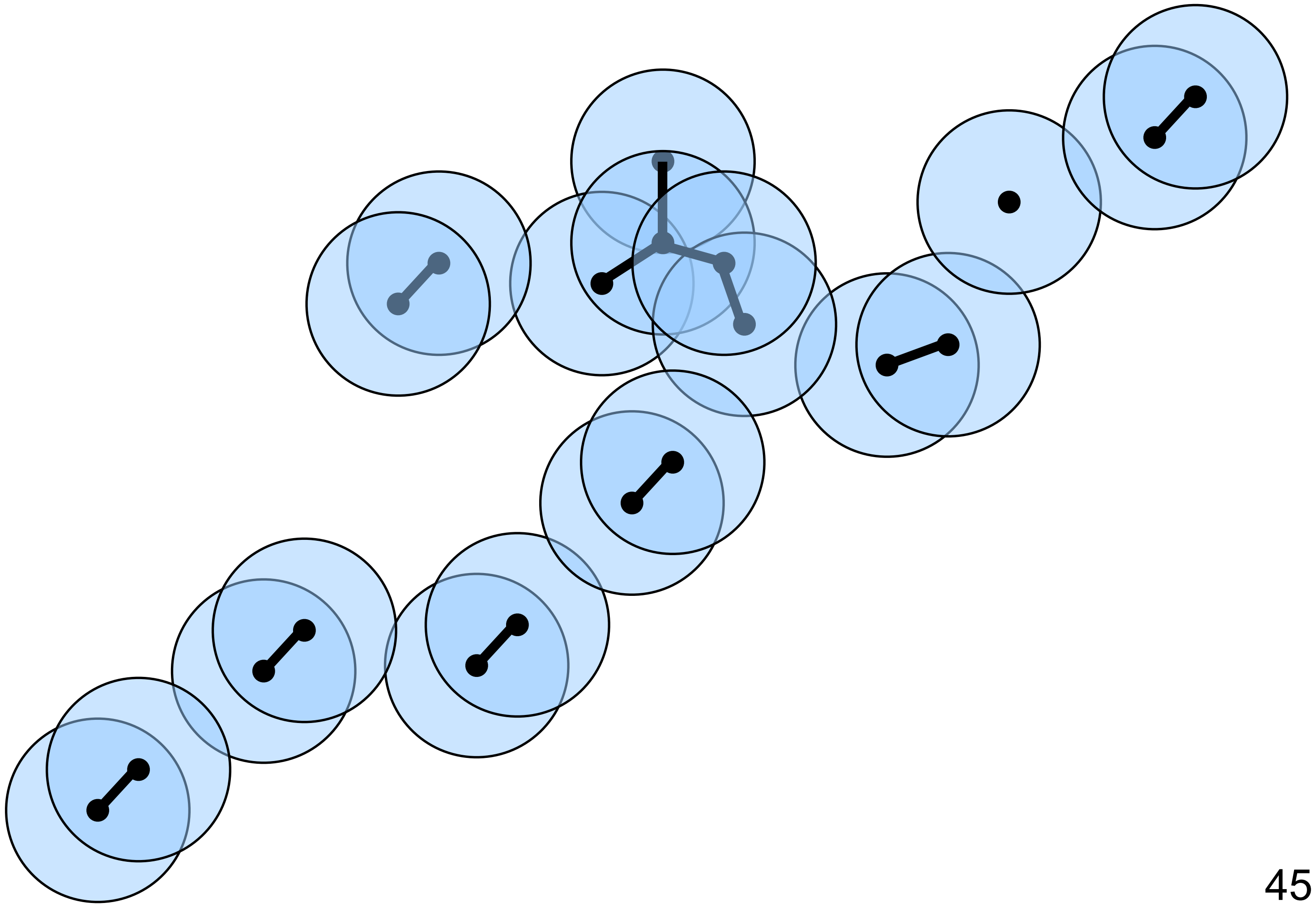

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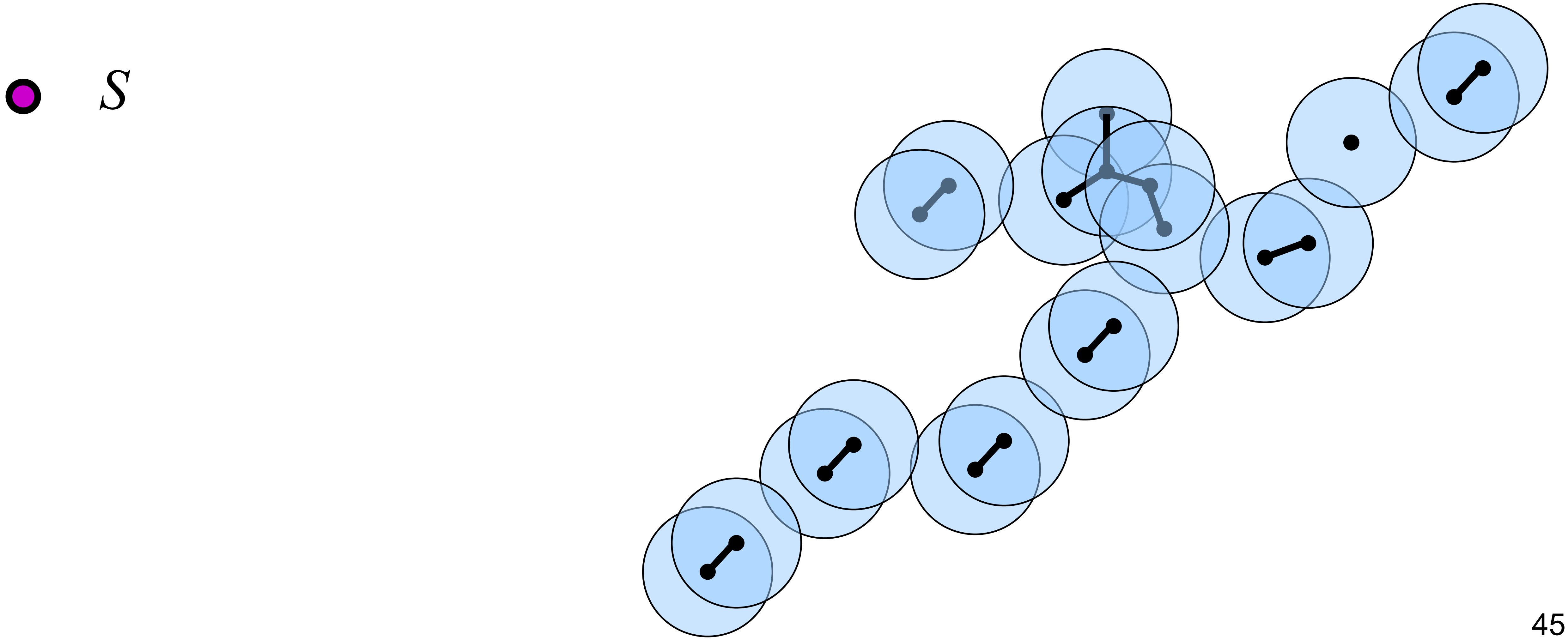

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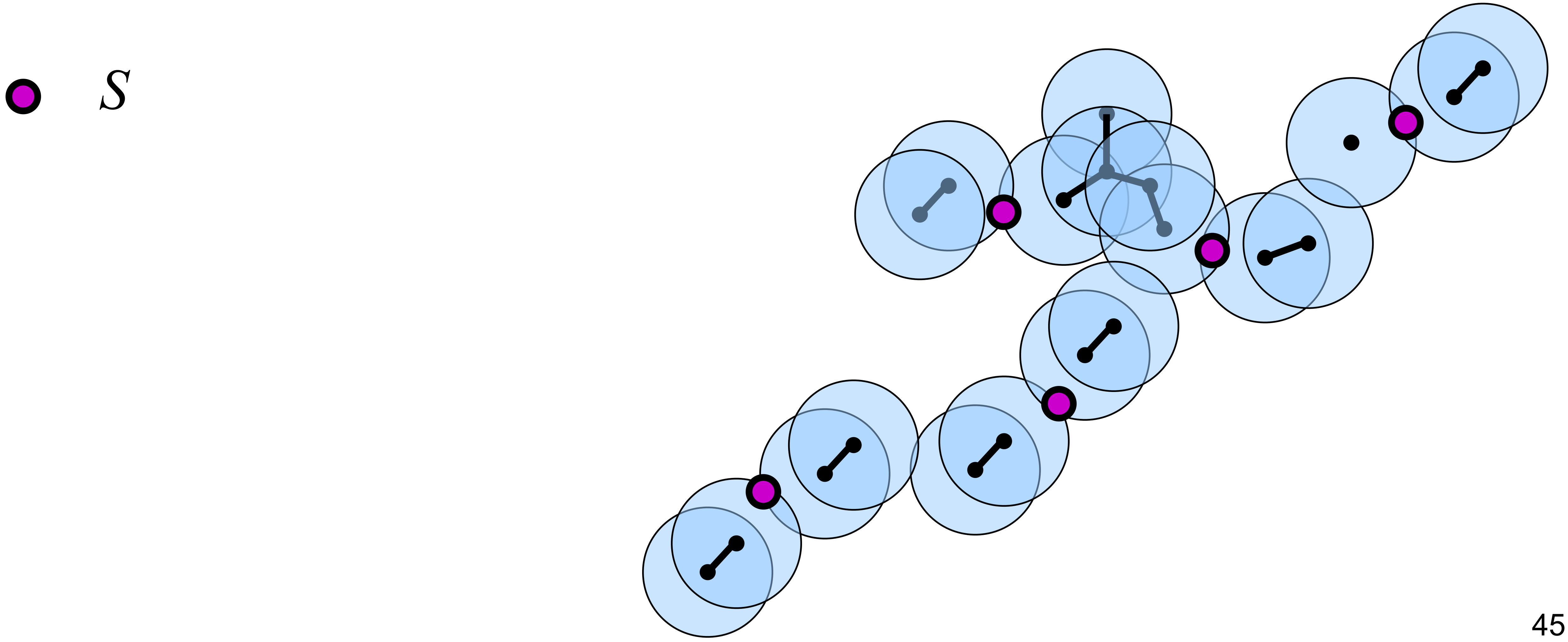

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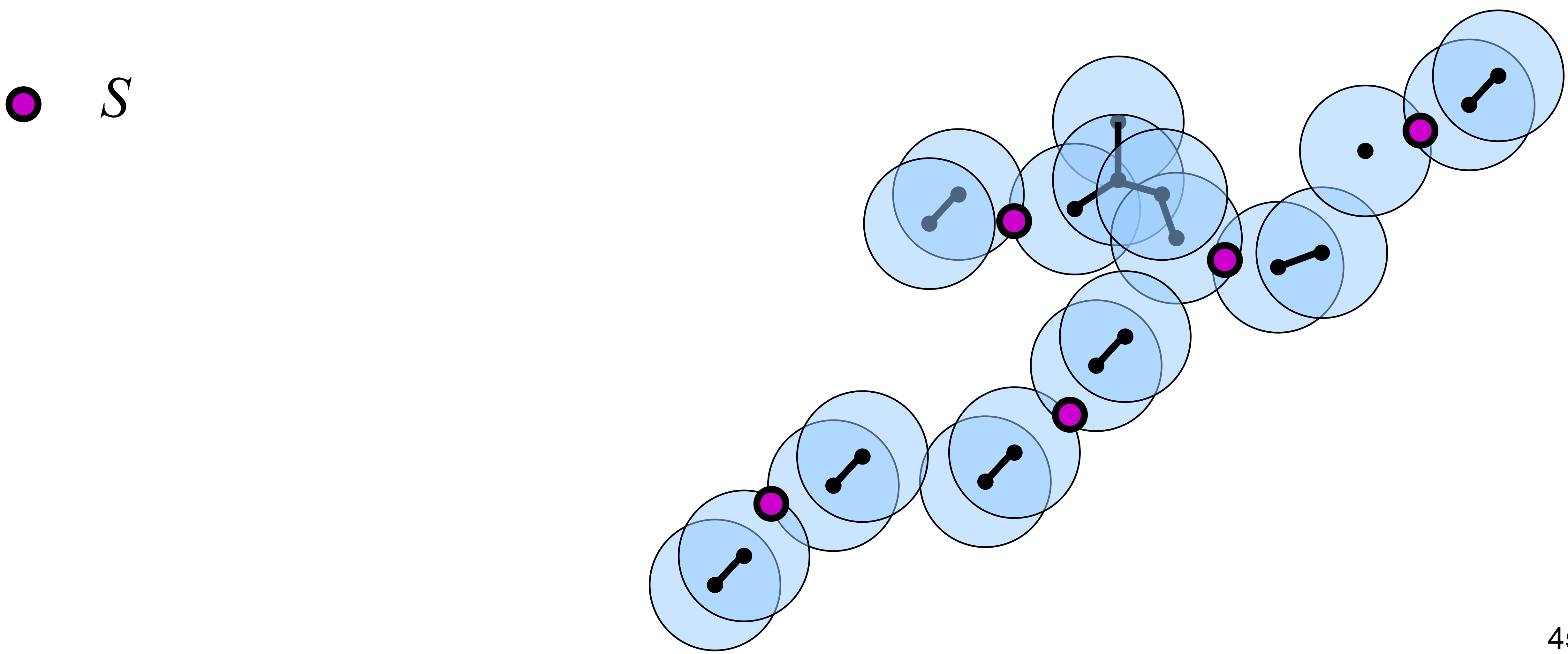

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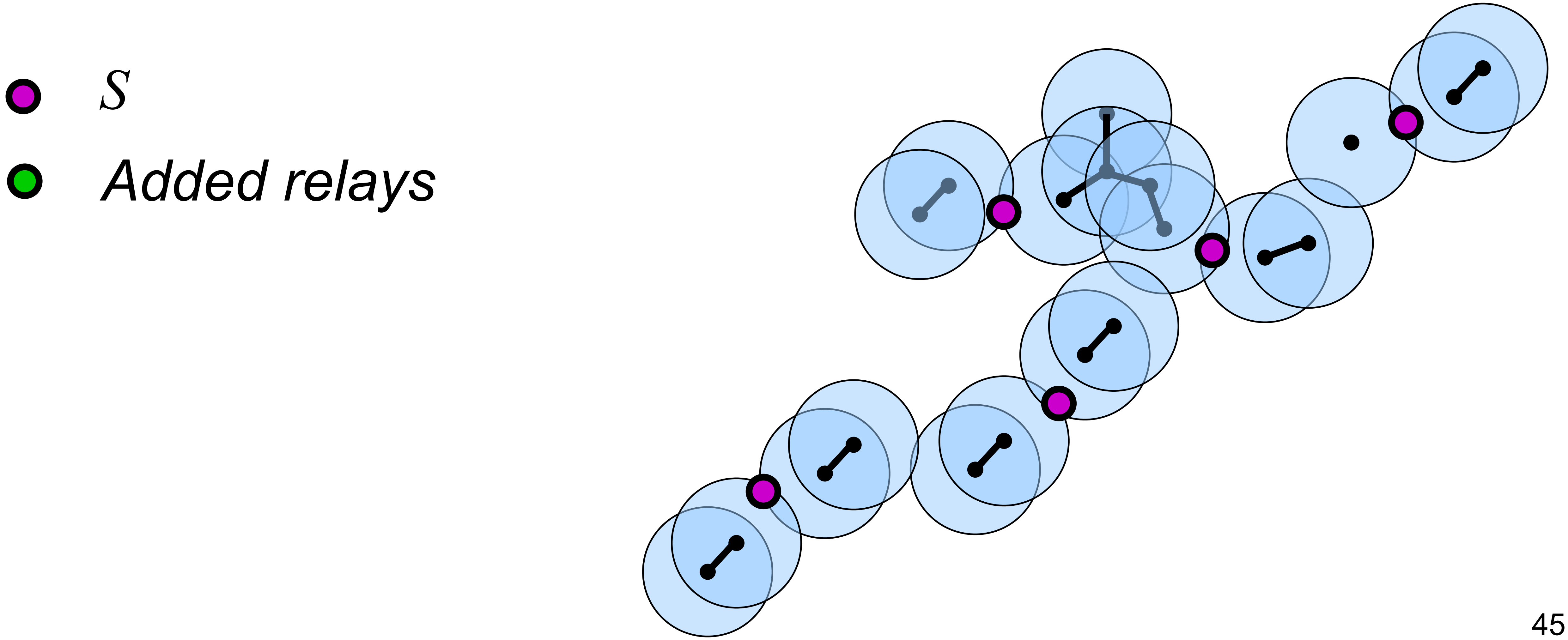

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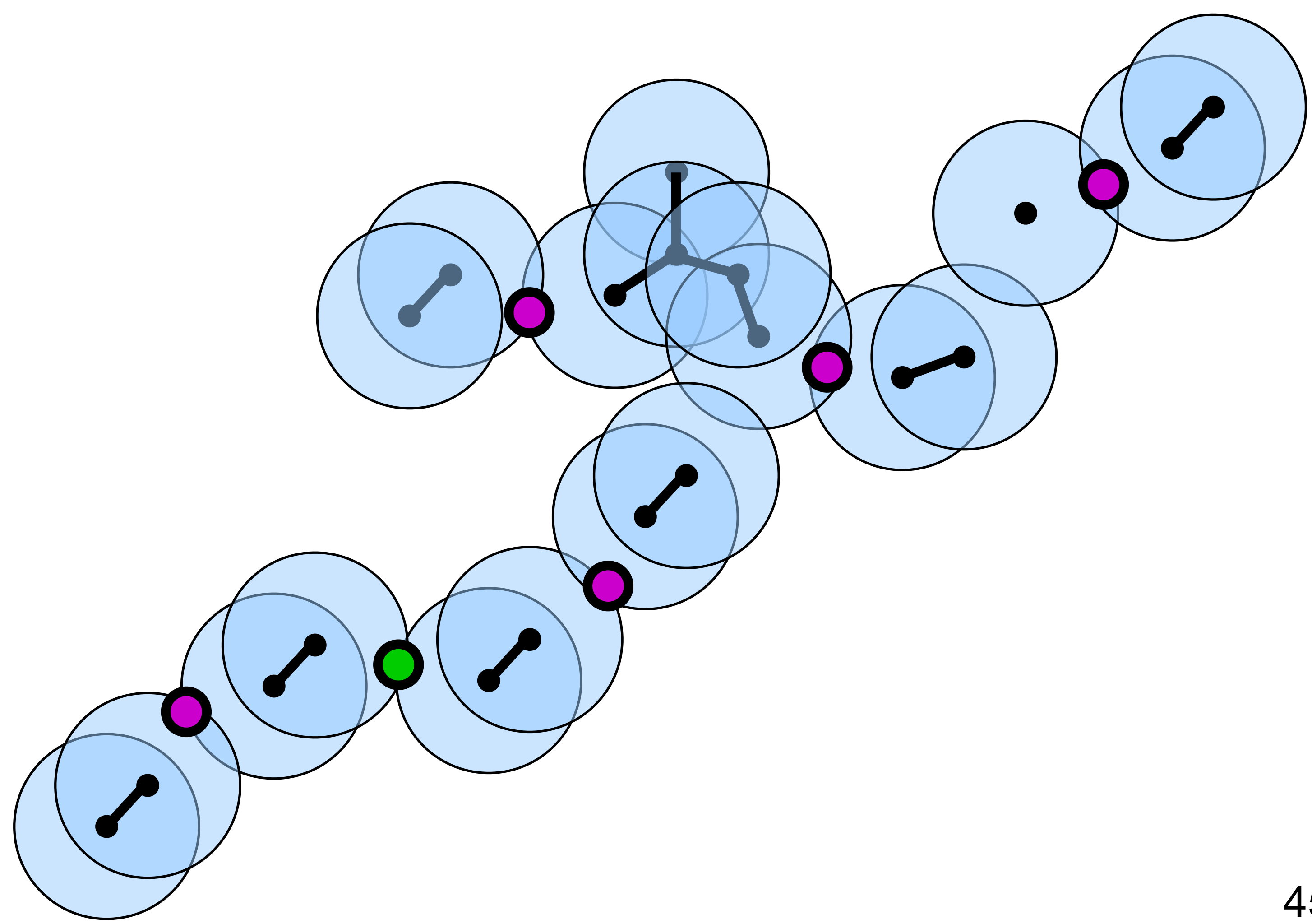

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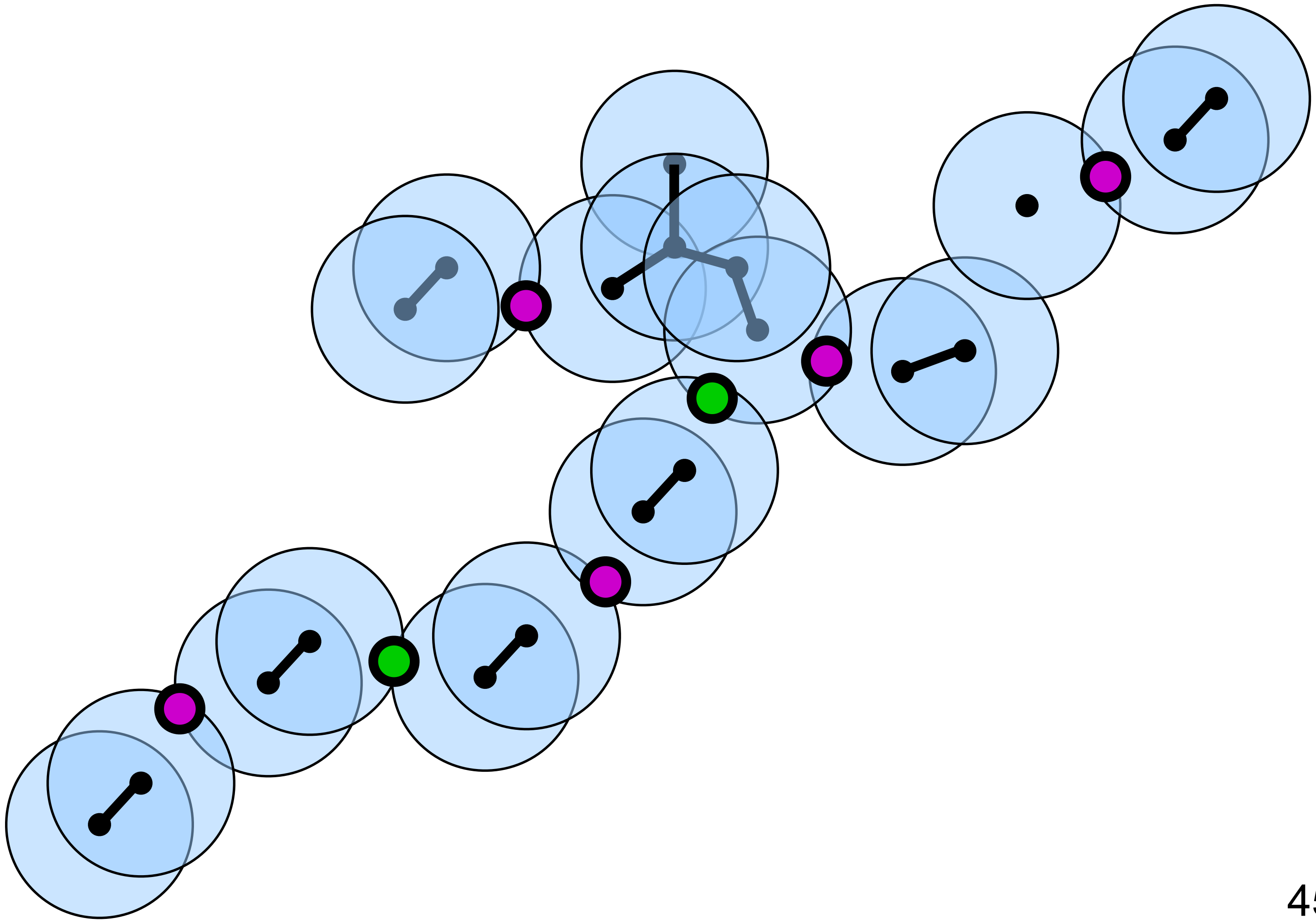
- S (magenta points)
- *Added relays* (green points)



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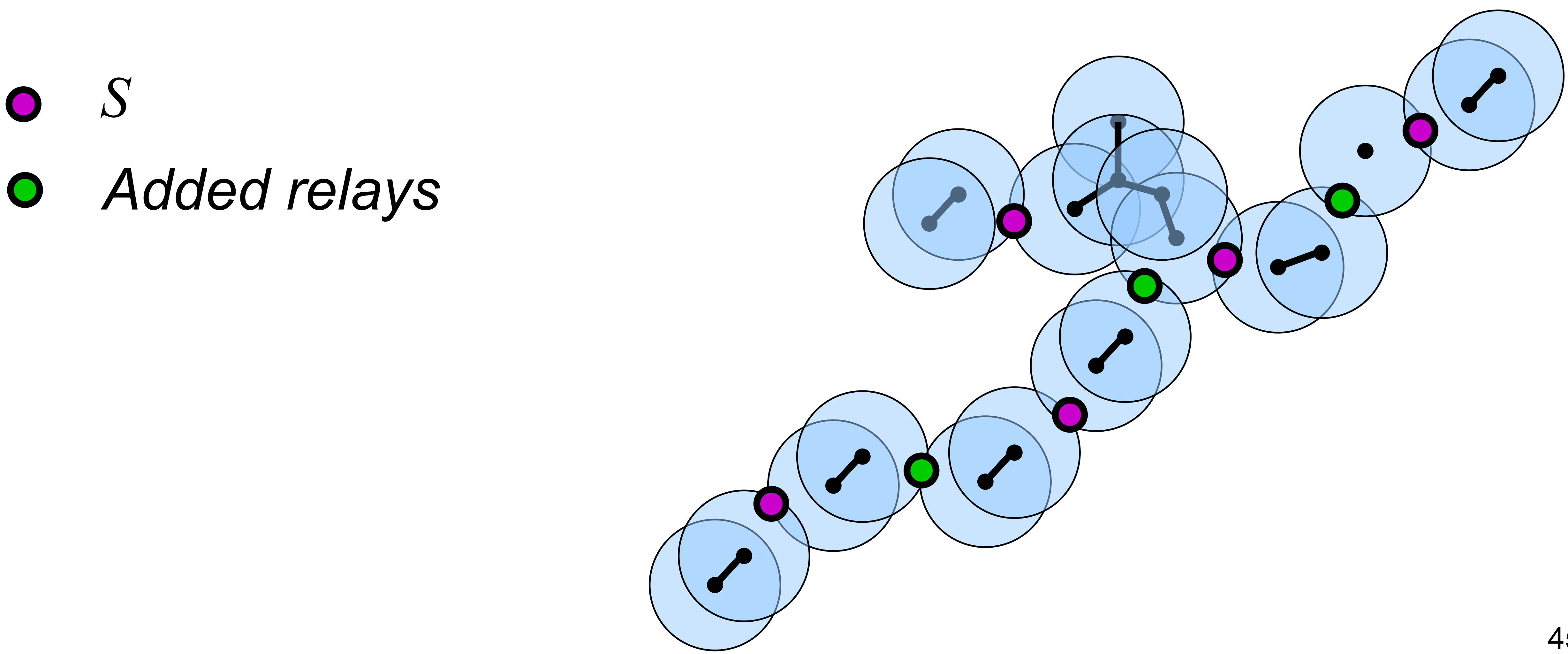

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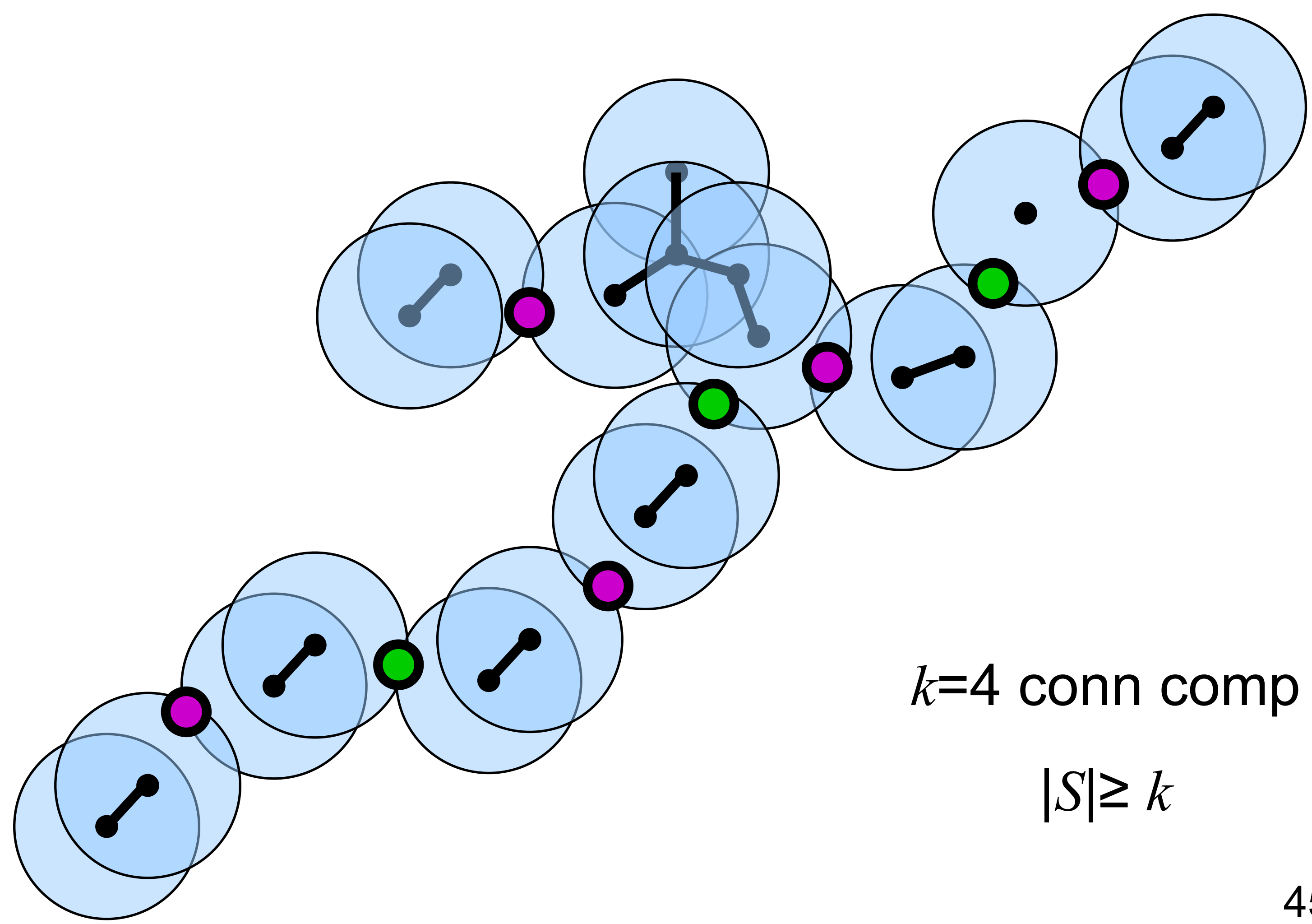

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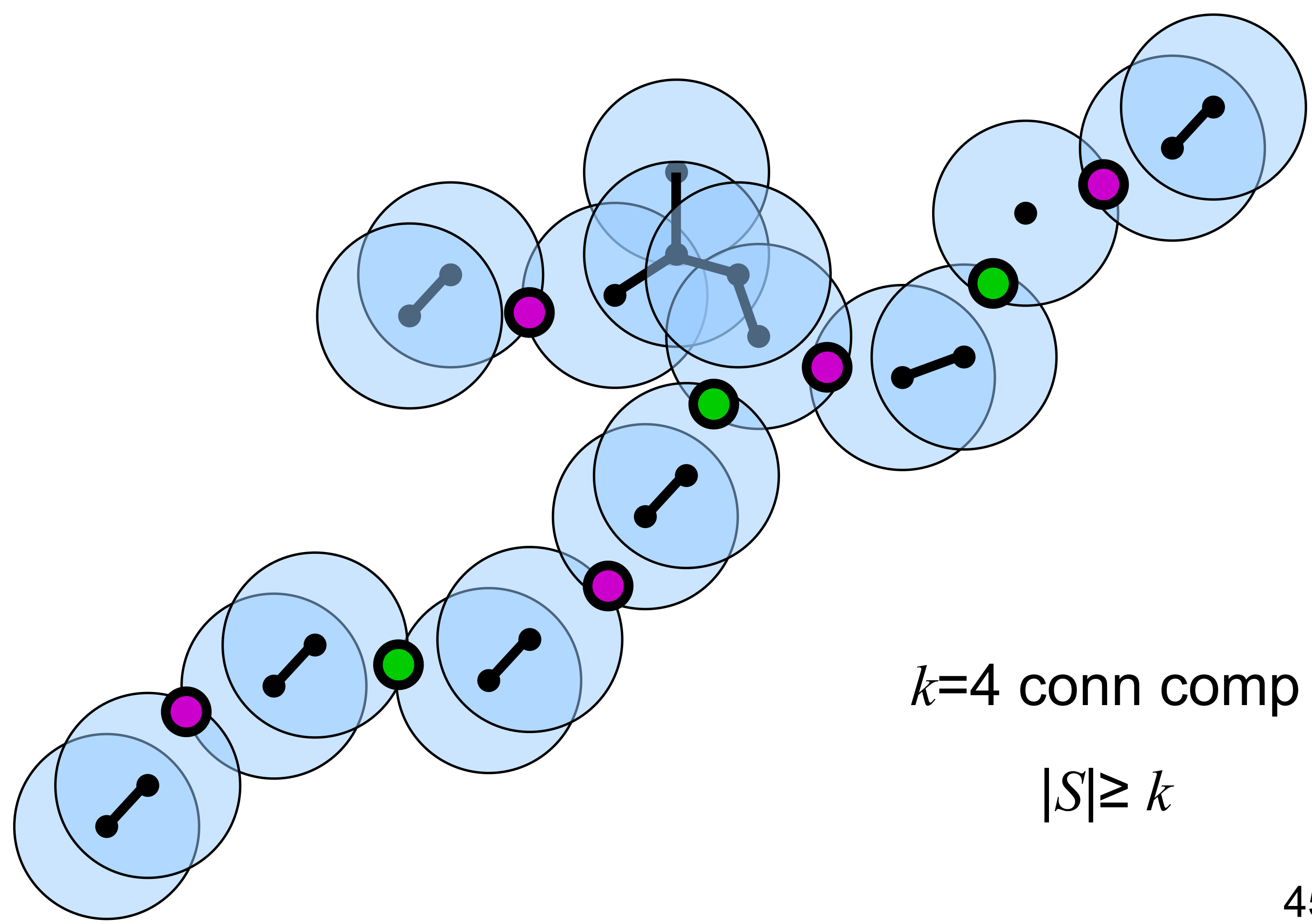

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$|S| \geq k$

Each added relay reduces the number of connected components by 1

Thus, at most $k\text{-}1 \leq |S|\text{-}1$ additional relays needed to connect the relays within C

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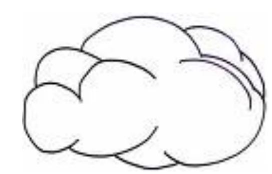

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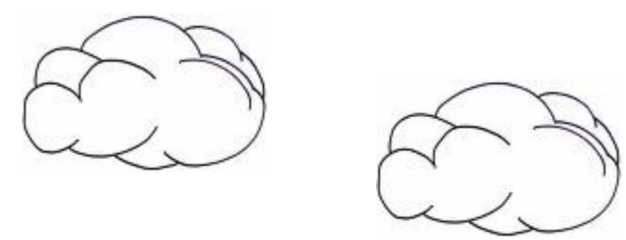

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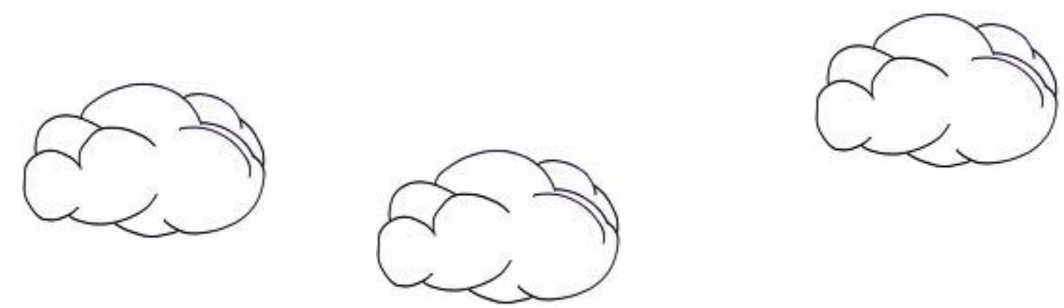

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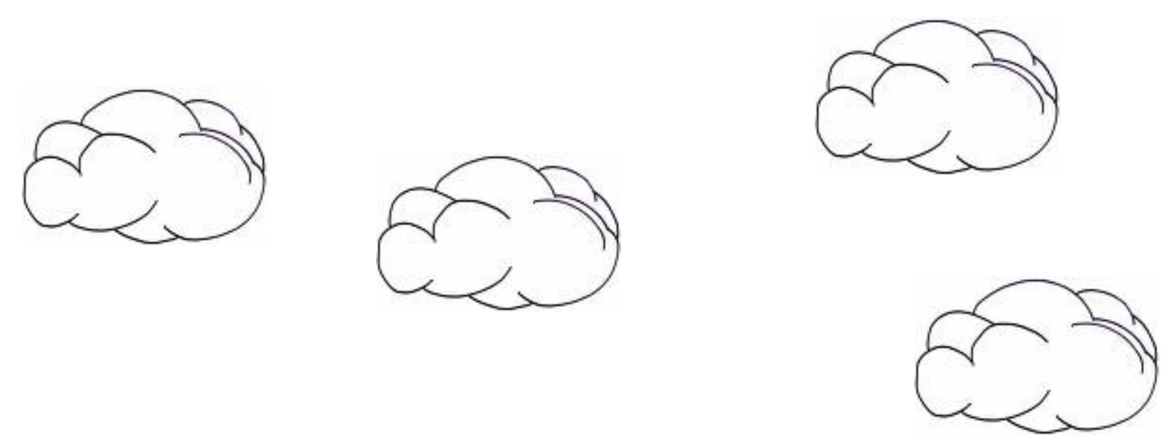

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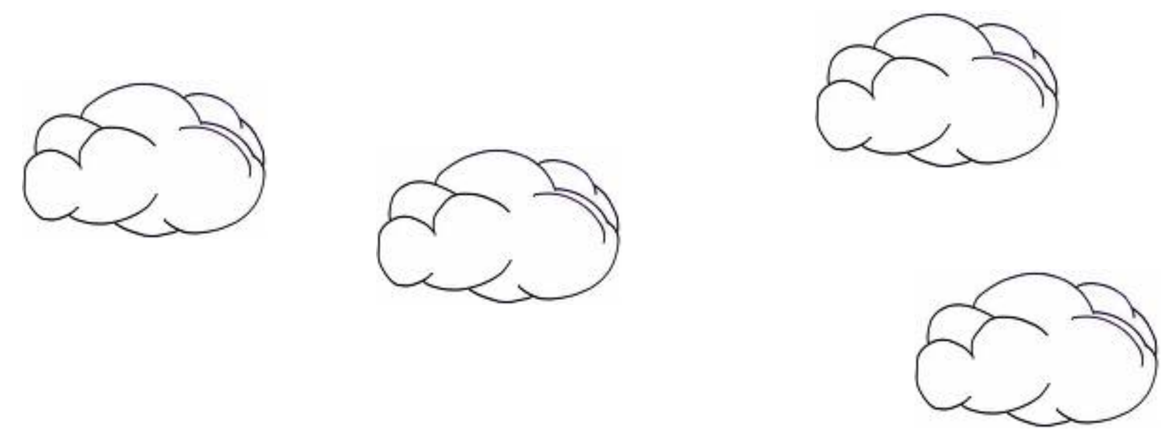
Minimum Steiner Forest for the Neighborhoods in P



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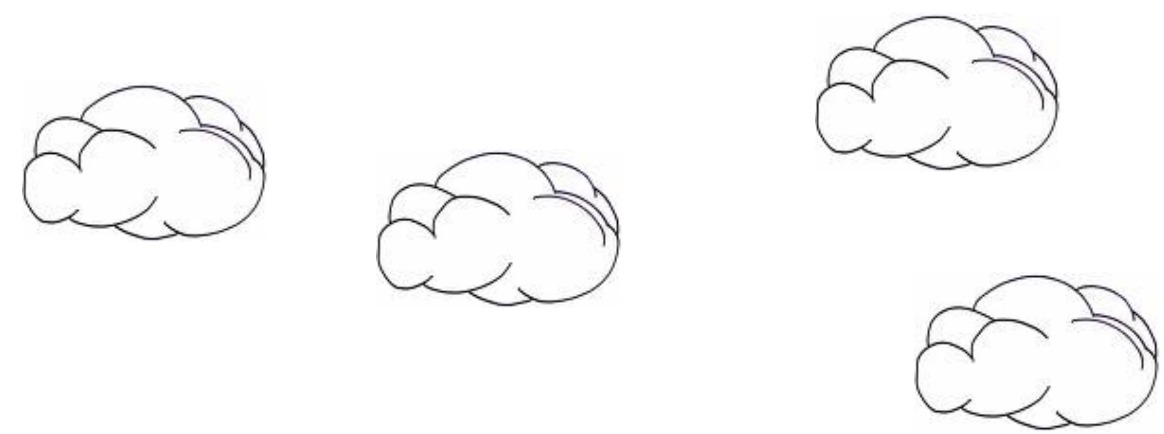

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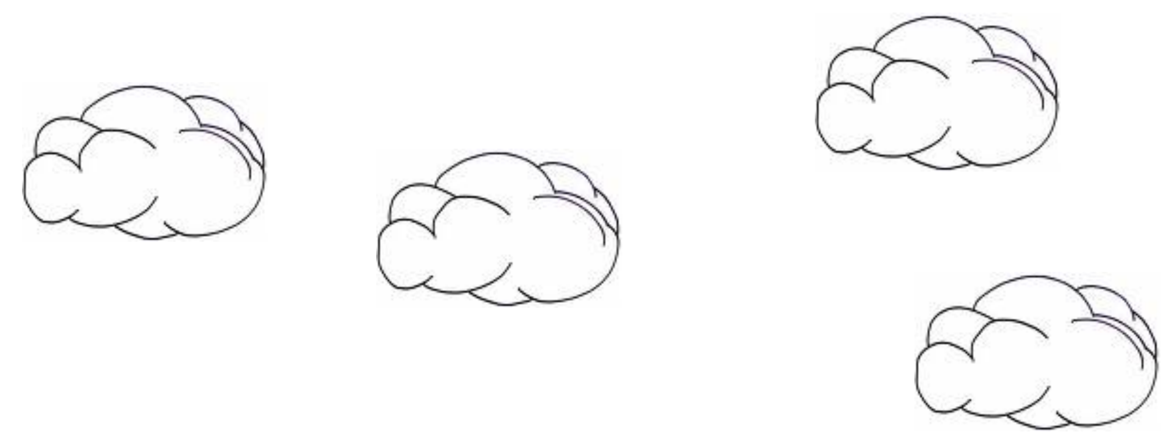

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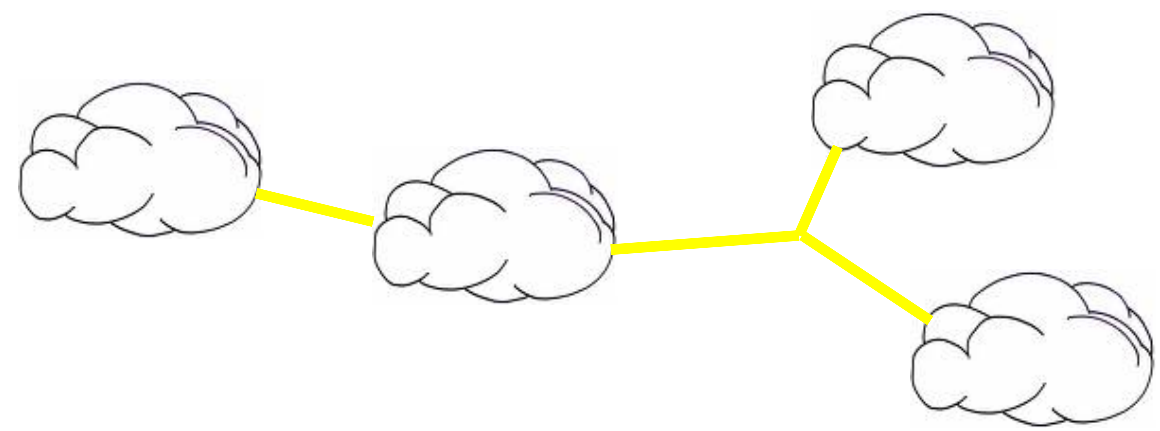

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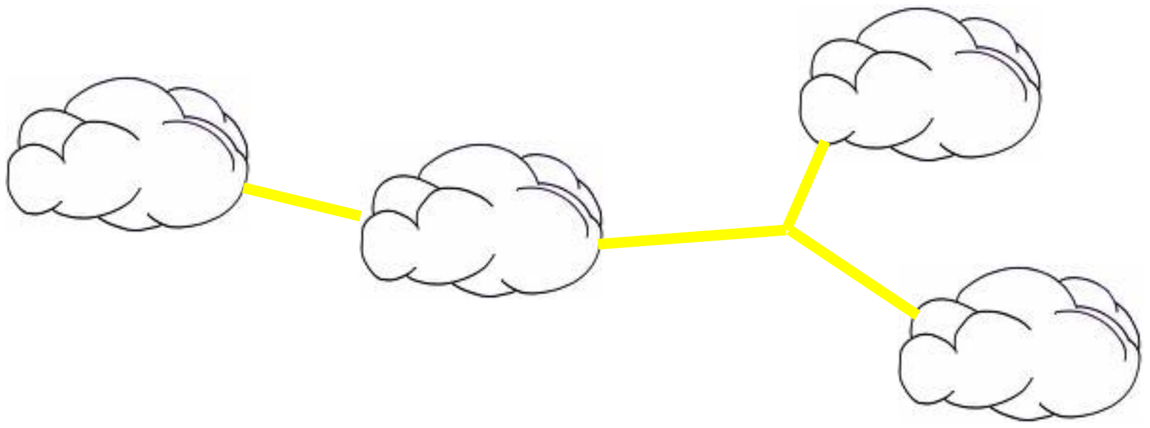

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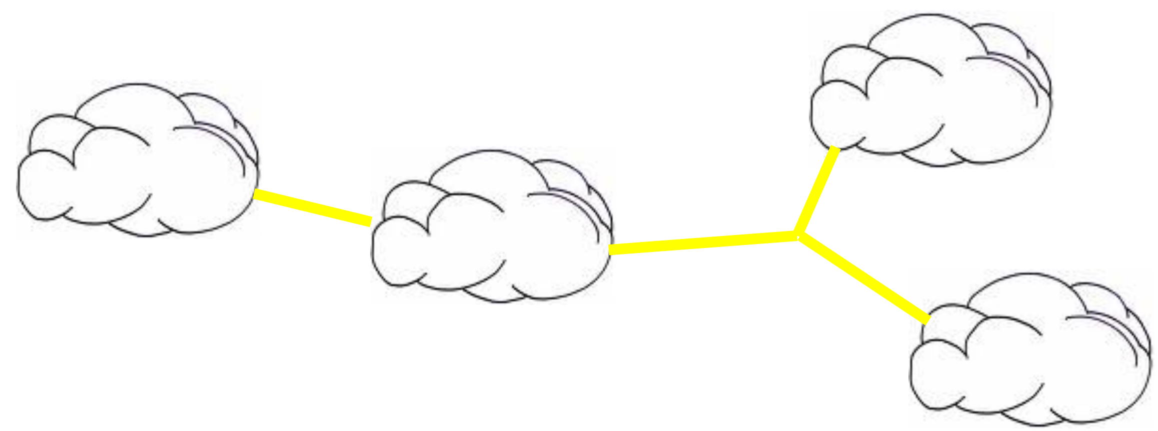

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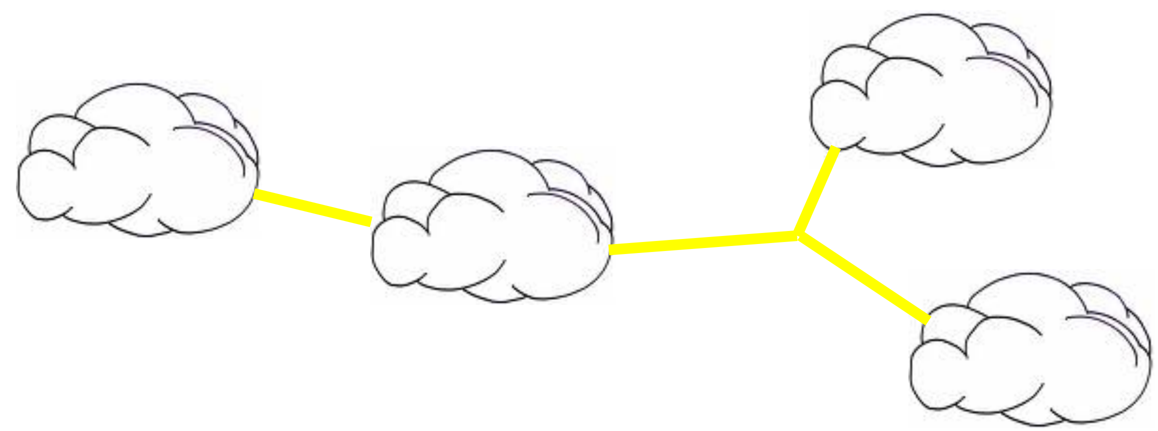

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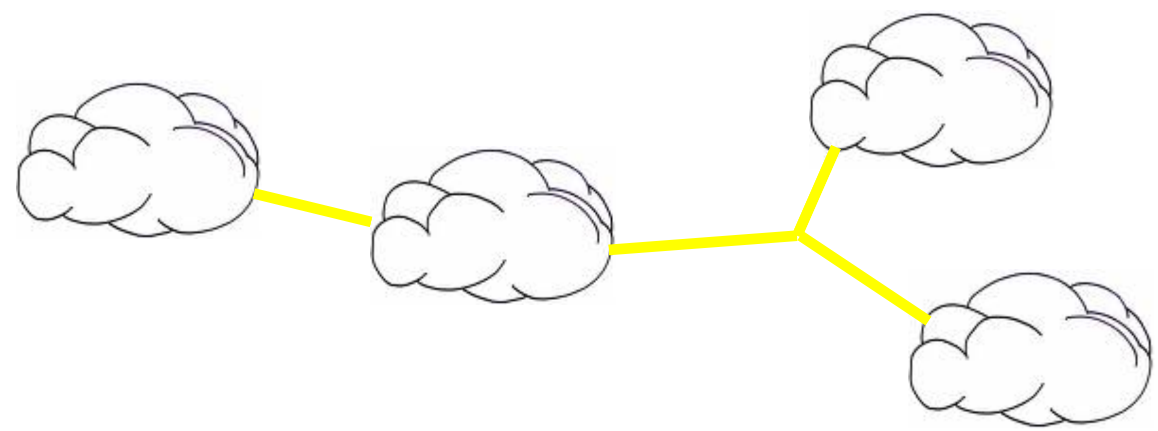

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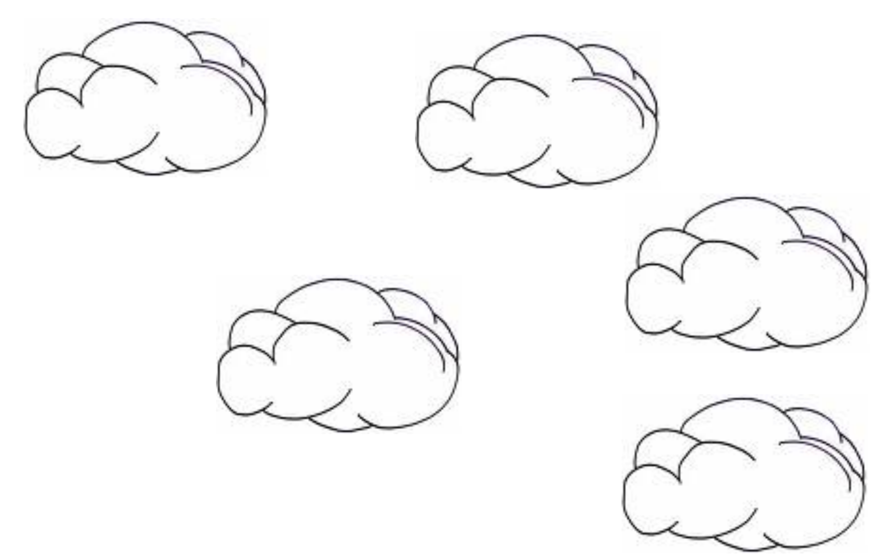

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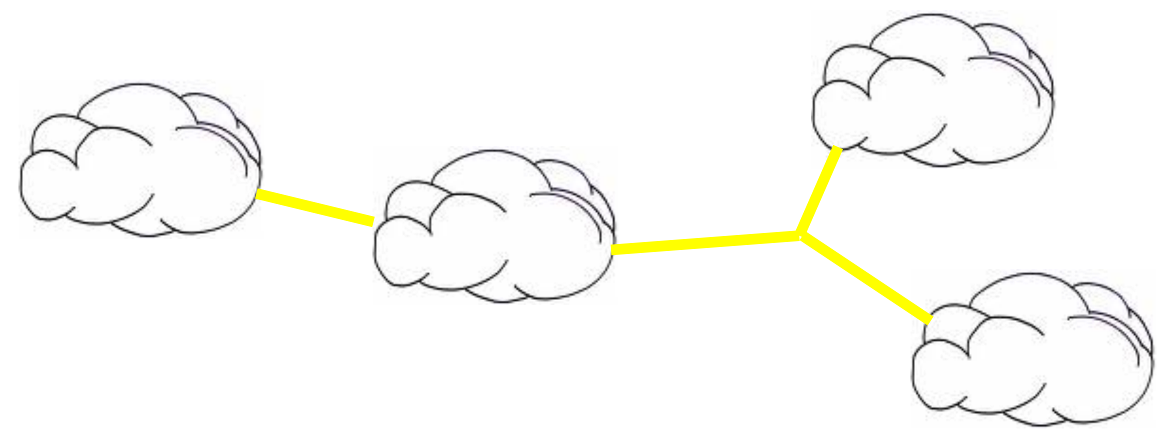

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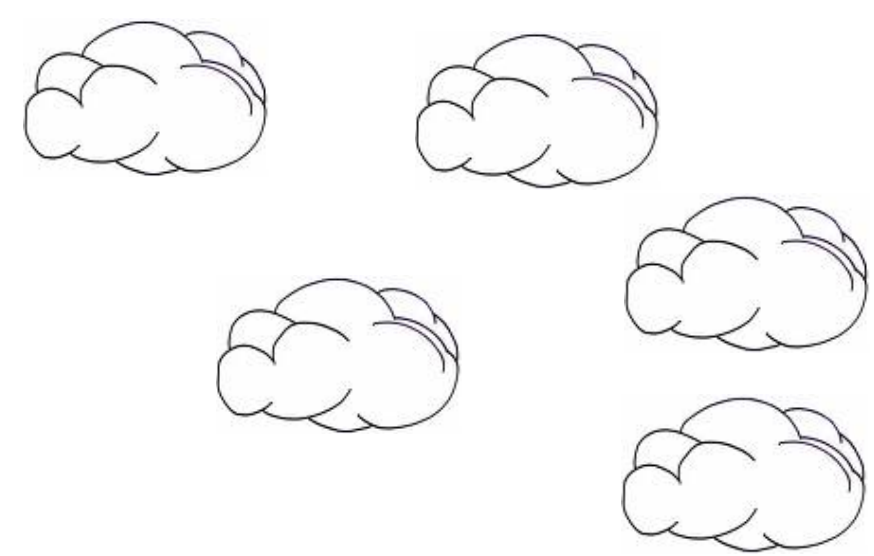
MSFN($\mathcal{P}$) : min spanning tree in graph on $\mathcal{P}$,



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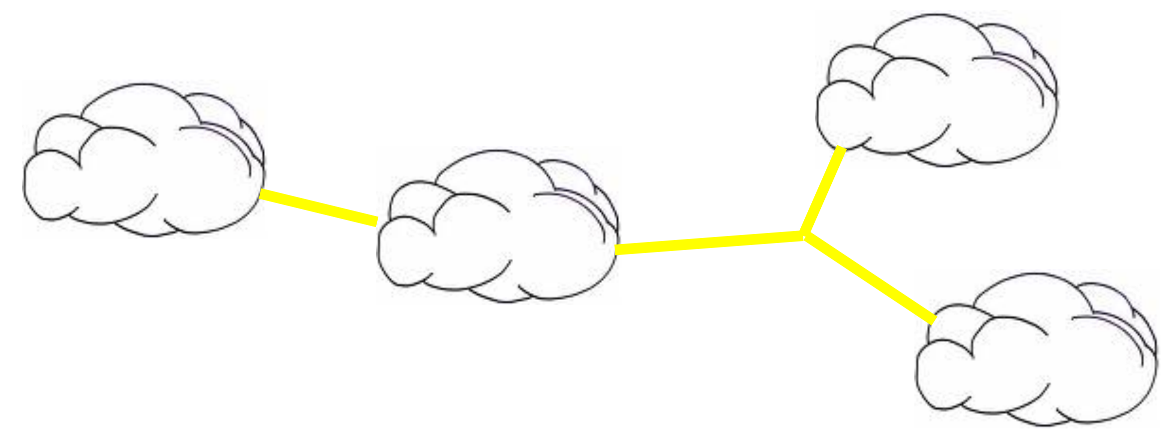
$G_{\mathcal{P}}=(\mathcal{P},E(G))$ is connected

Counts only length *outside* $\mathcal{P}$

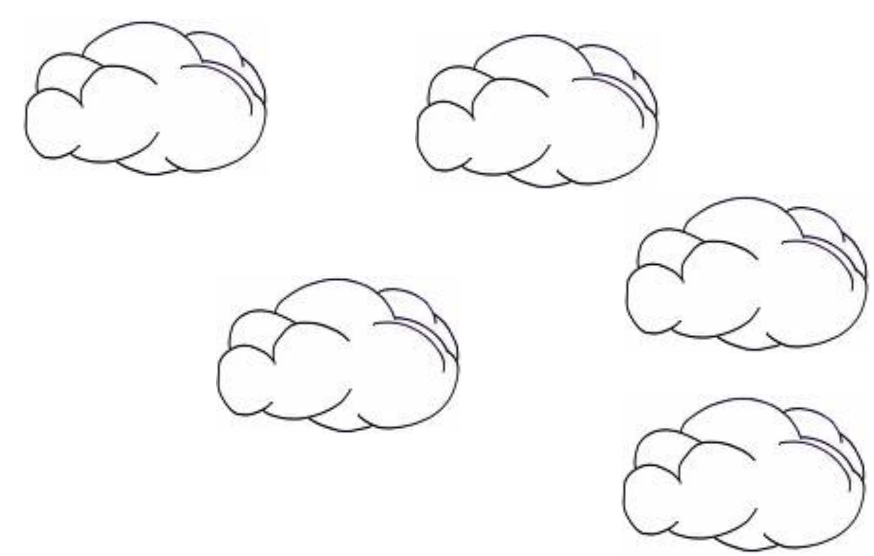
(vs. MStTN)



Minimum Spanning Forest for the Neighborhoods in $\mathcal{P}$

MSFN($\mathcal{P}$) : min spanning tree in graph on $\mathcal{P}$,

edge weights = distance between sets



Steiner Forests and Spanning Trees with Neighborhoods

Let $\mathcal{P}$ = family of planar subsets (neighborhoods)

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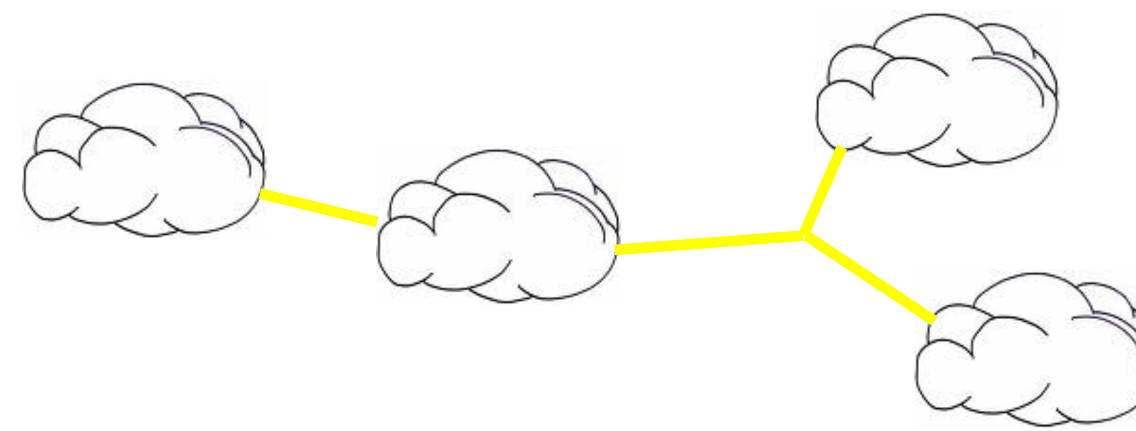

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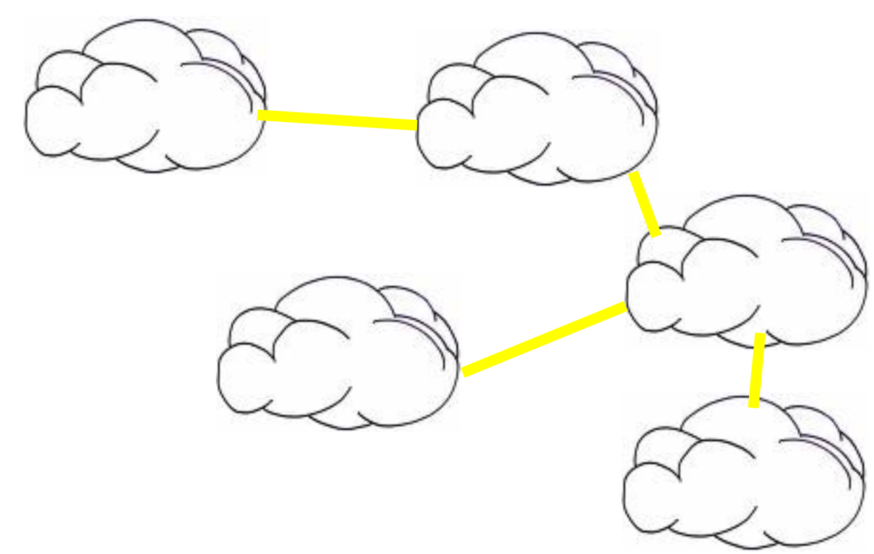

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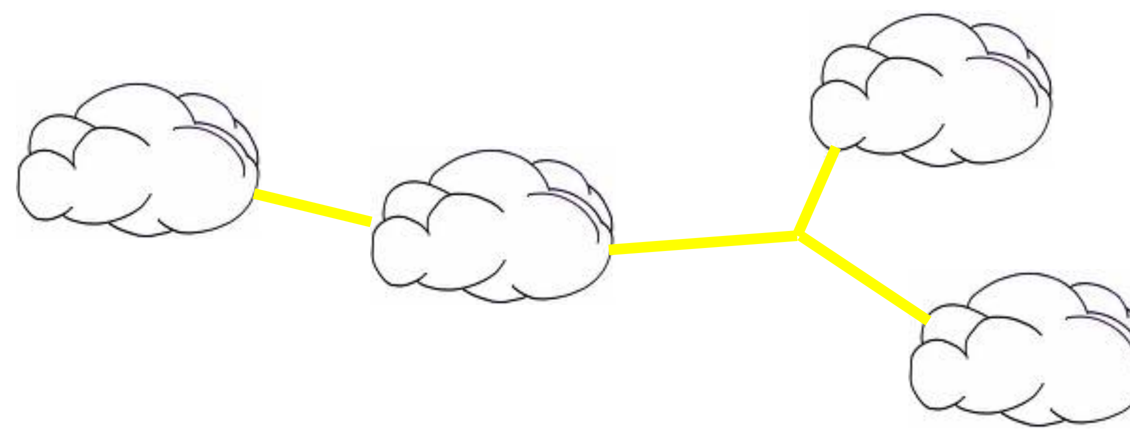

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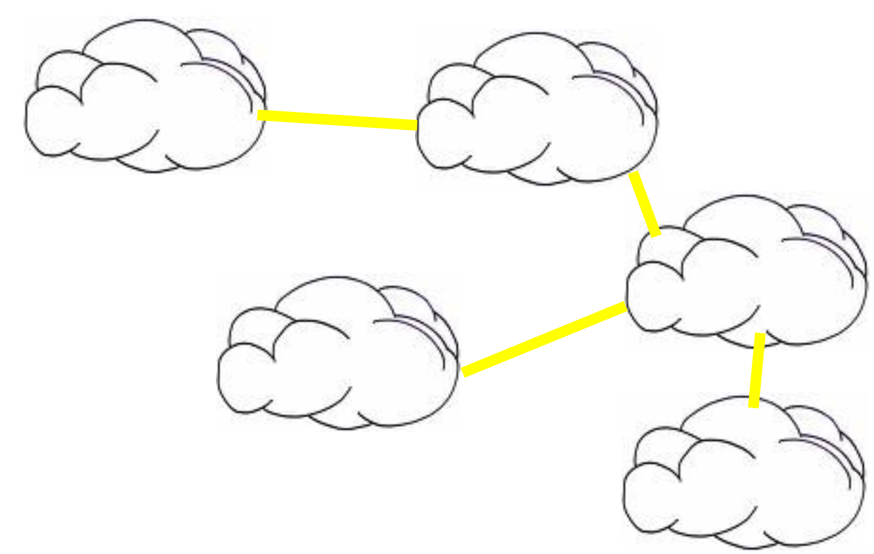

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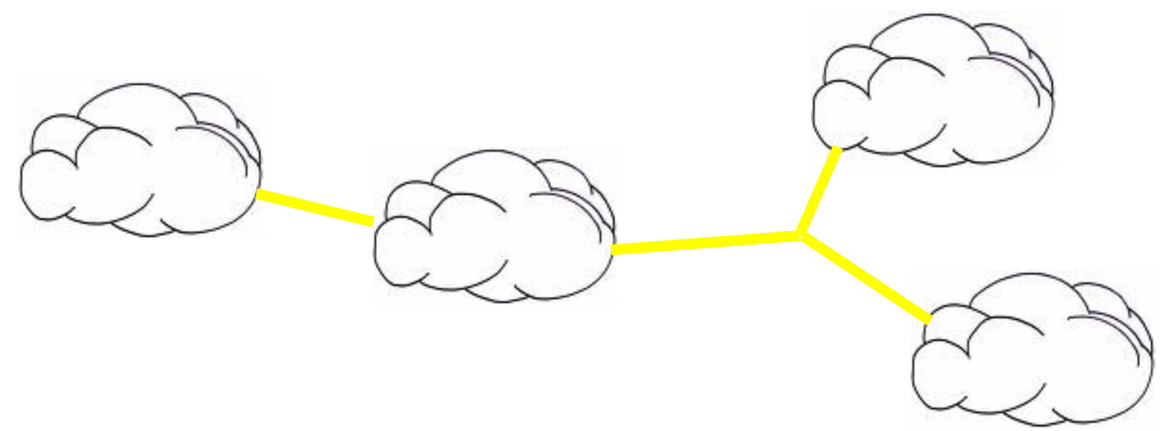

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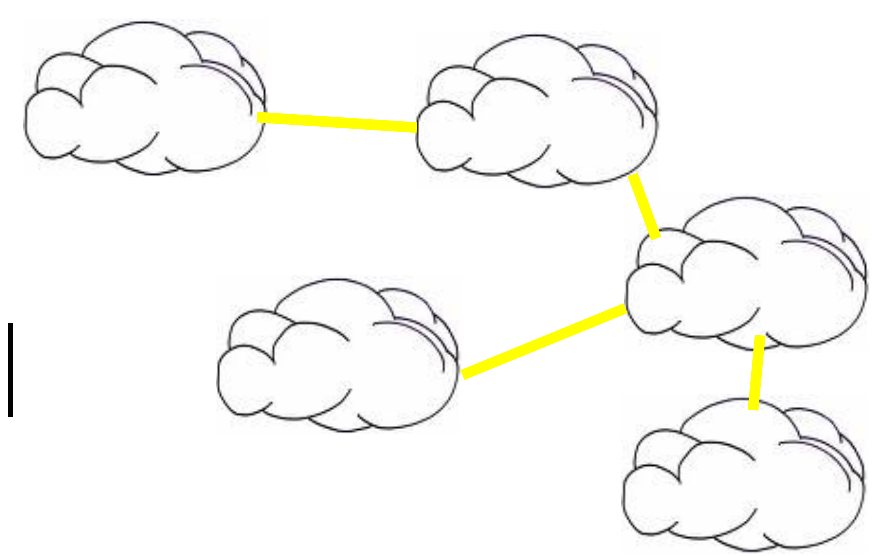

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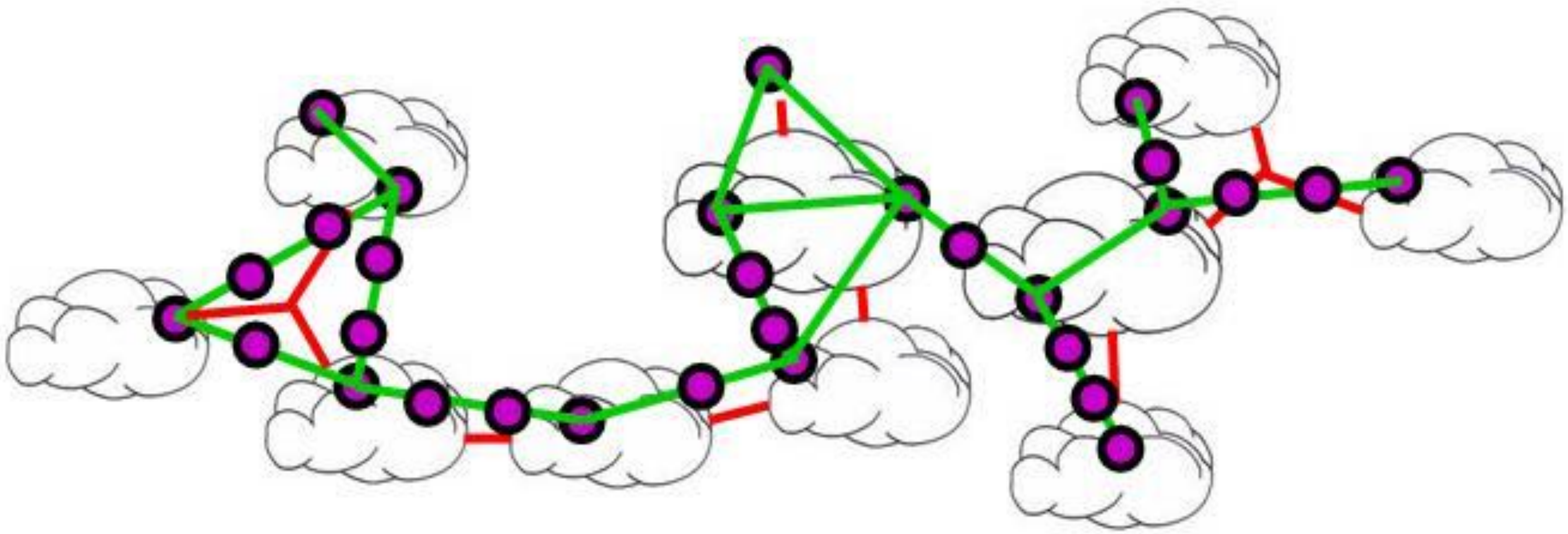

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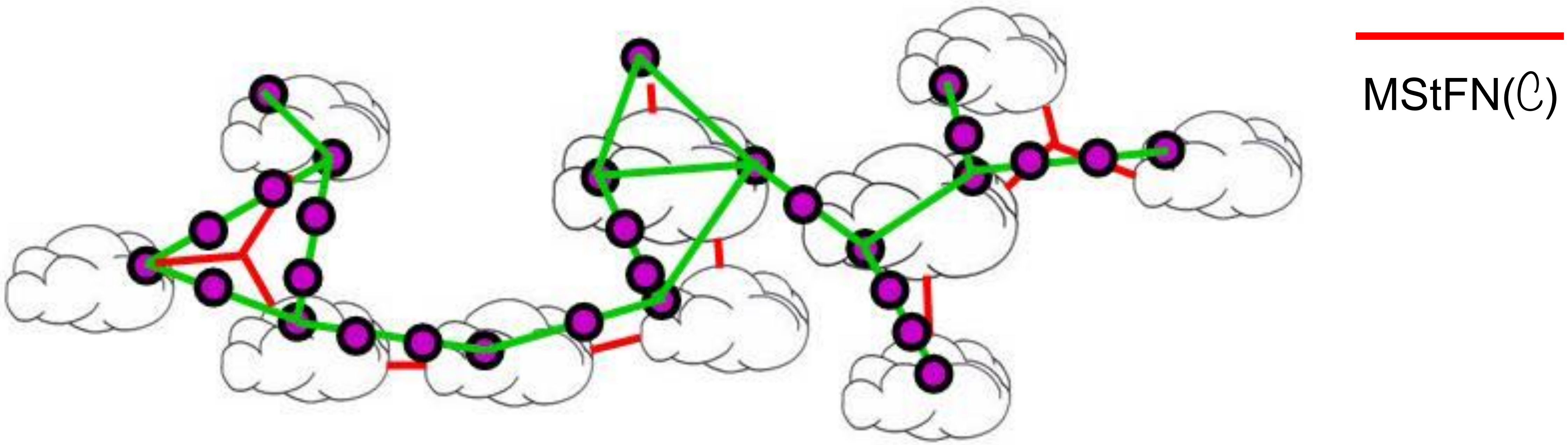

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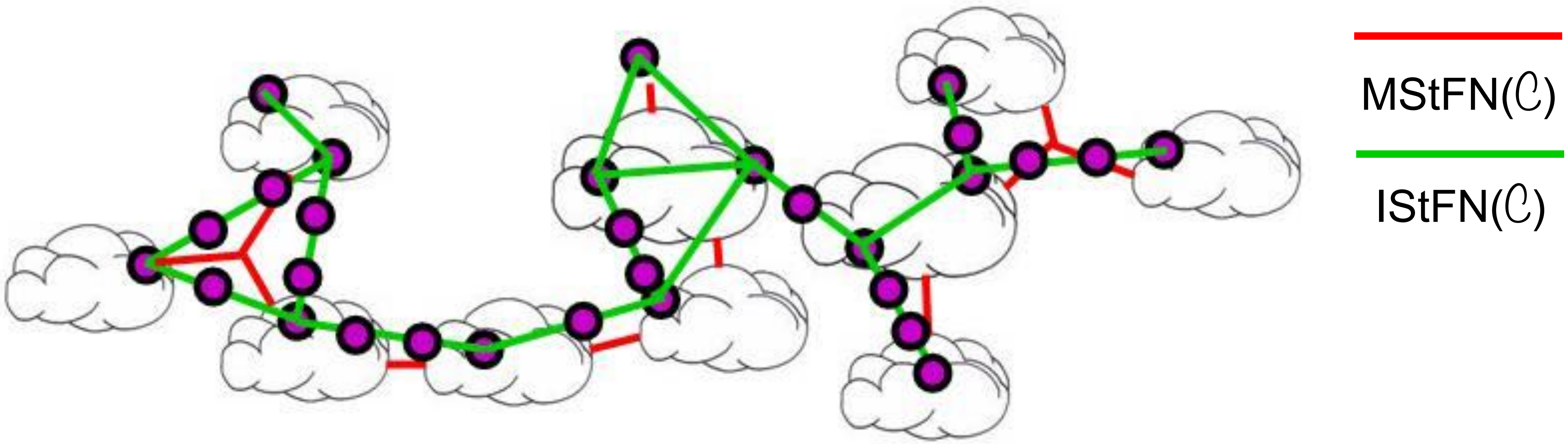

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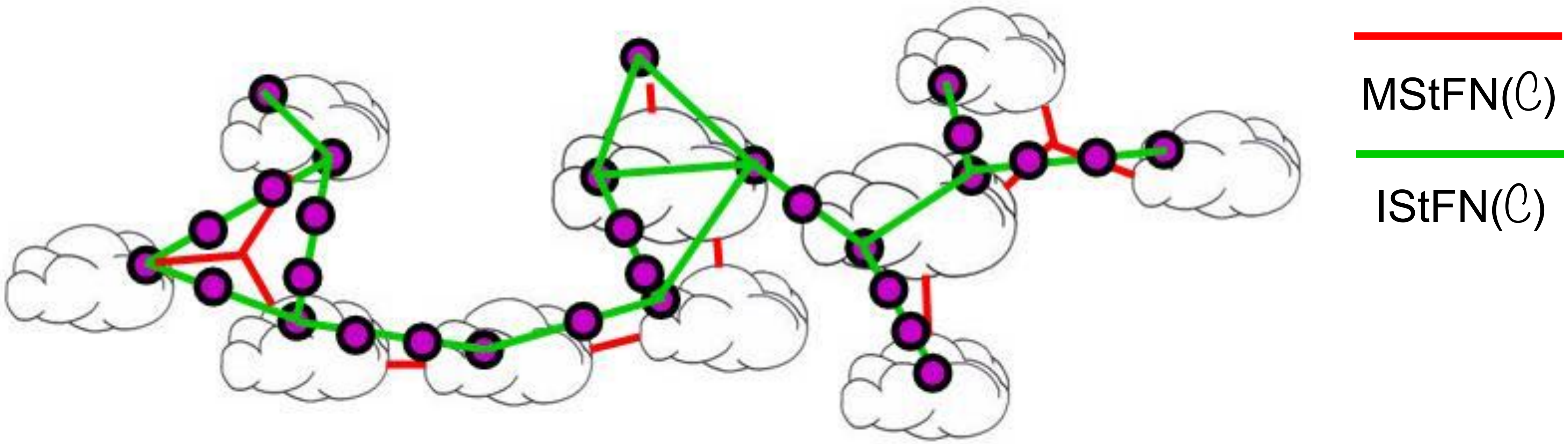

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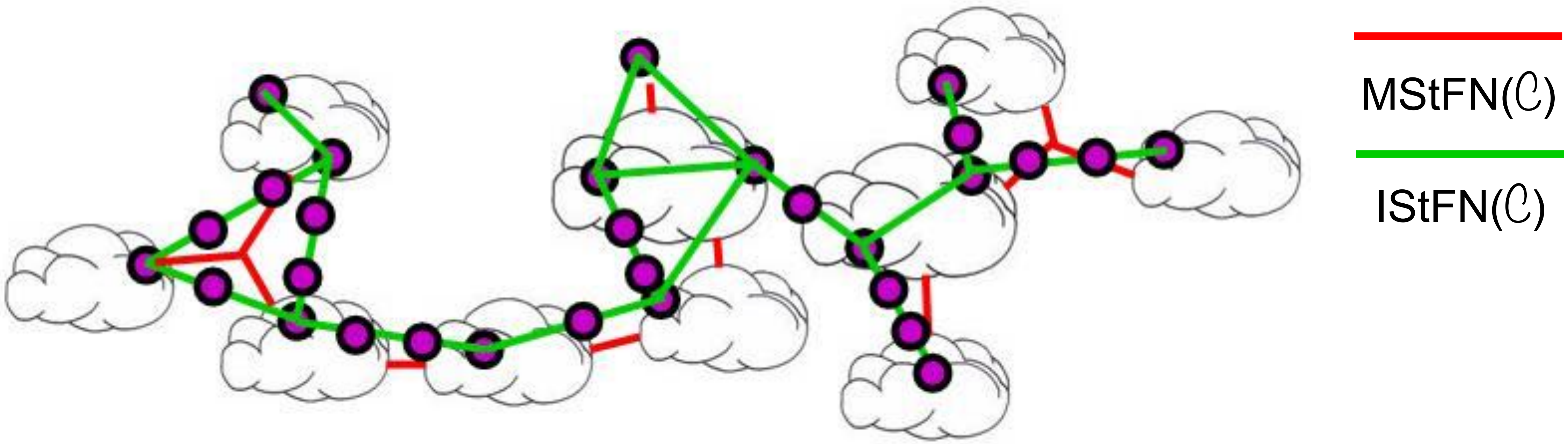

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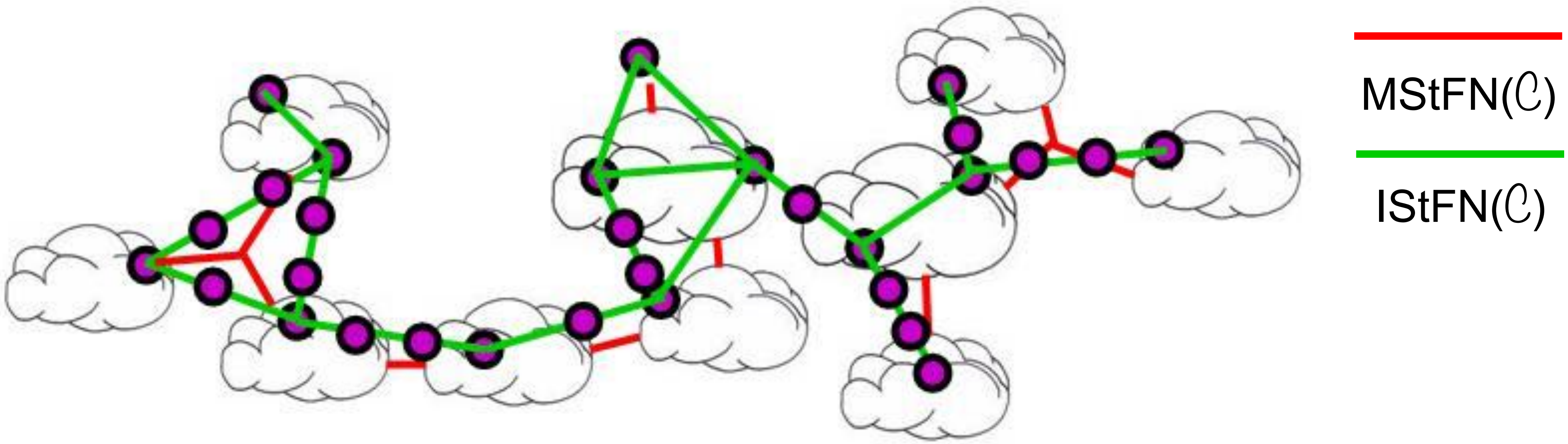

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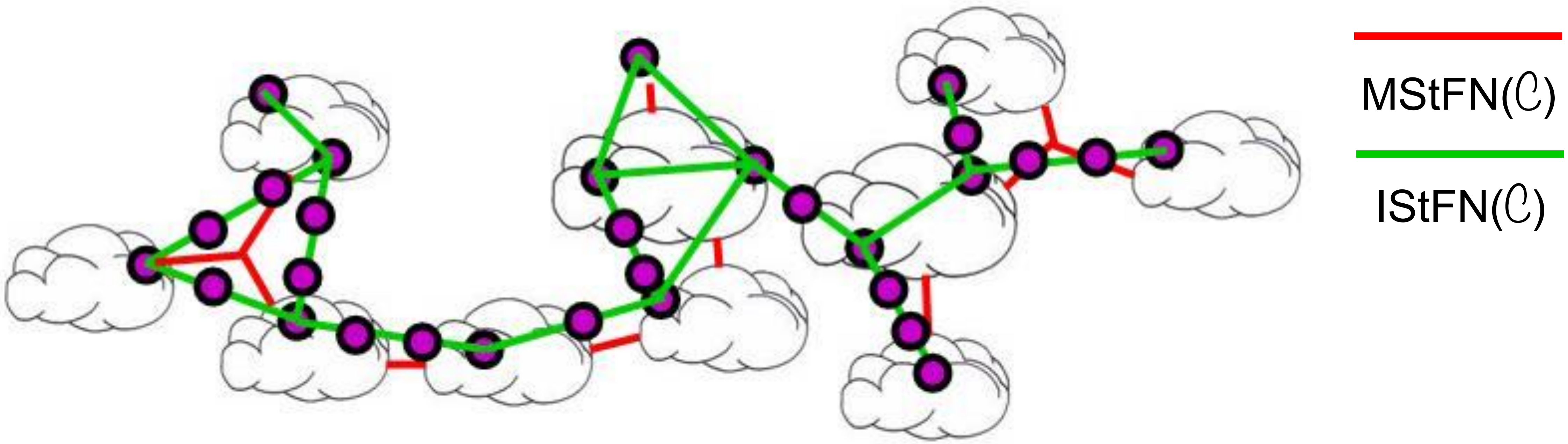

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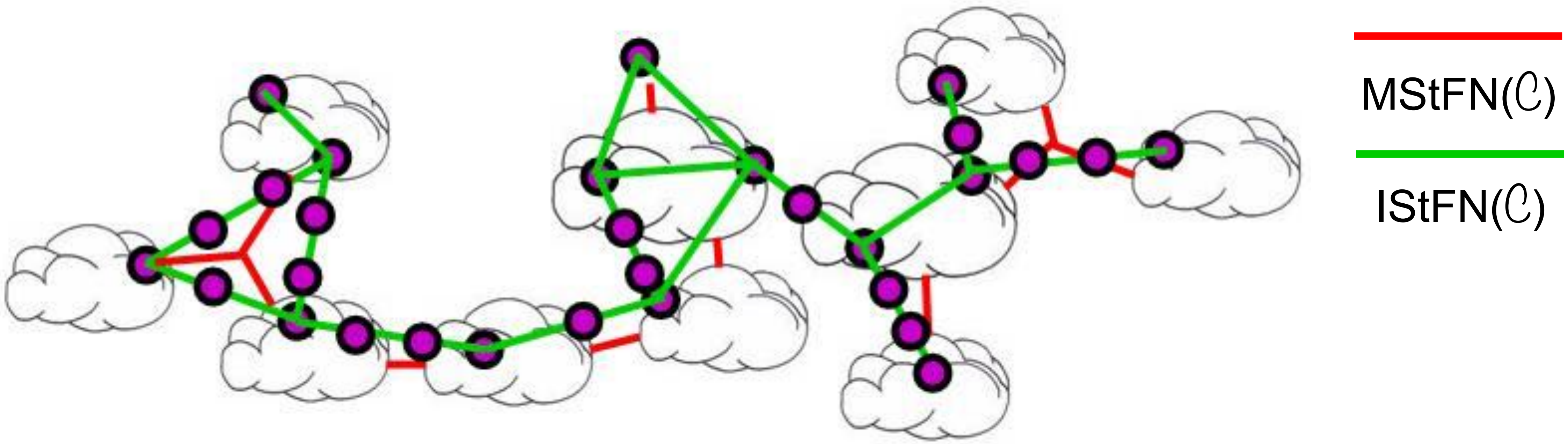

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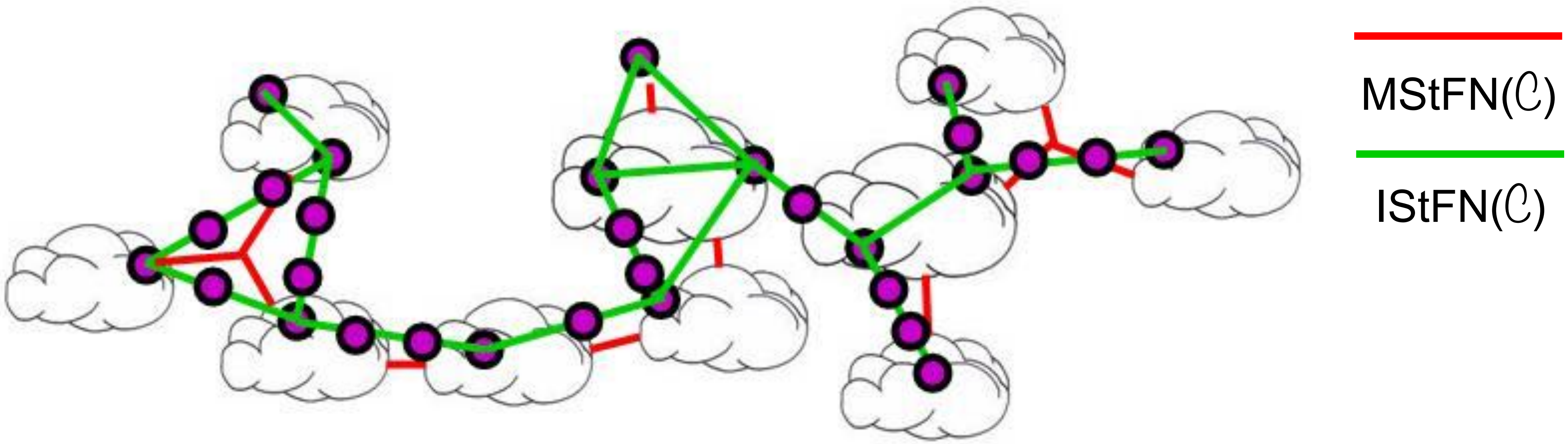

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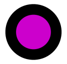

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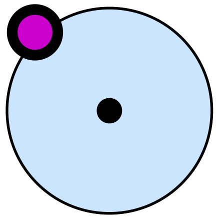

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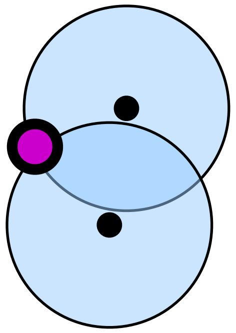

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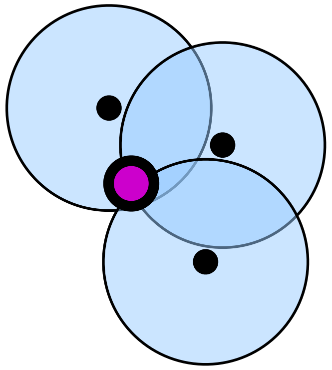

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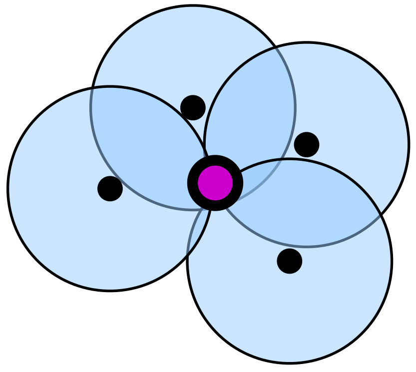

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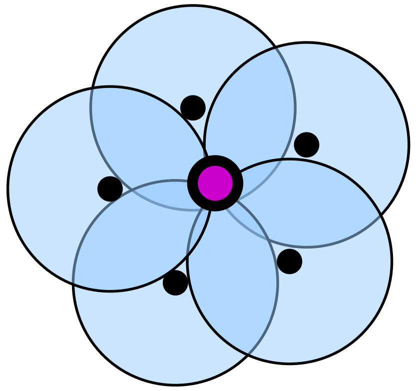

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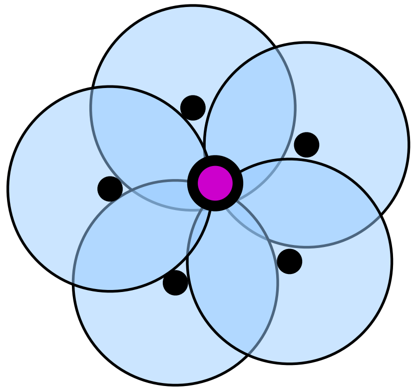

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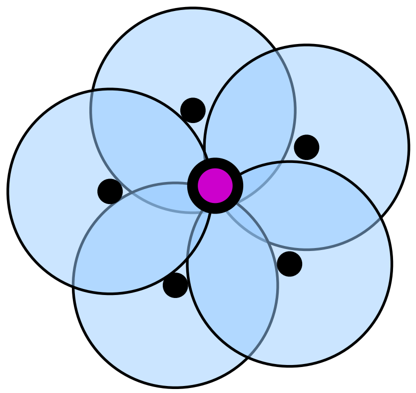

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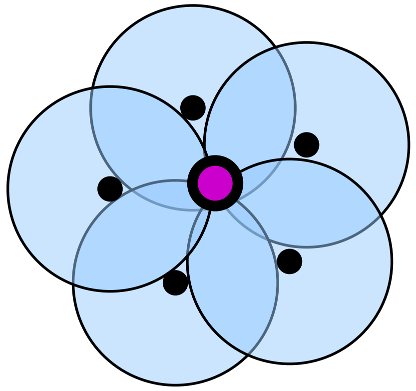

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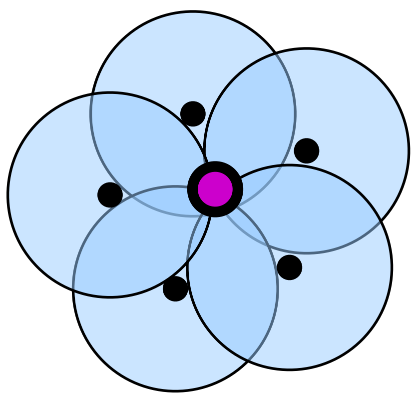

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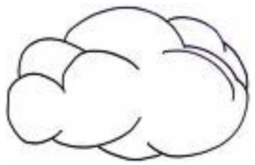

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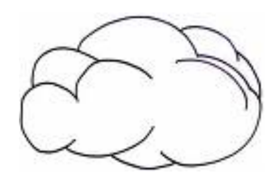

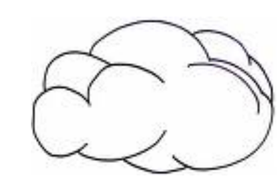

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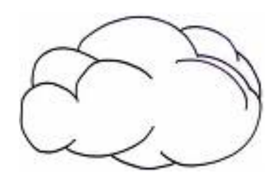

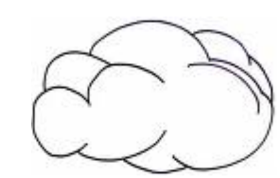

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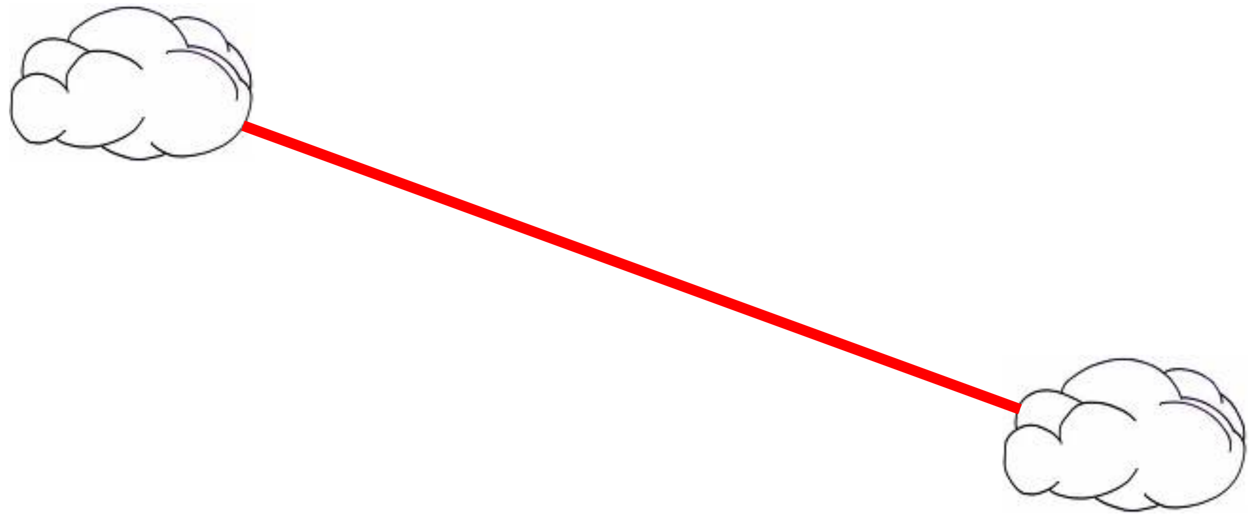

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- Compute MSFN($\mathcal{C}$).
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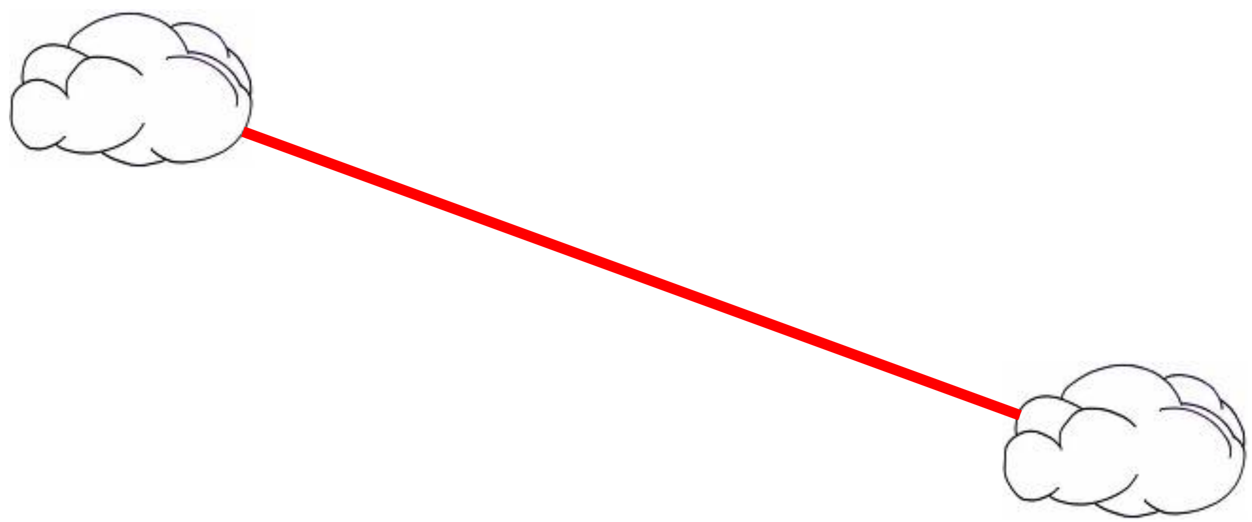

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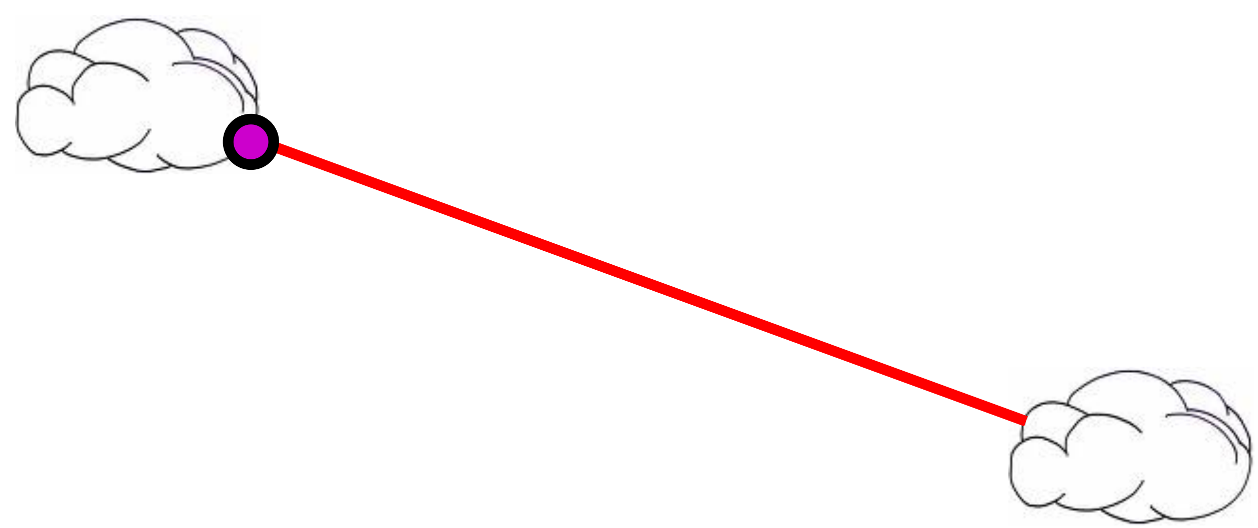

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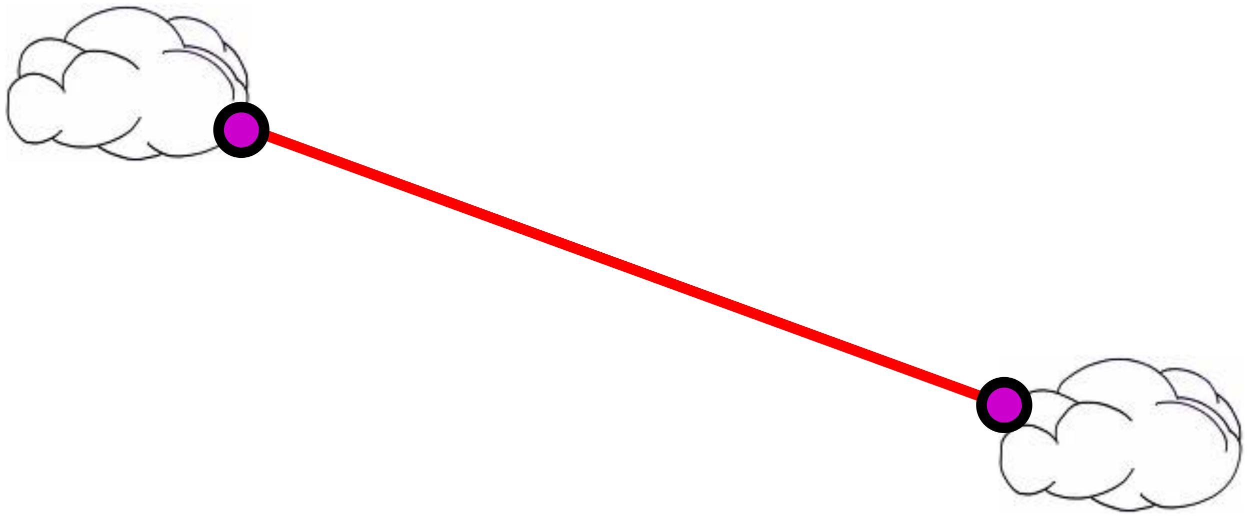

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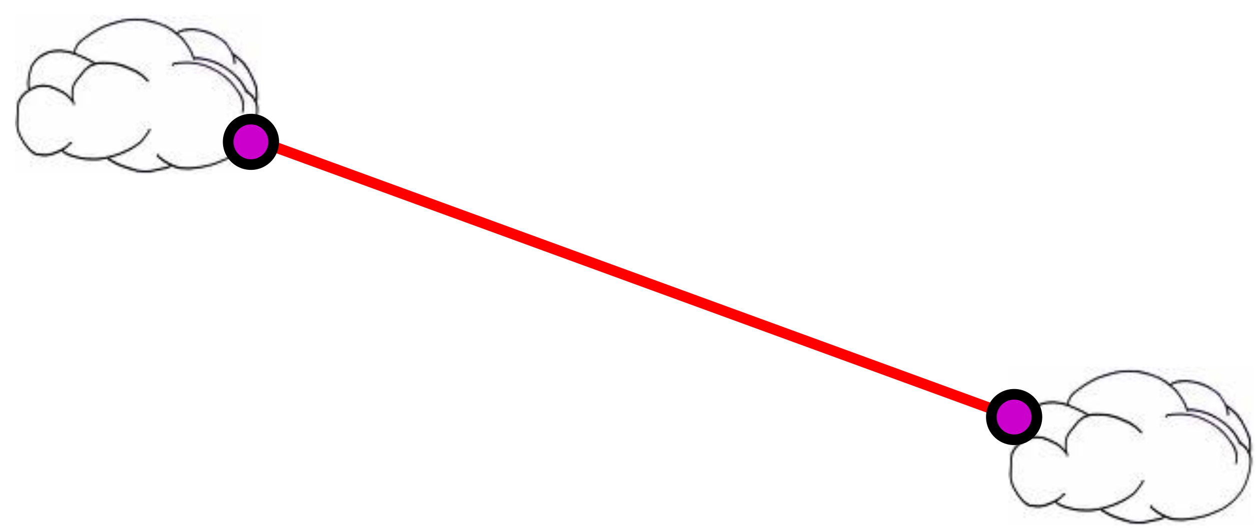

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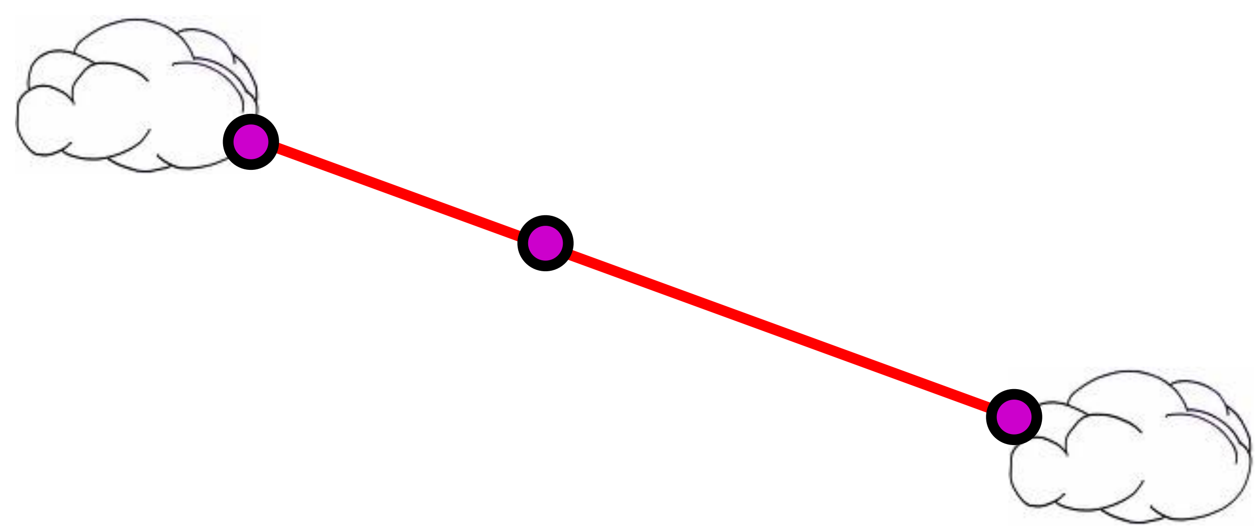

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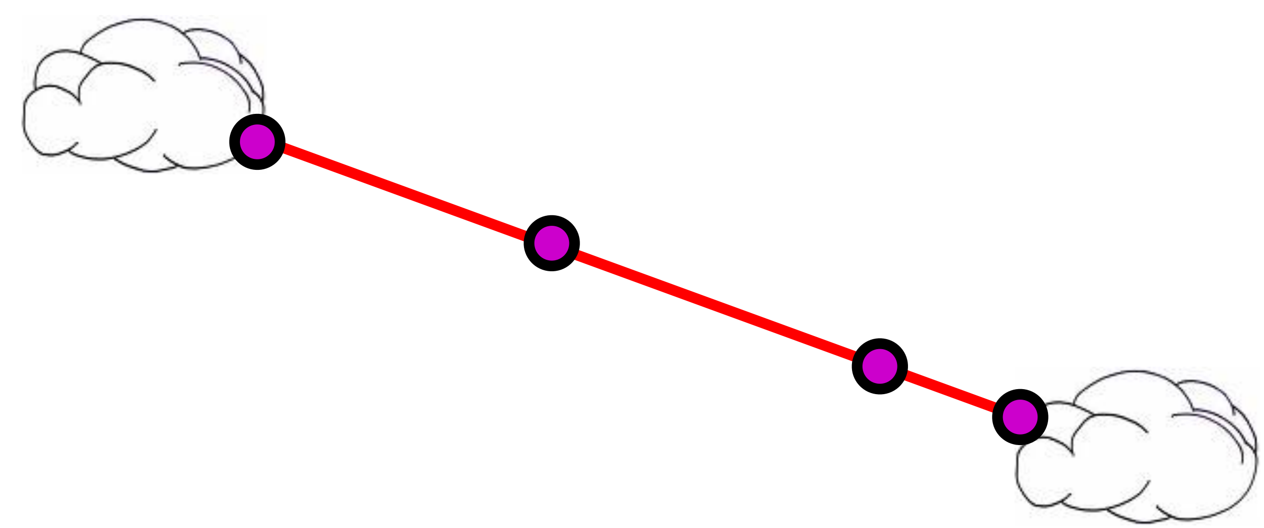

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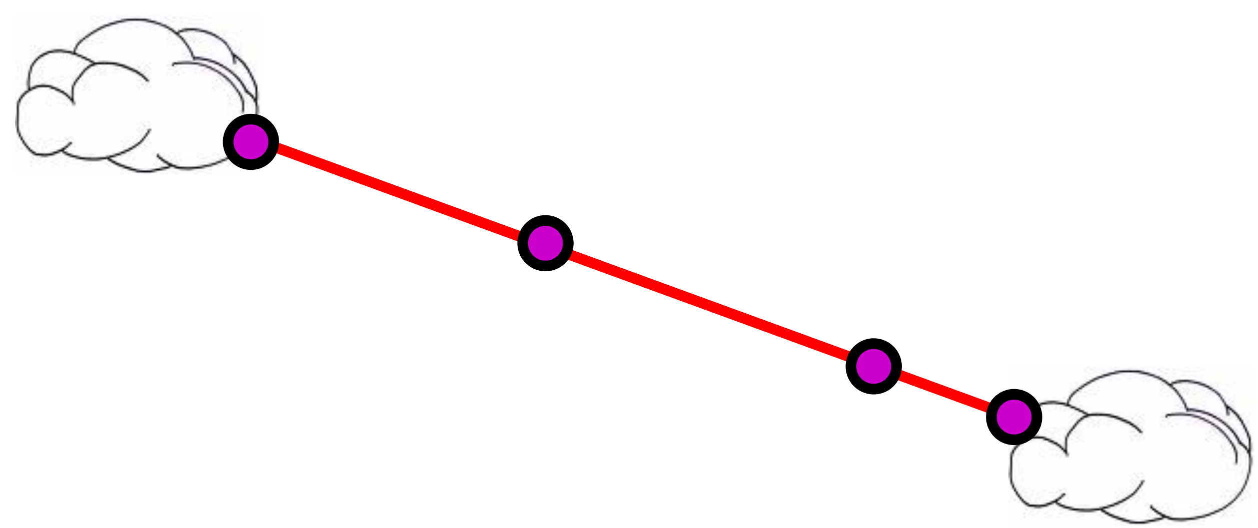

$$|R''| \leq 2(|\mathrm{C}|-1) + \sum_e \left\lfloor \frac{|e|}{r} \right\rfloor$$

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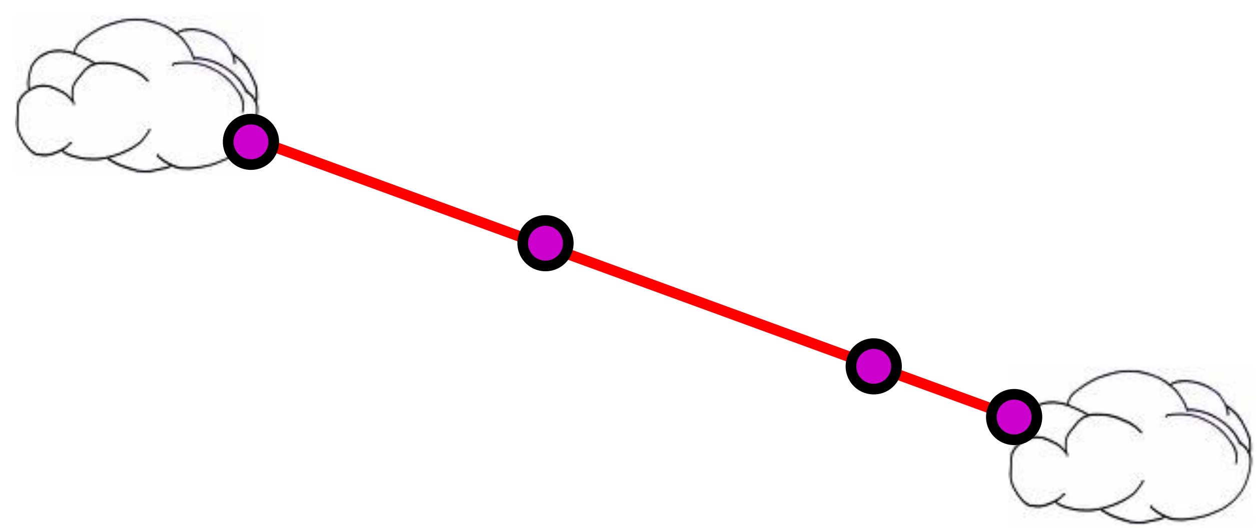

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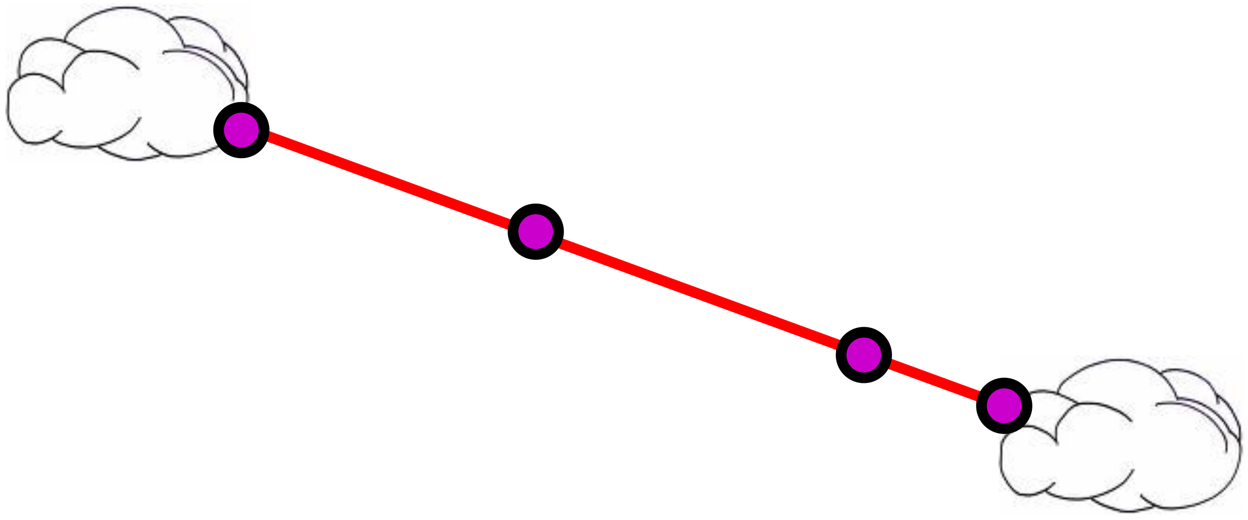

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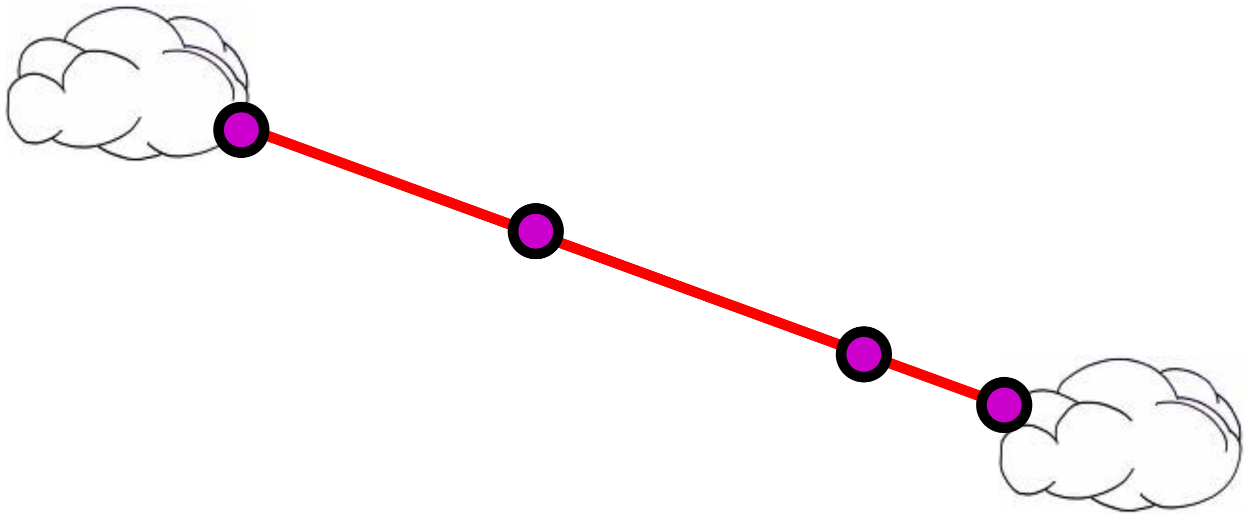

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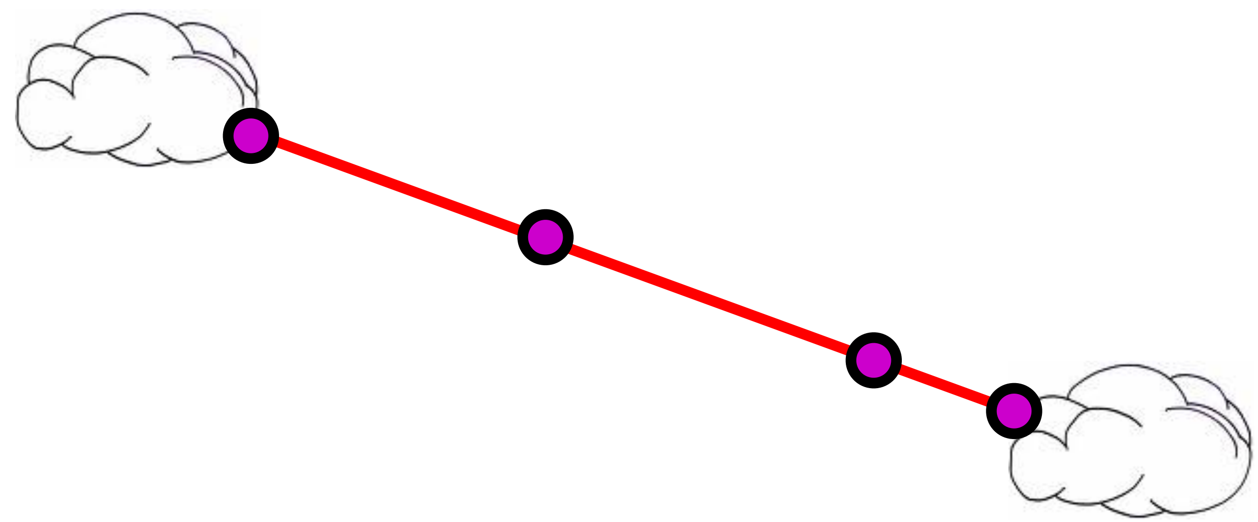

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- Running time:
  - Build Delaunay of n sensors $O(n \log n)$
  - Identify blobs and clouds $O(n)$
  - Greedy to stab blobs (≤5 per relay) $O(n)$
  - MSFN(C) is subgraph of Delaunay $O(n \log n)$
  - Total: $O(n \log n)$

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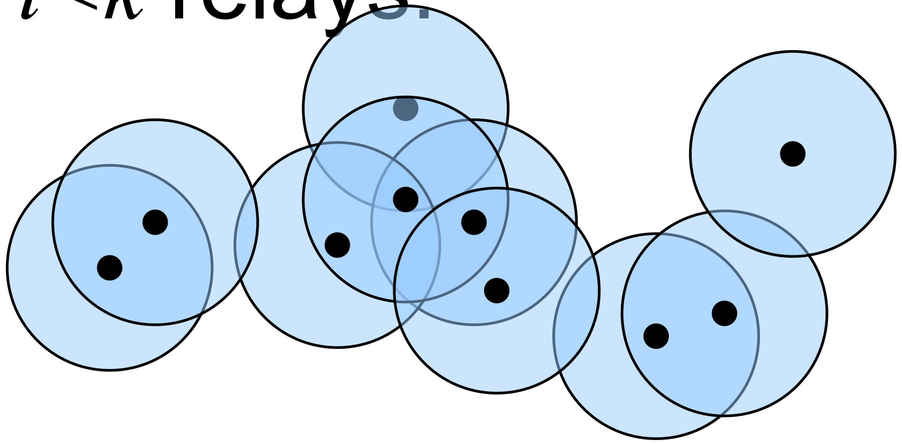

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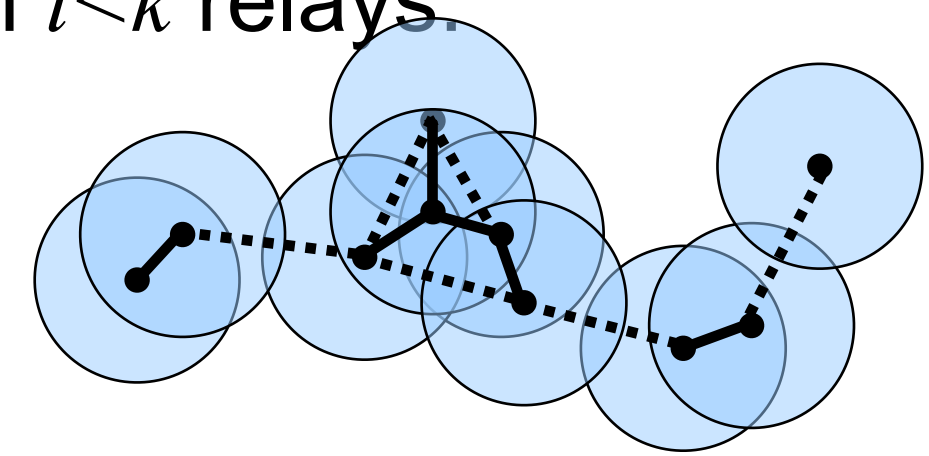

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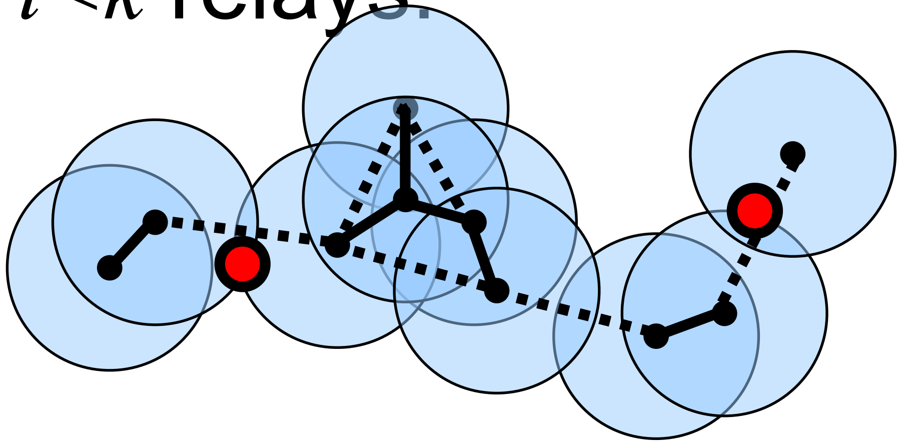

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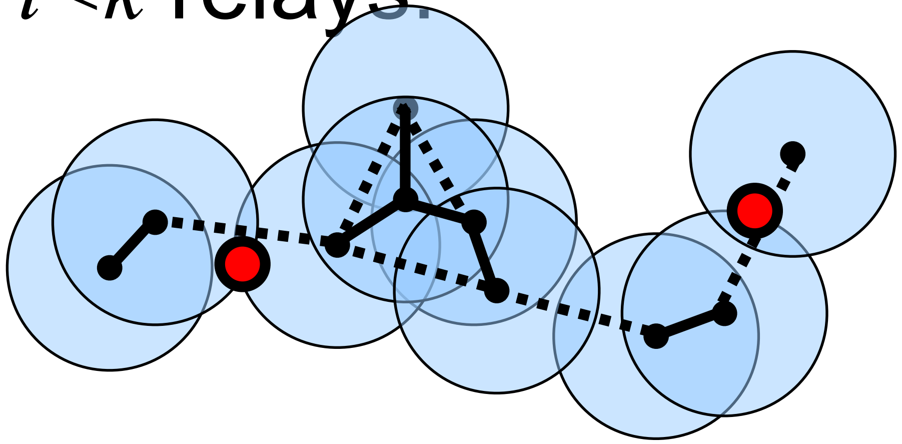

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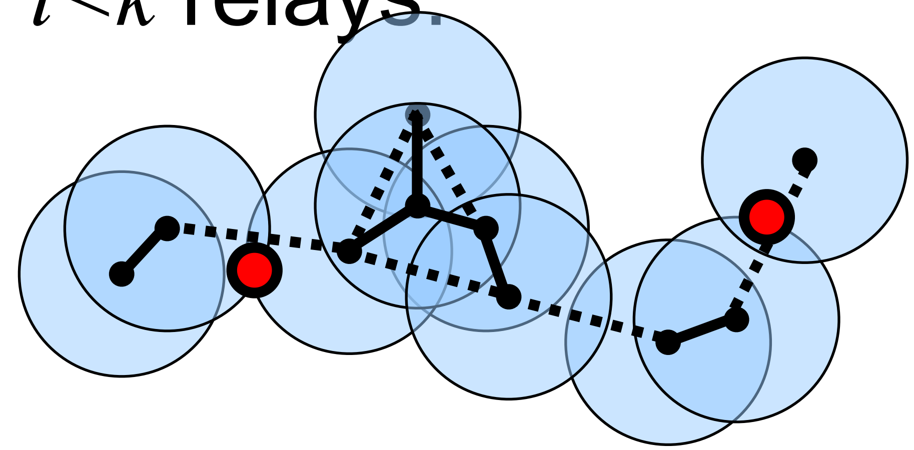

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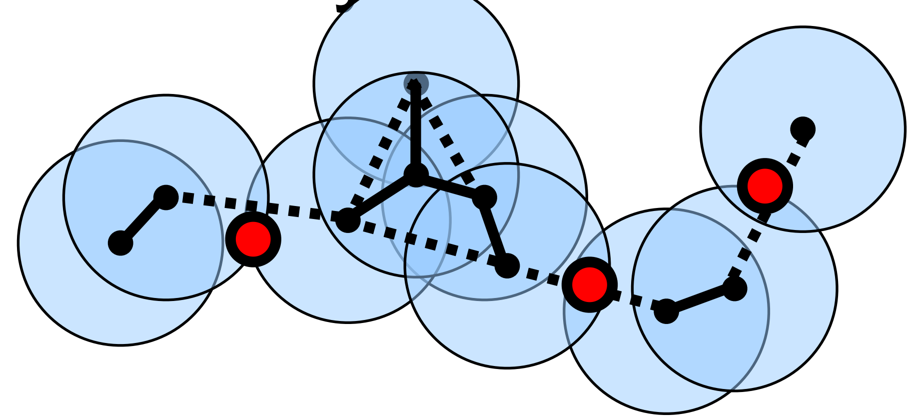

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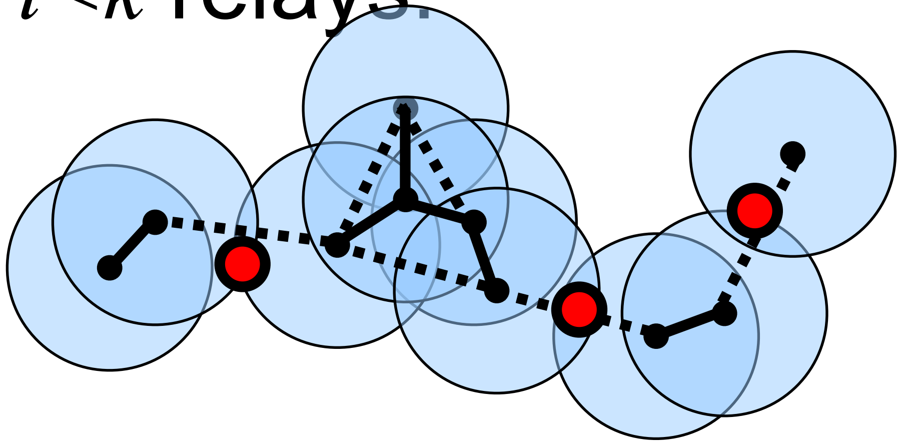

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Let $\mathcal{C}_{k+}$ be the set of clouds requiring ≥ k stabs

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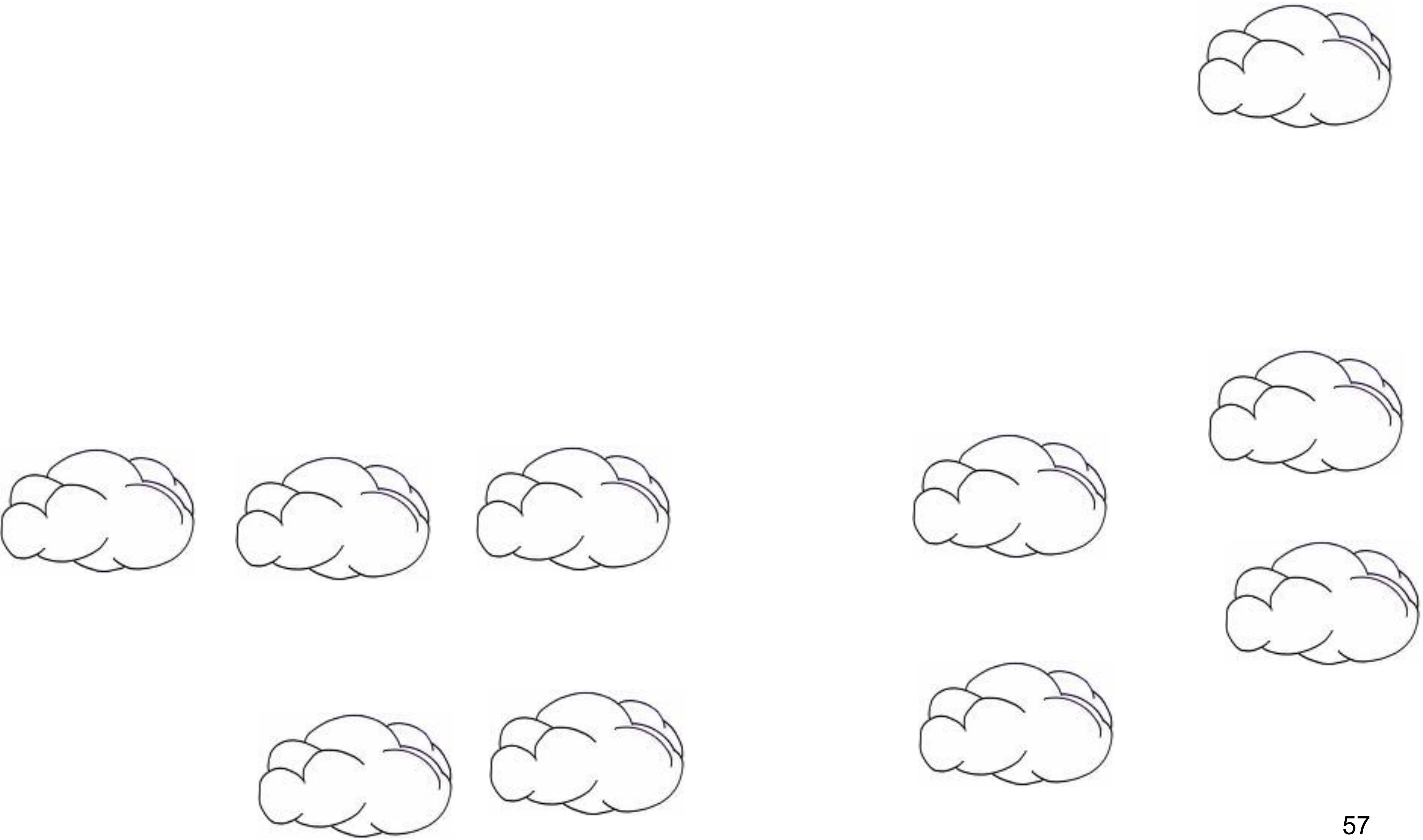

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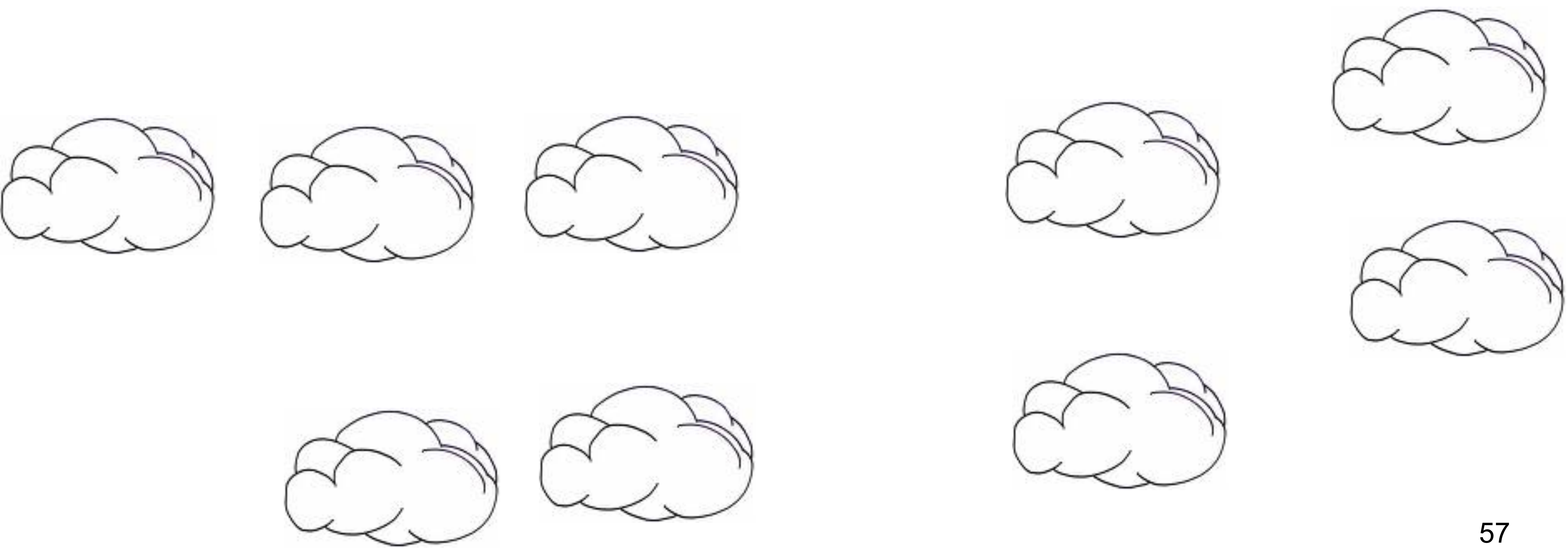

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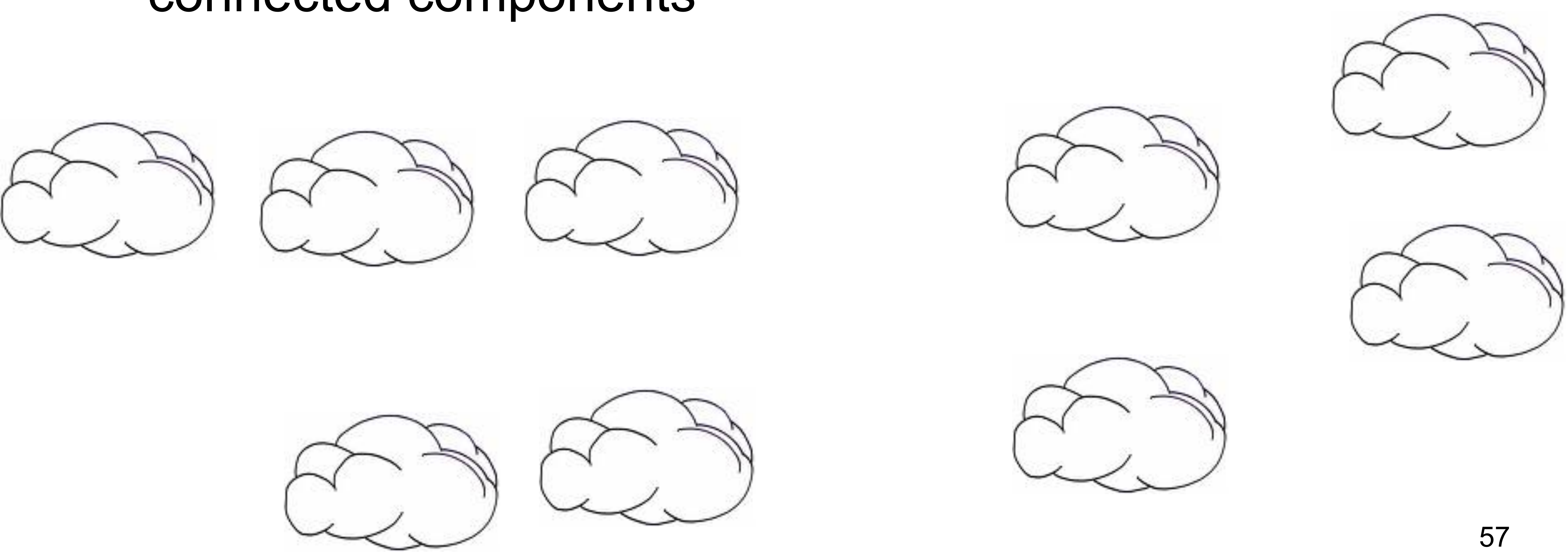

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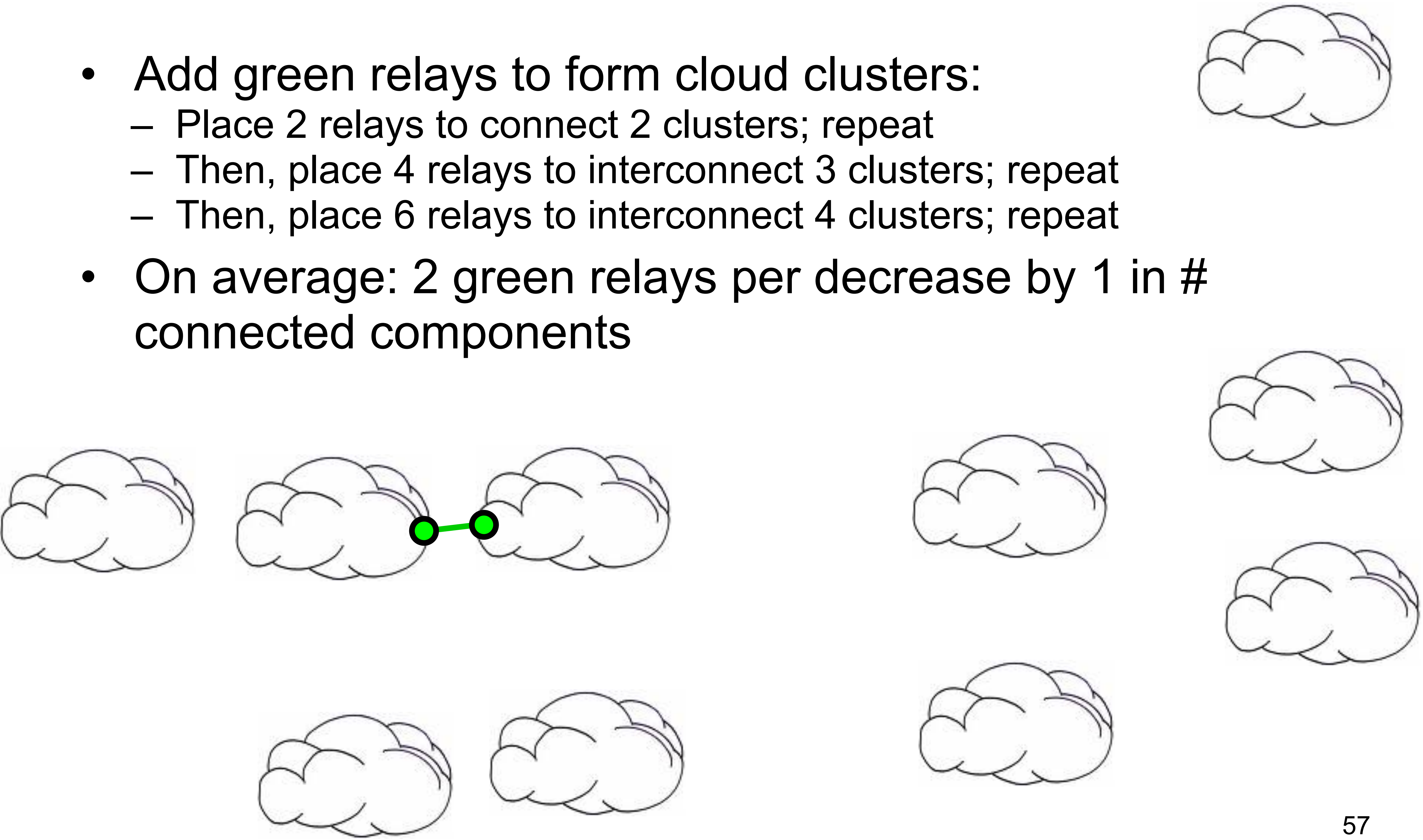

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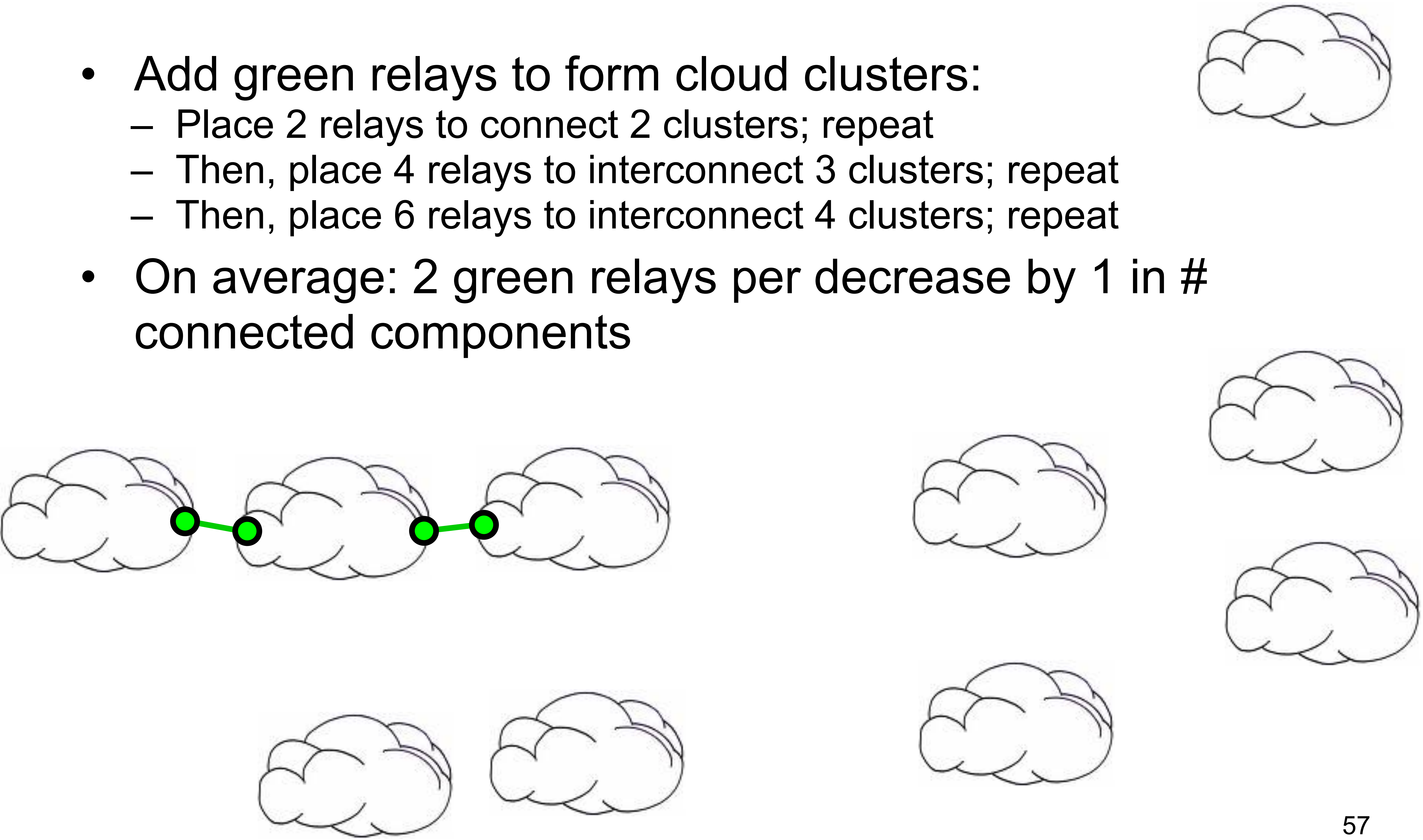

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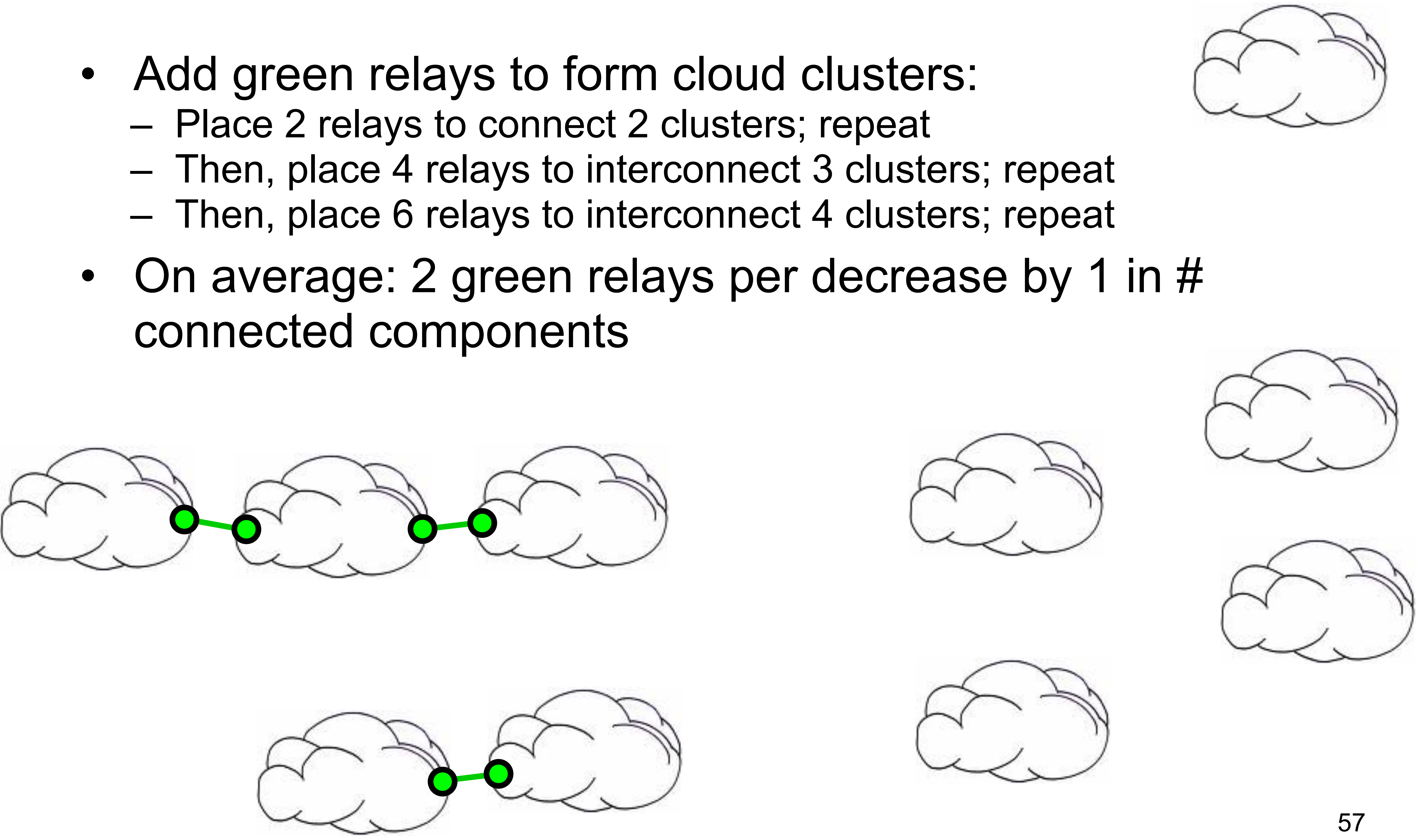

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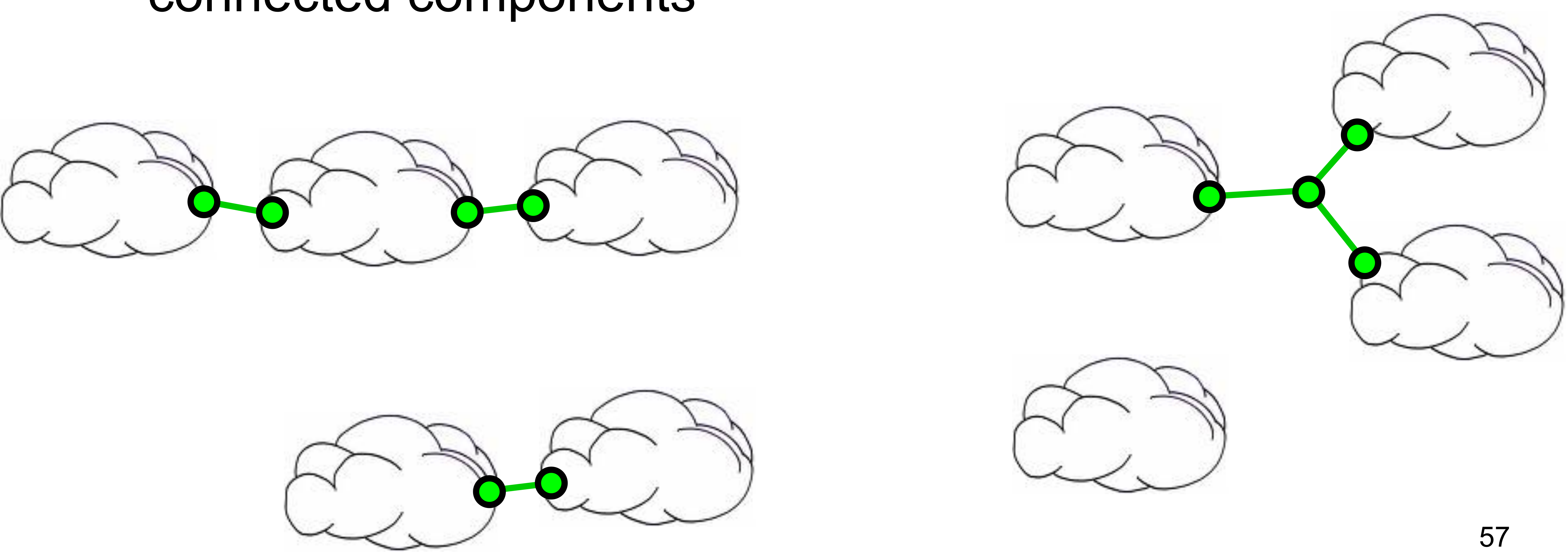

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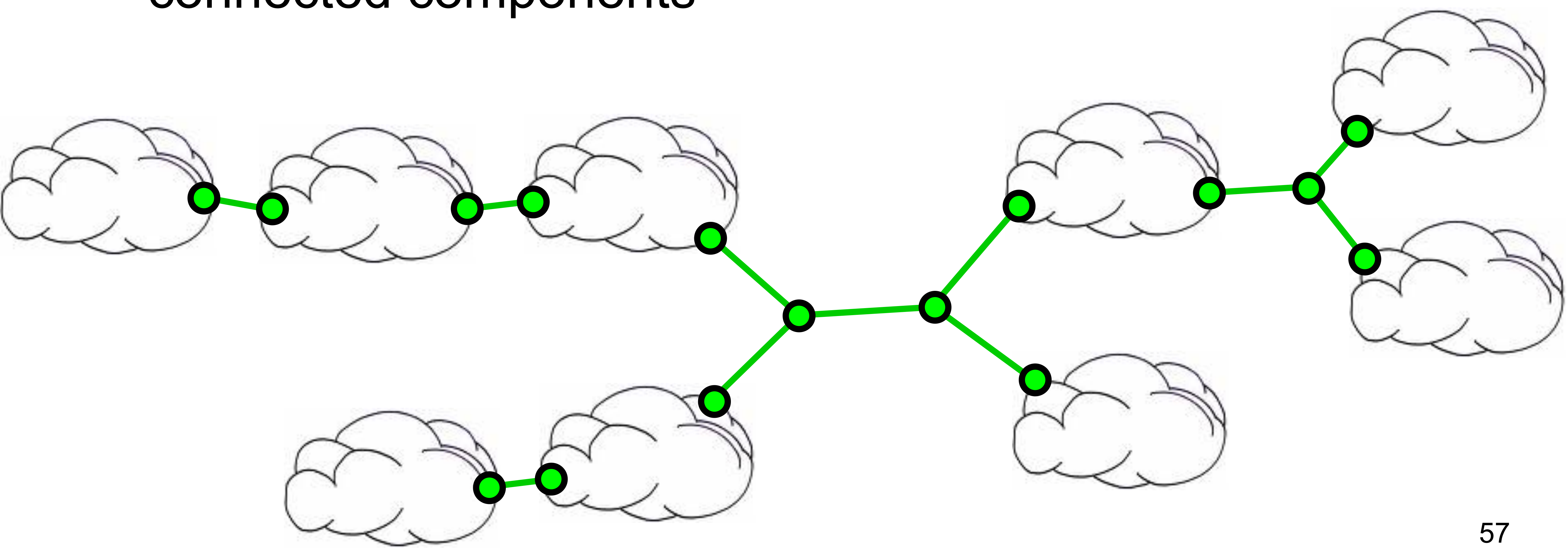

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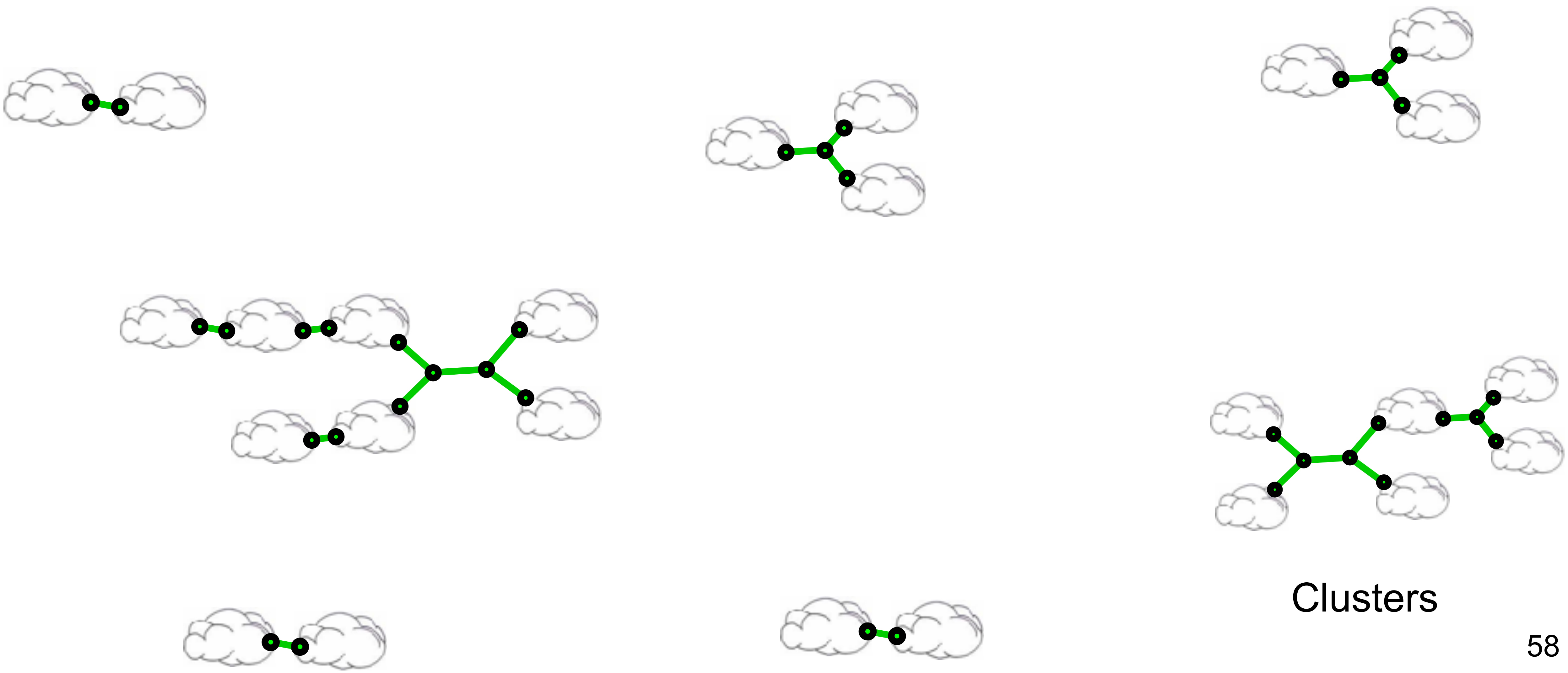

Clusters

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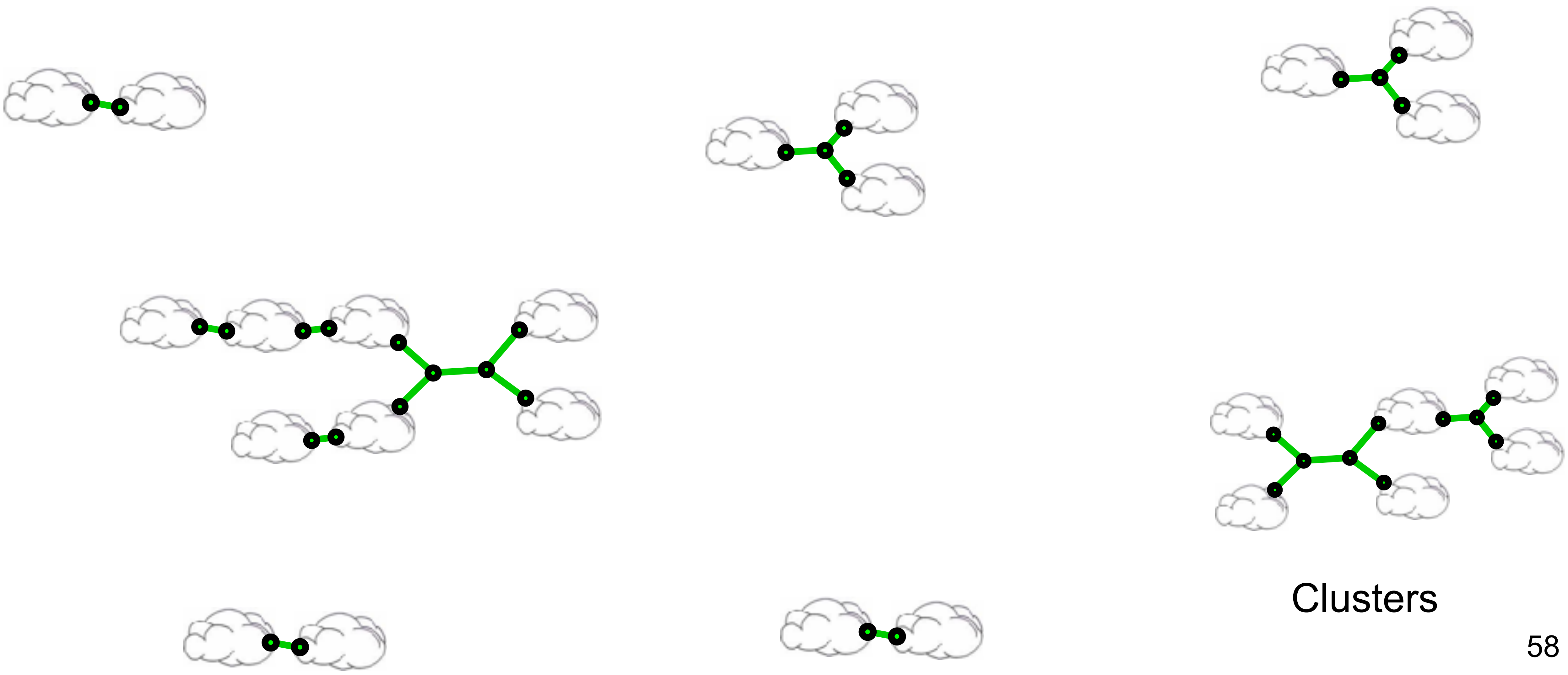

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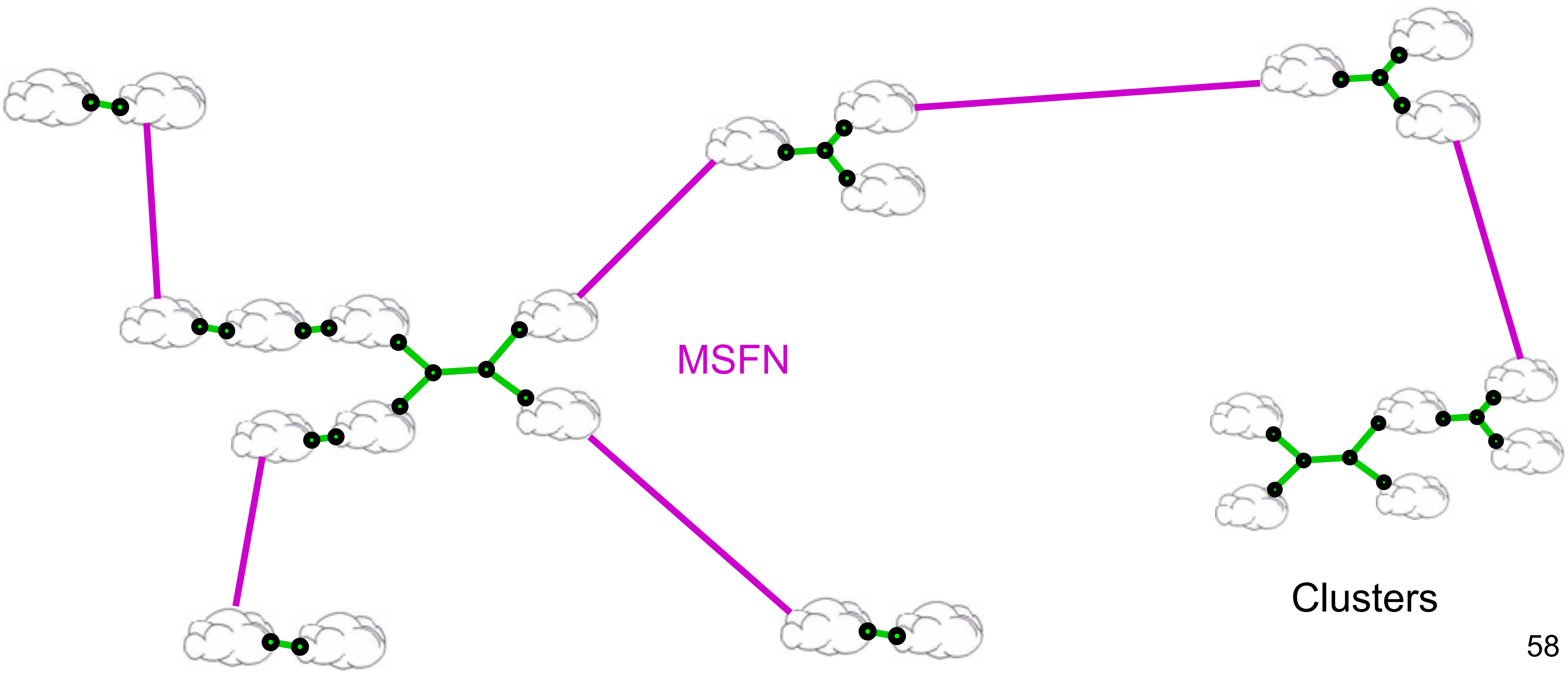

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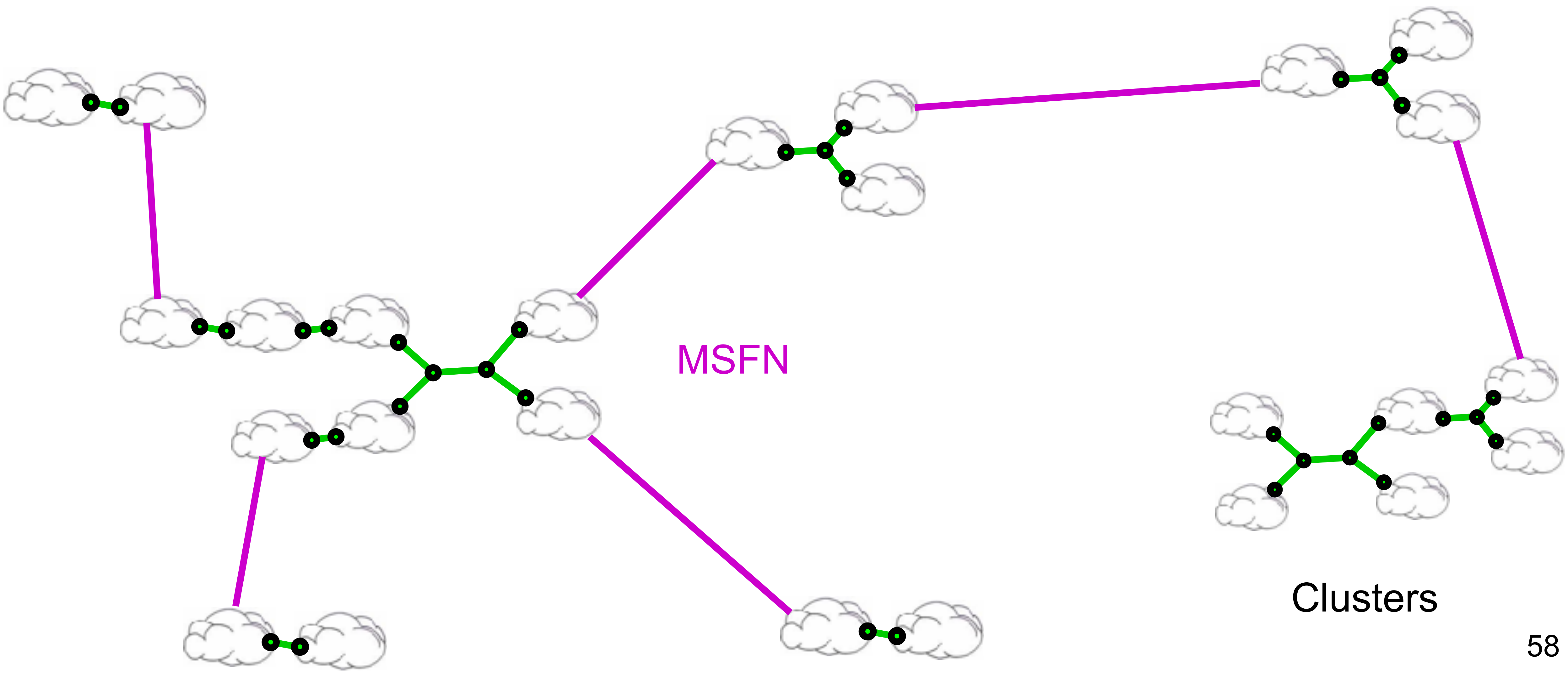

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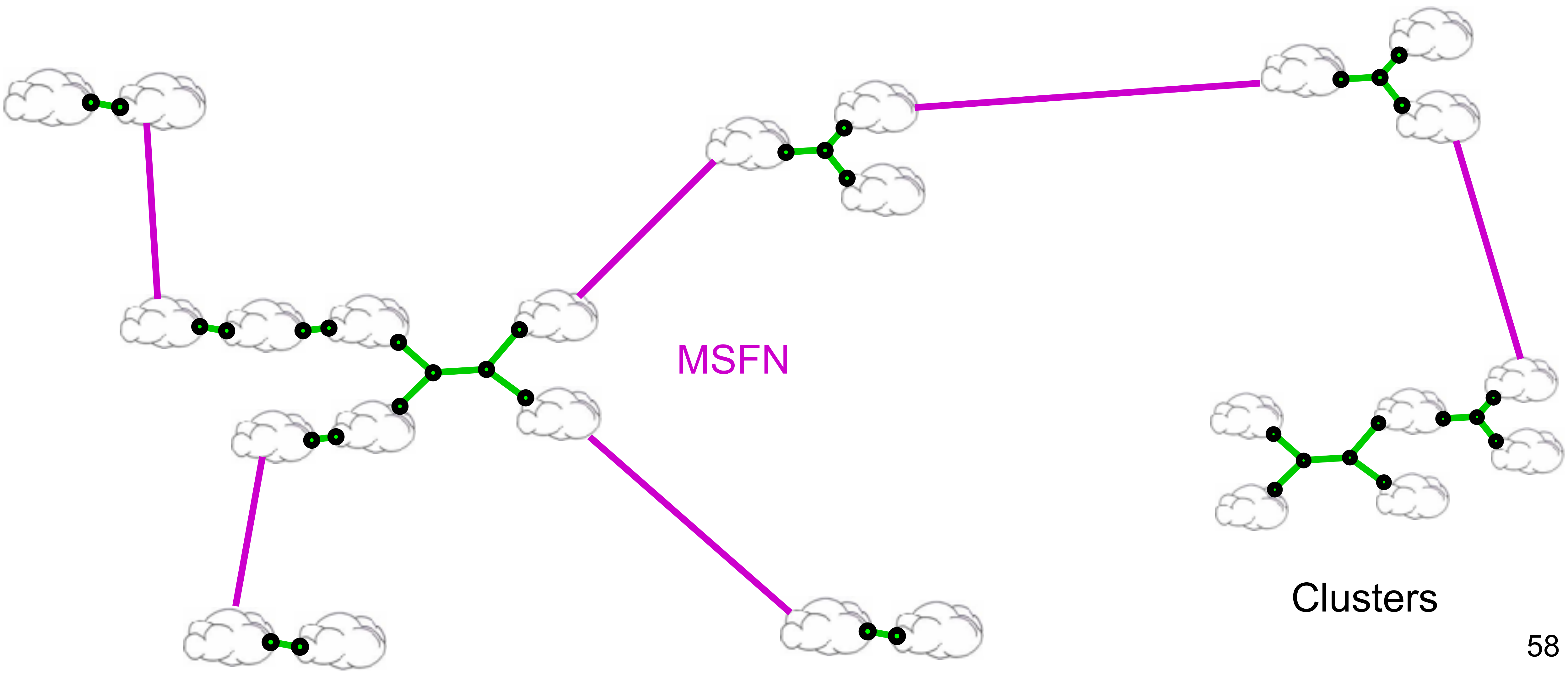

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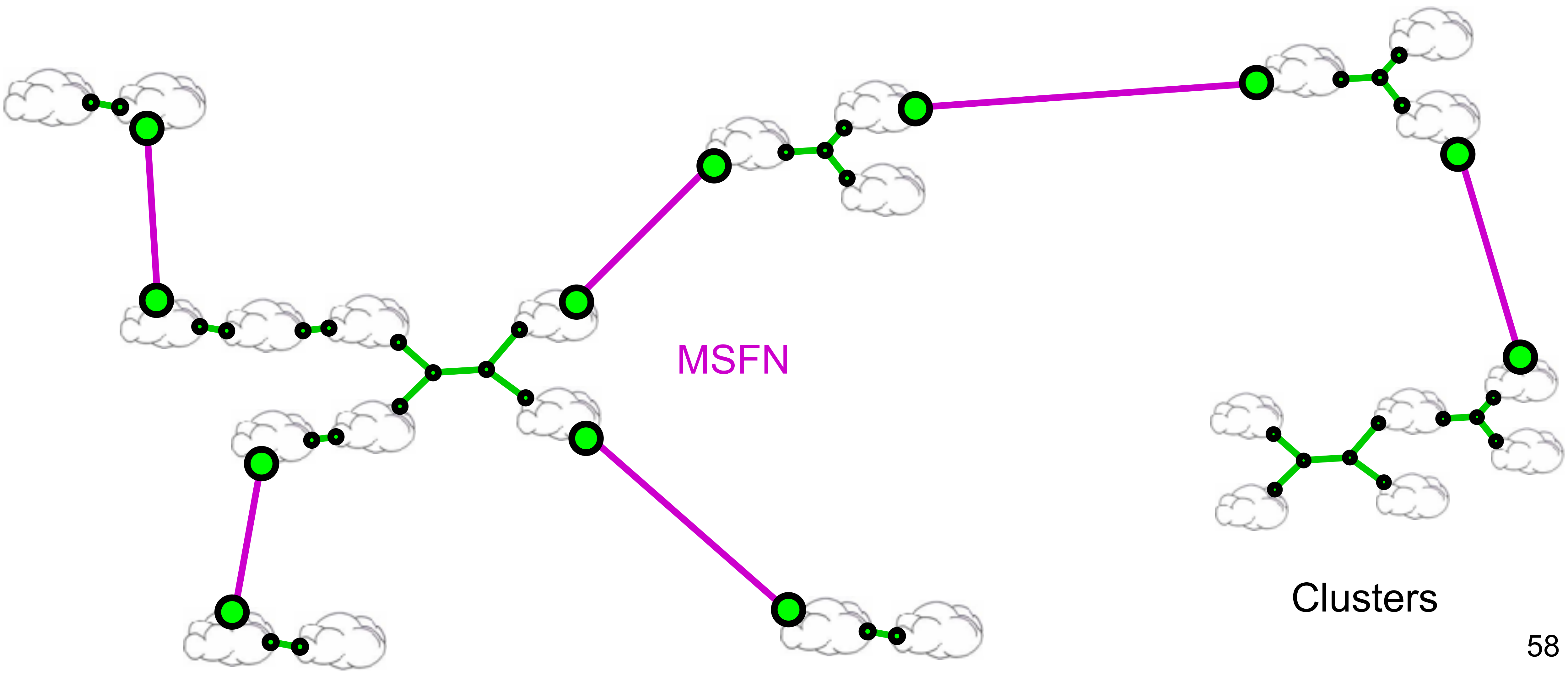

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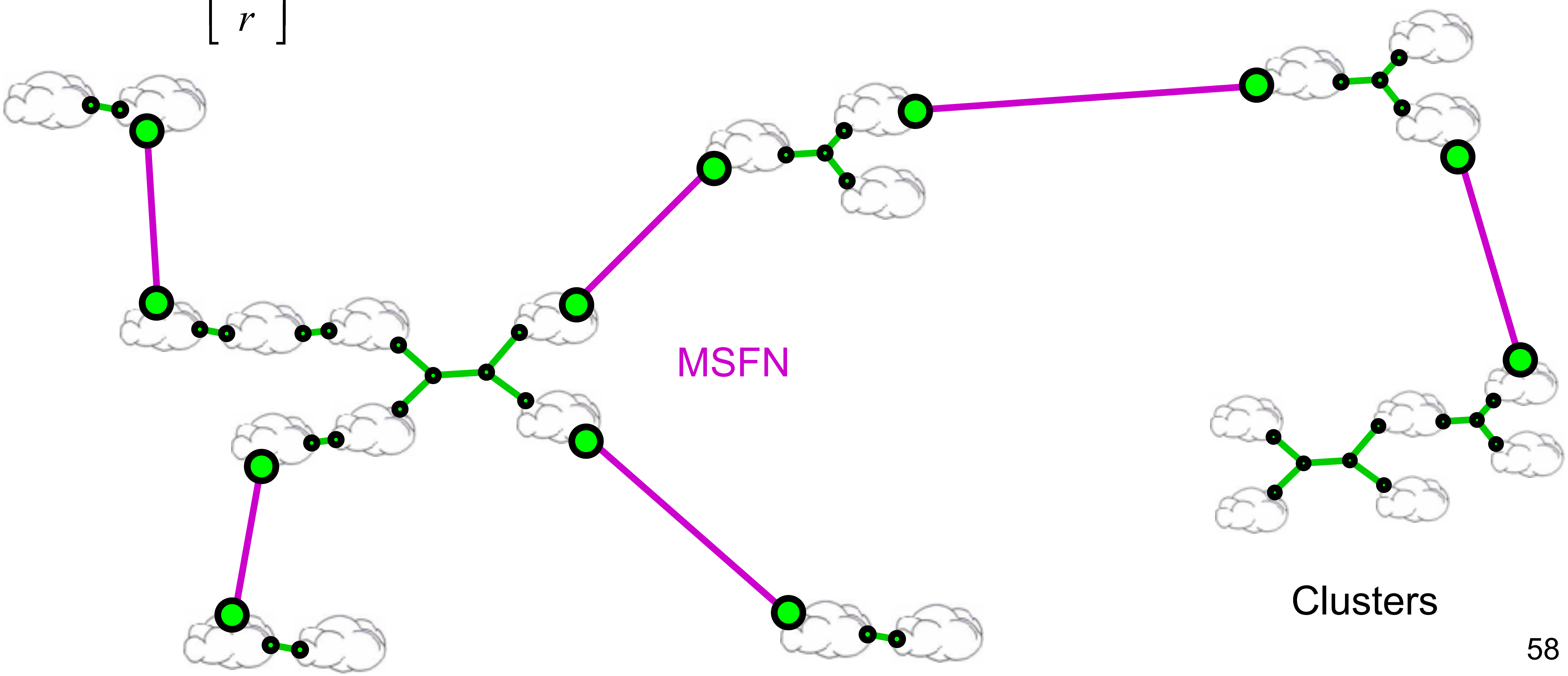

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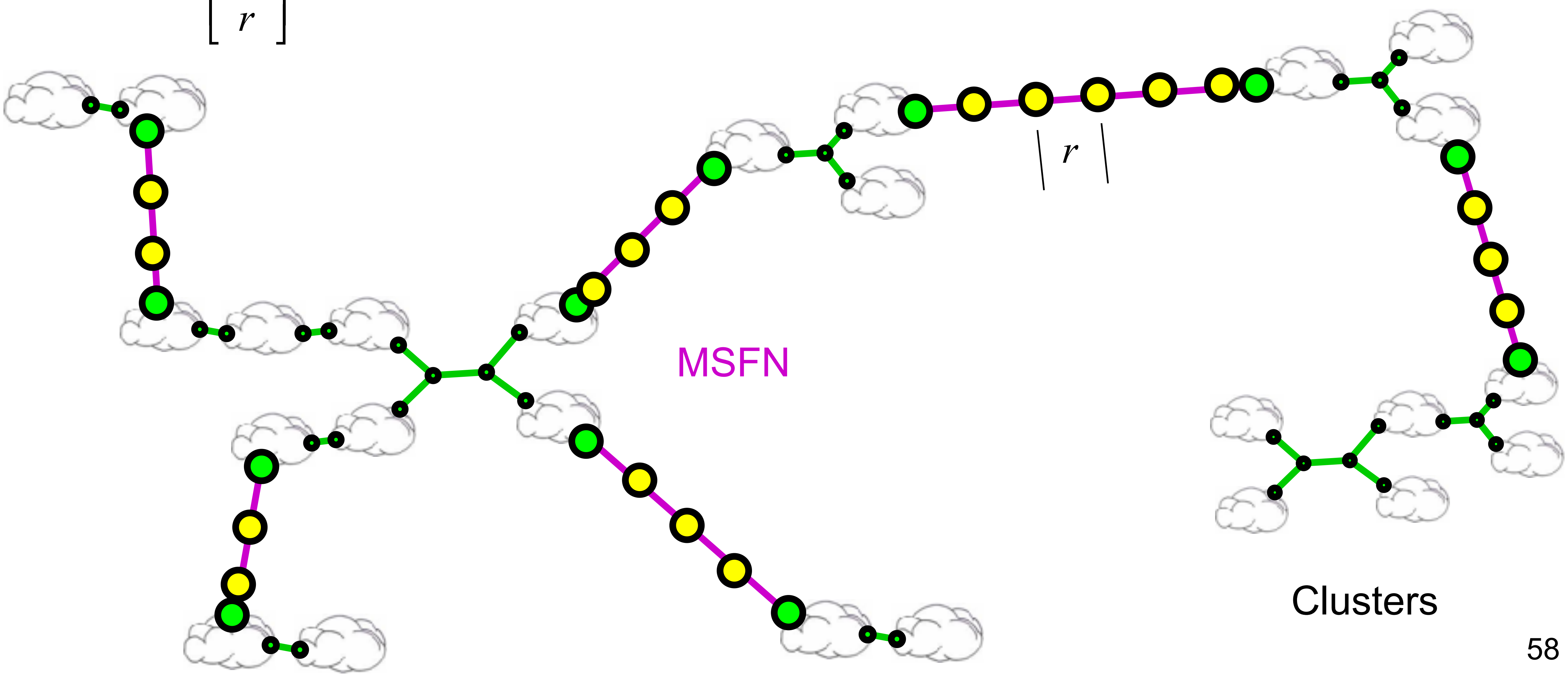

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Proof: Omitted.

(Combines Steiner tree ratio with degree arguments.).

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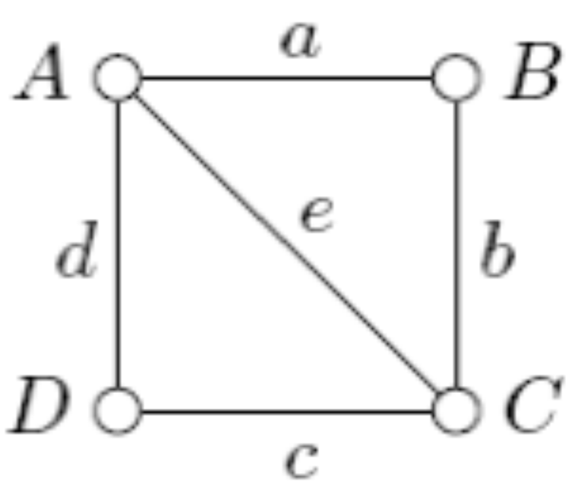

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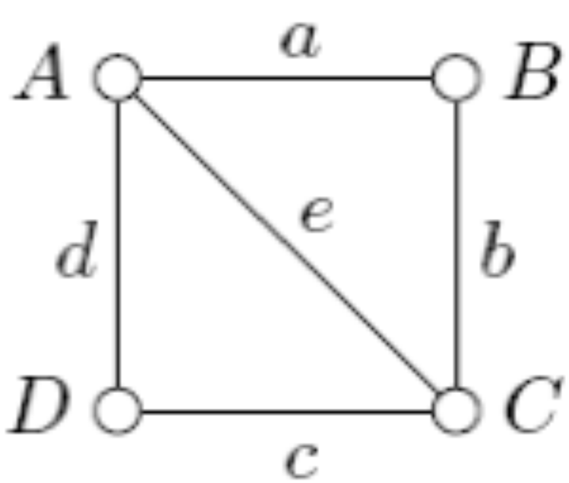

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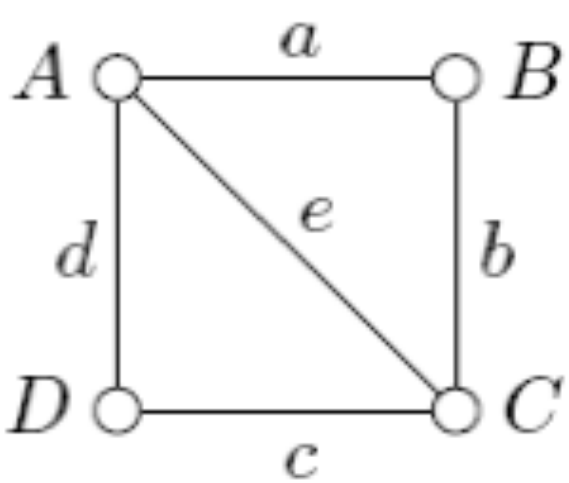

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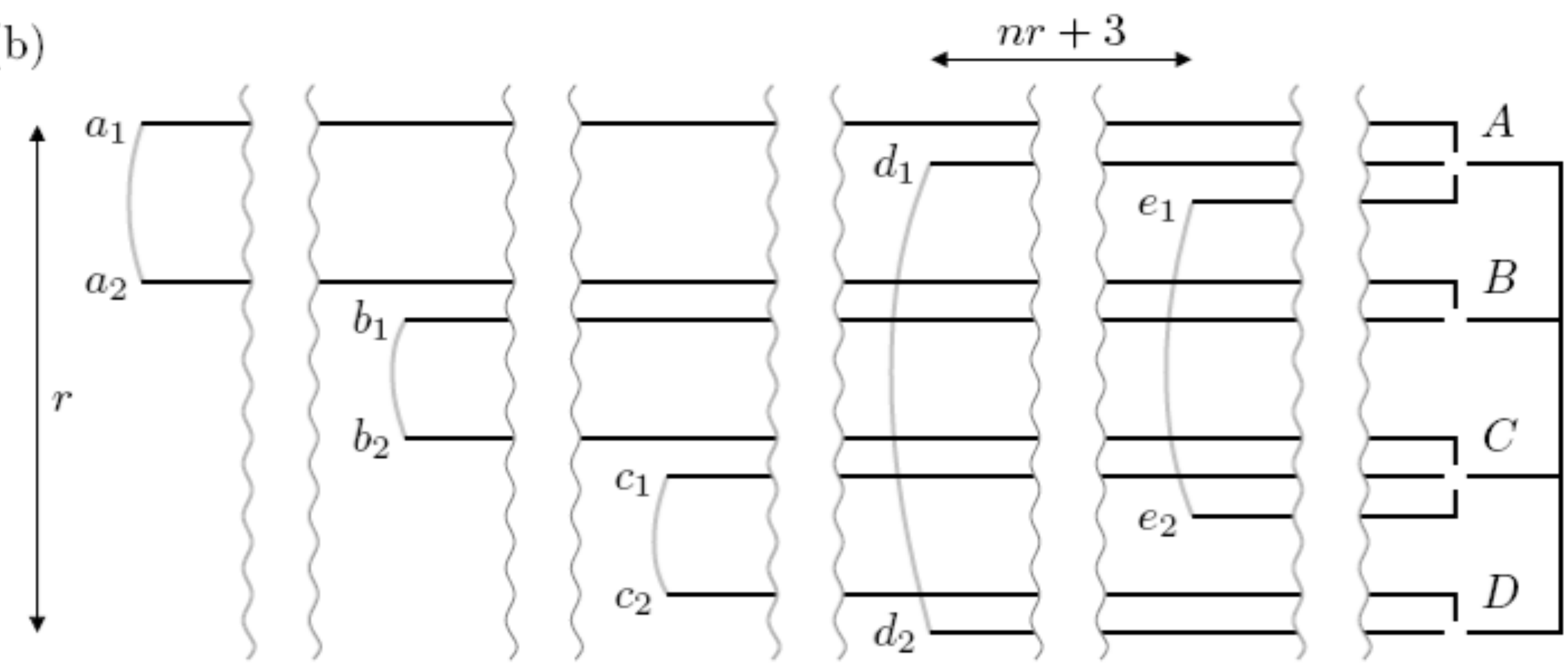

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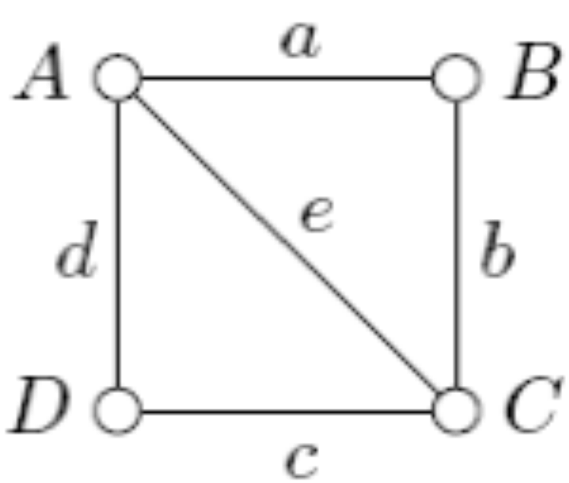

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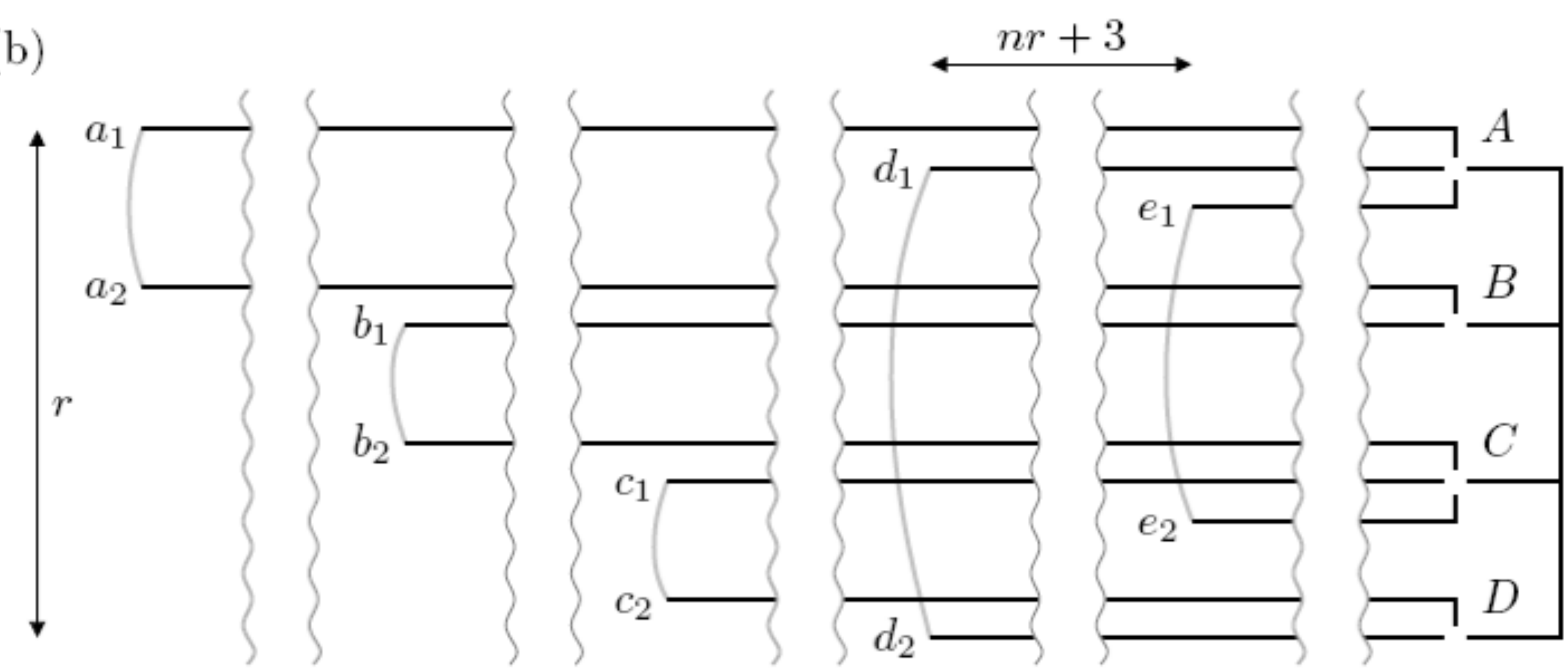

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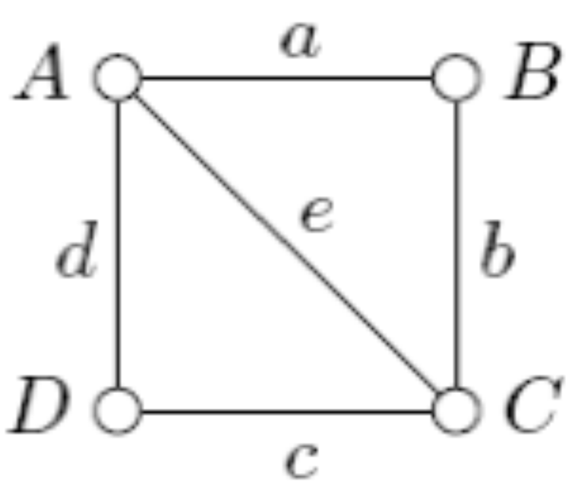

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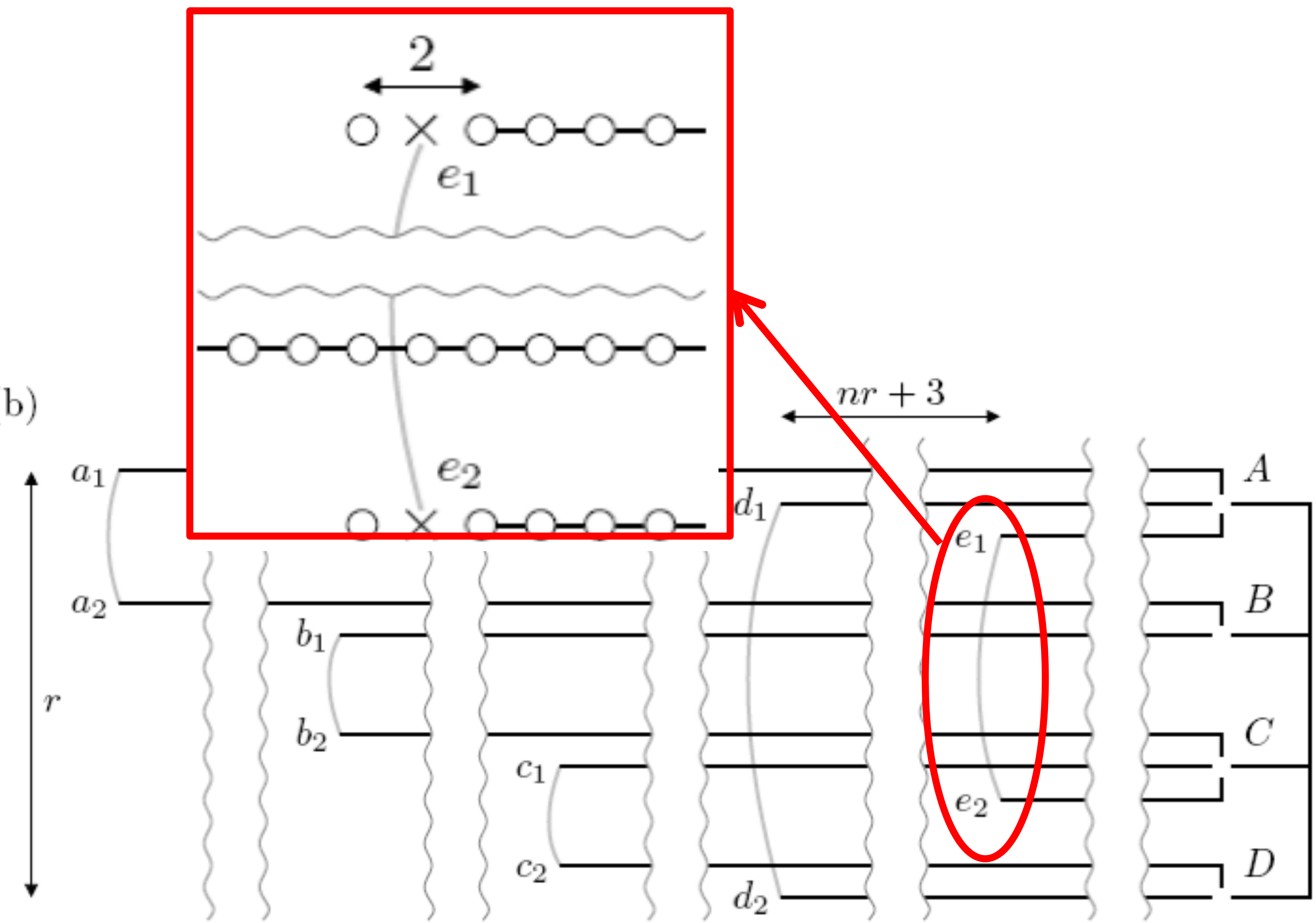

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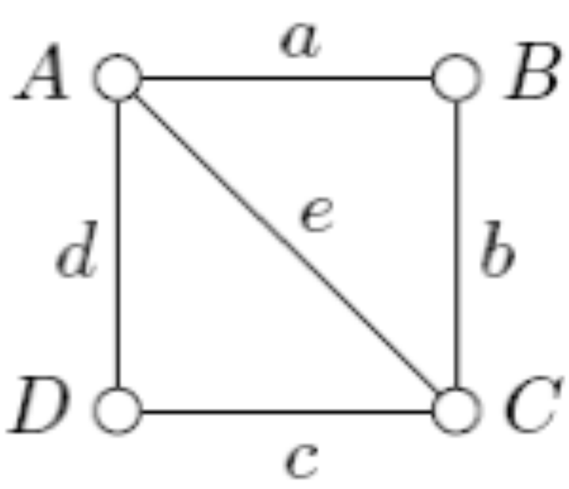

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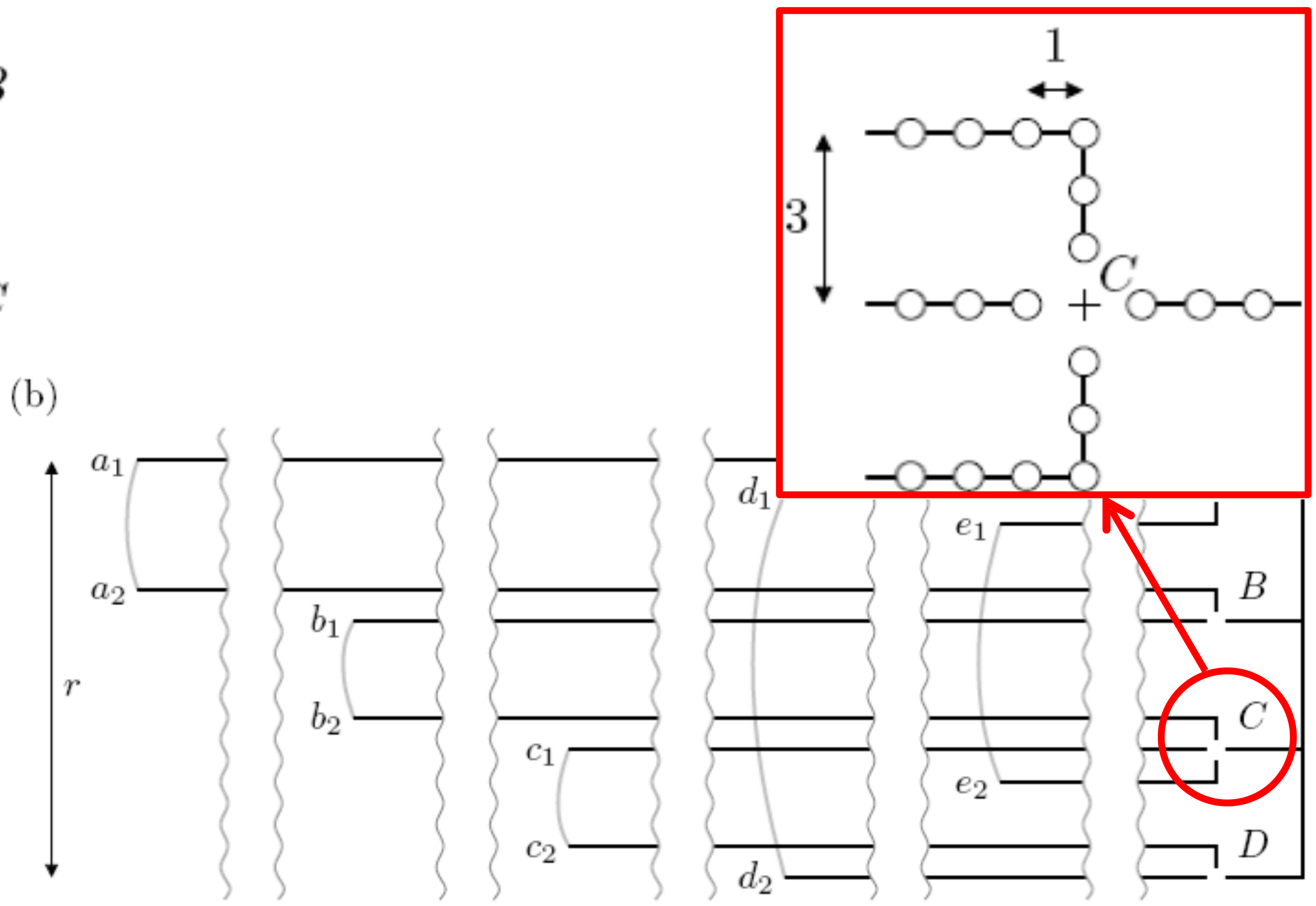

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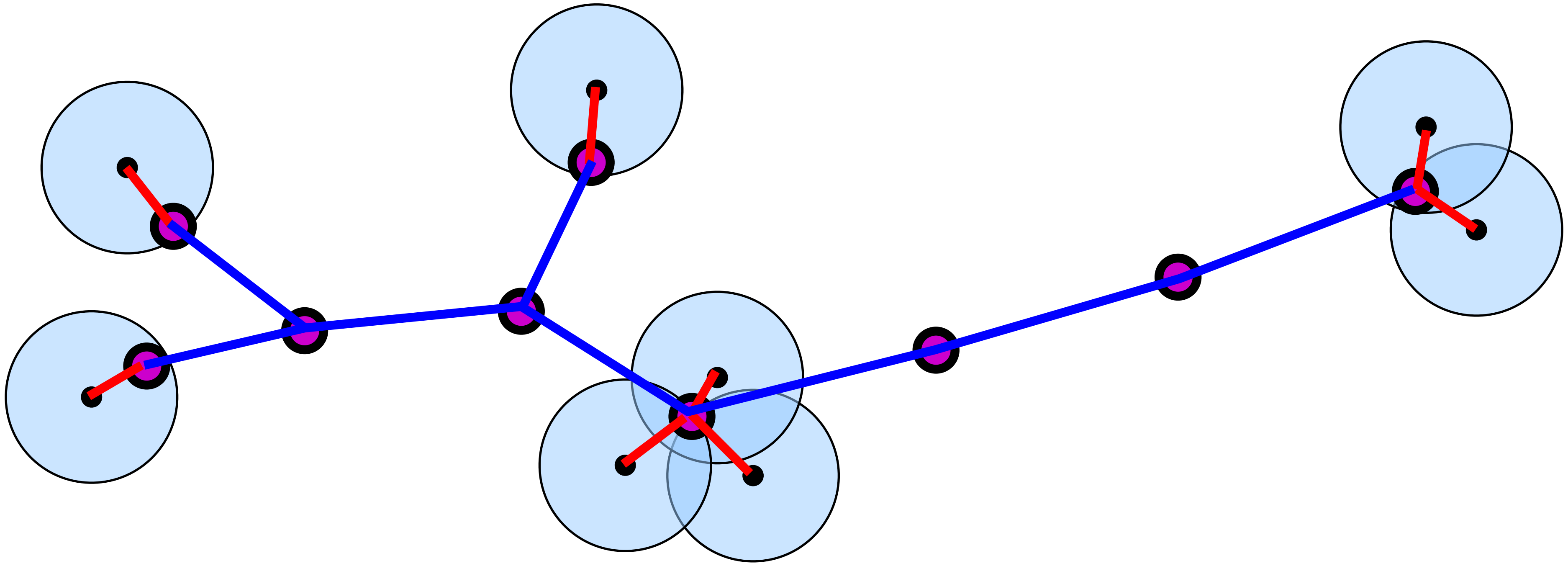

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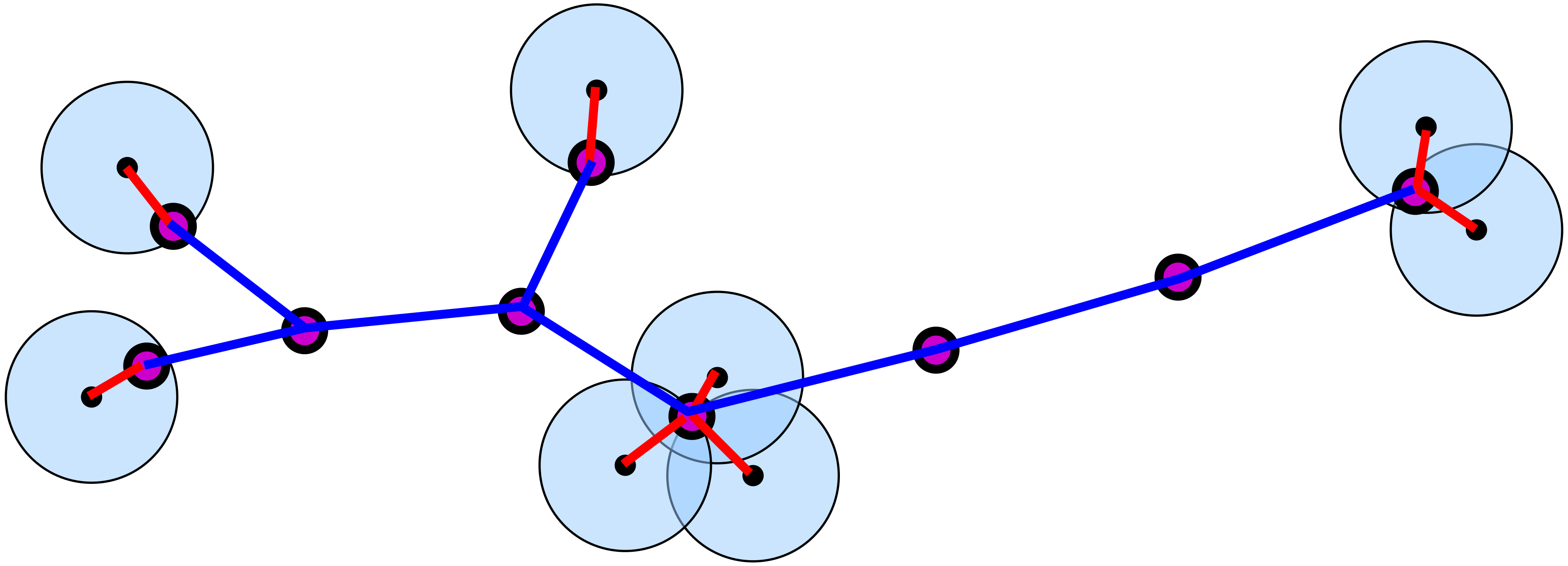
- Steiner spanning tree



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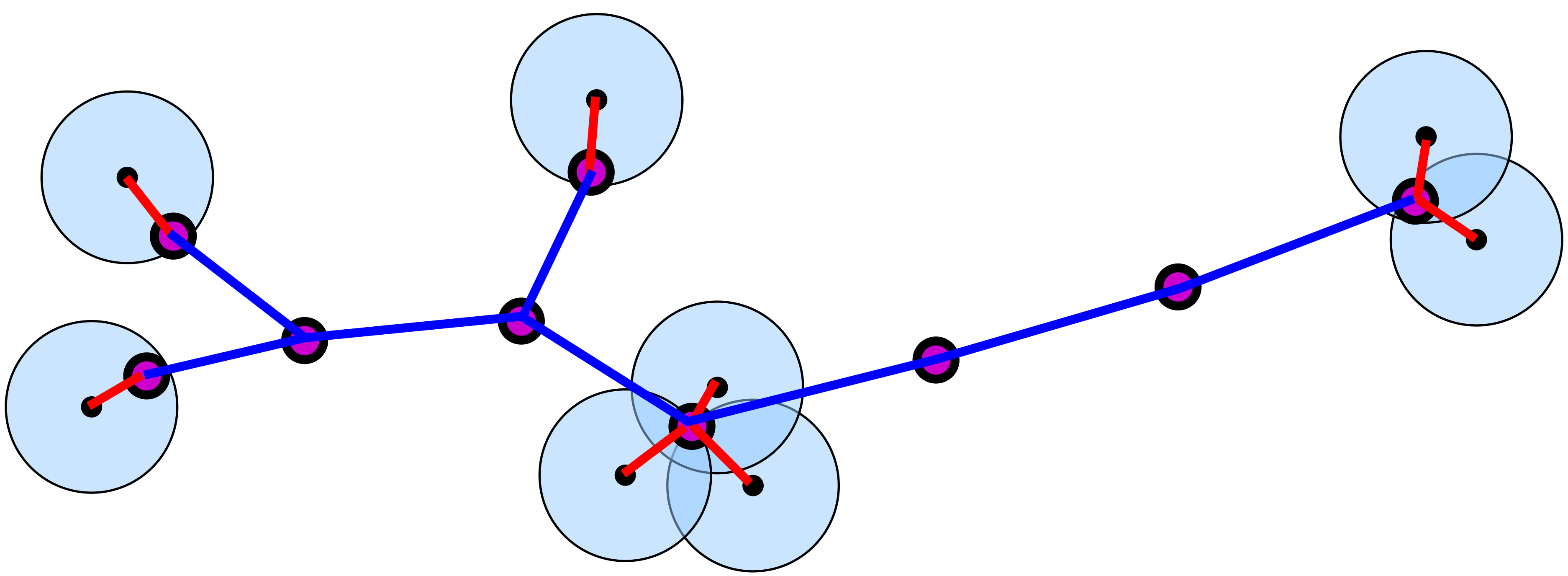

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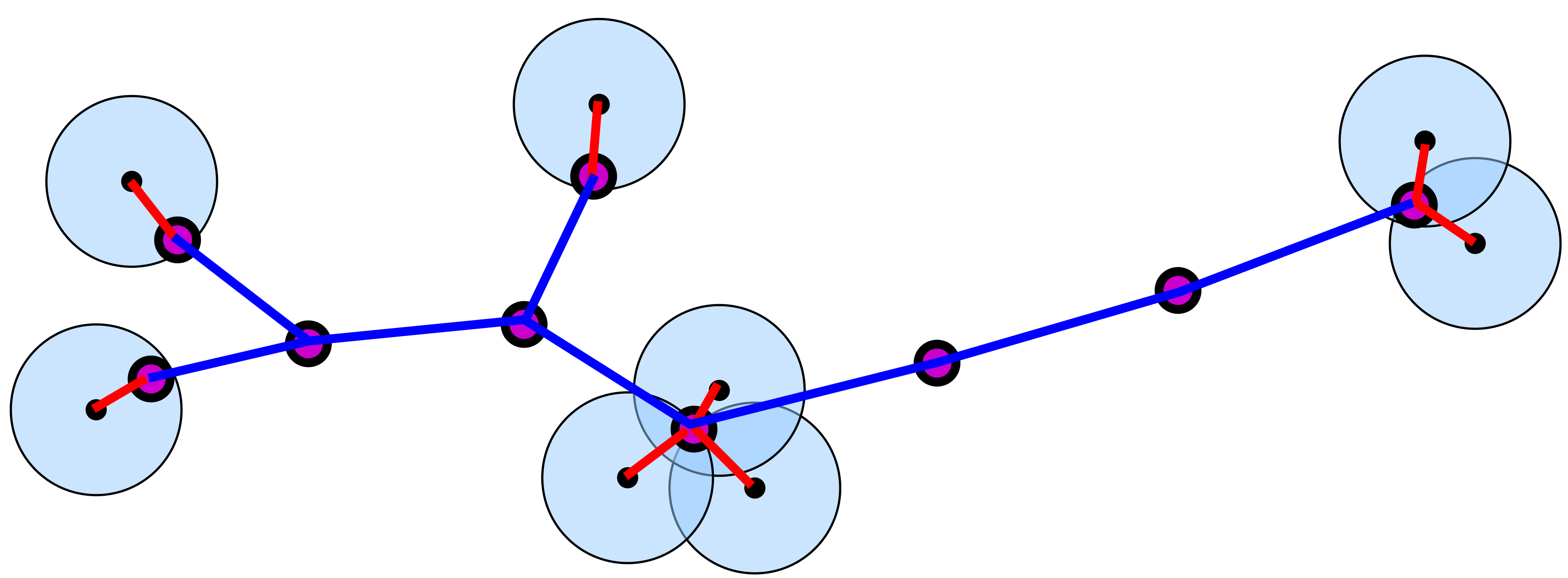

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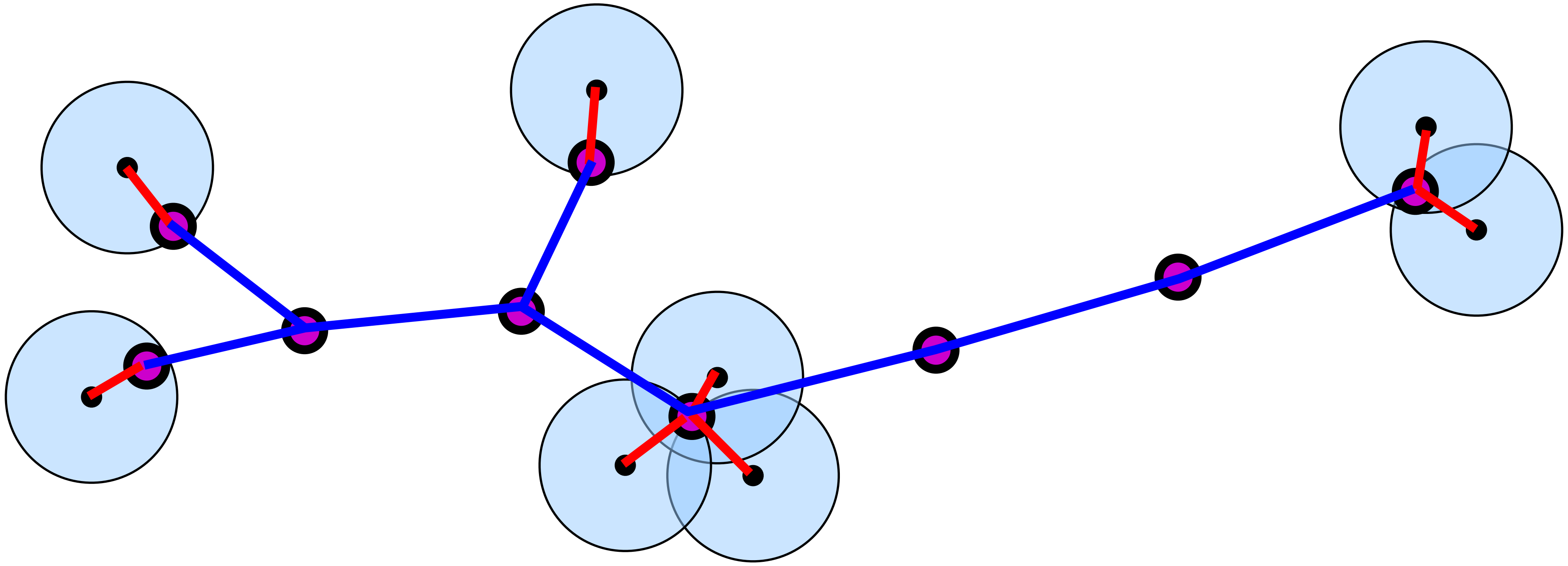
- Goal: Min # Steiner points (relays)



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- Optimize over all m-guillotine spanning trees using dynamic programming

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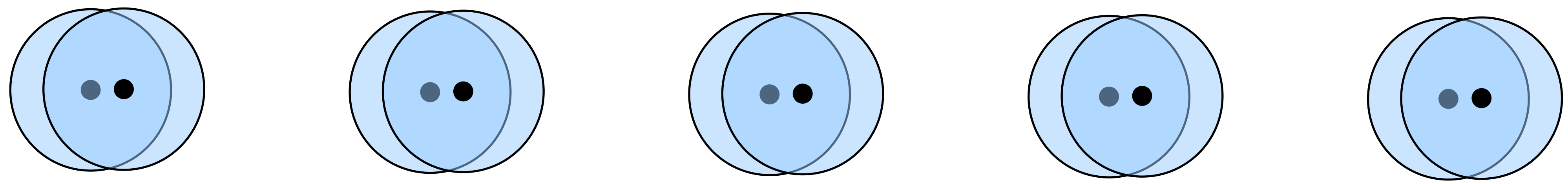

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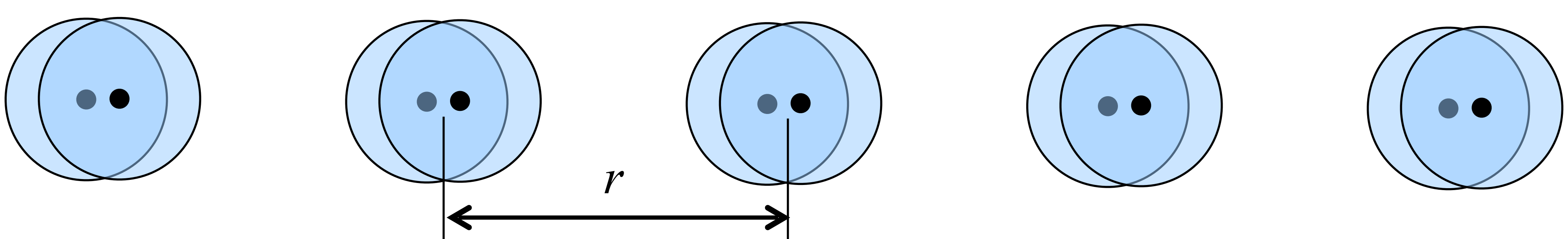

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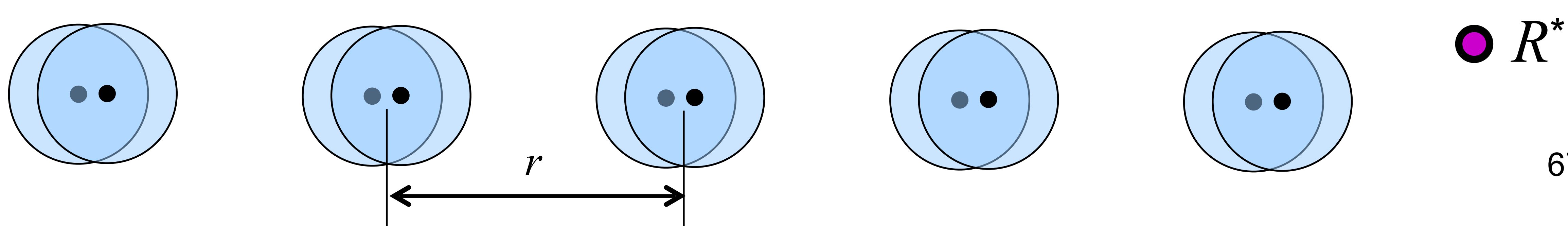

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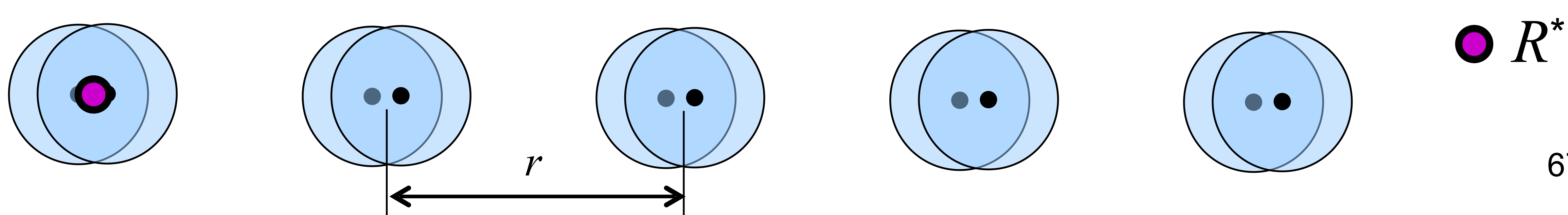

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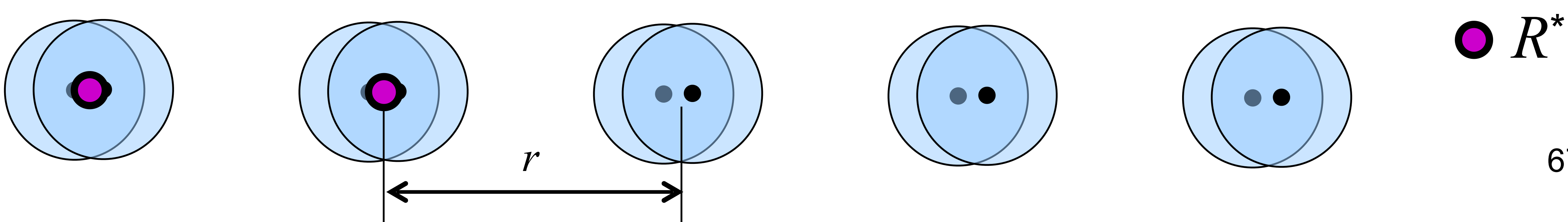

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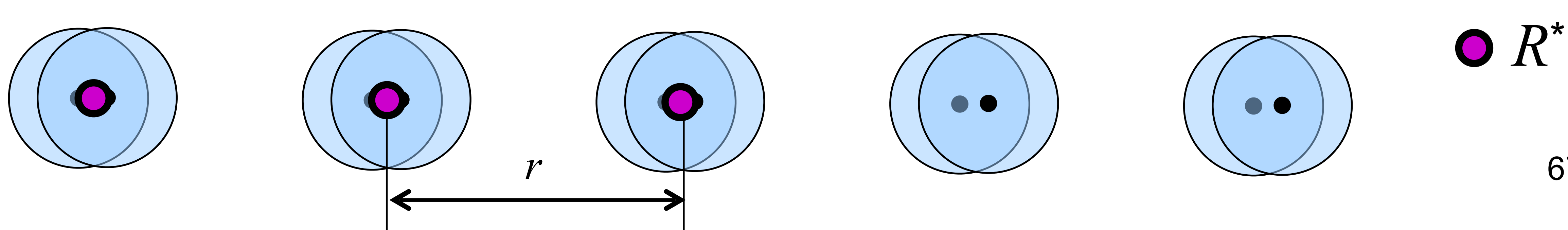

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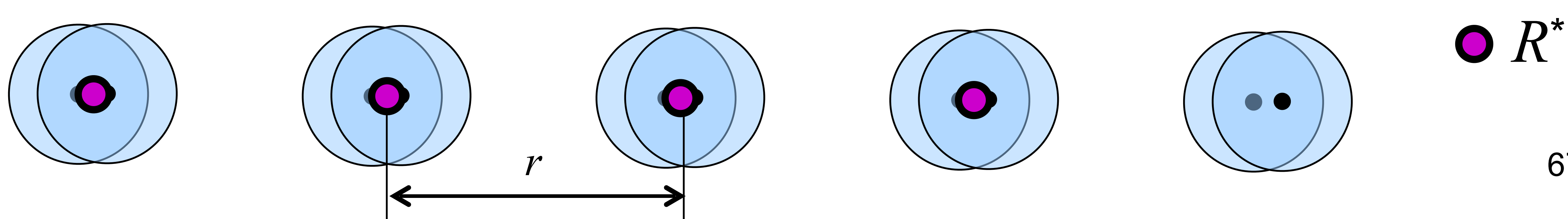

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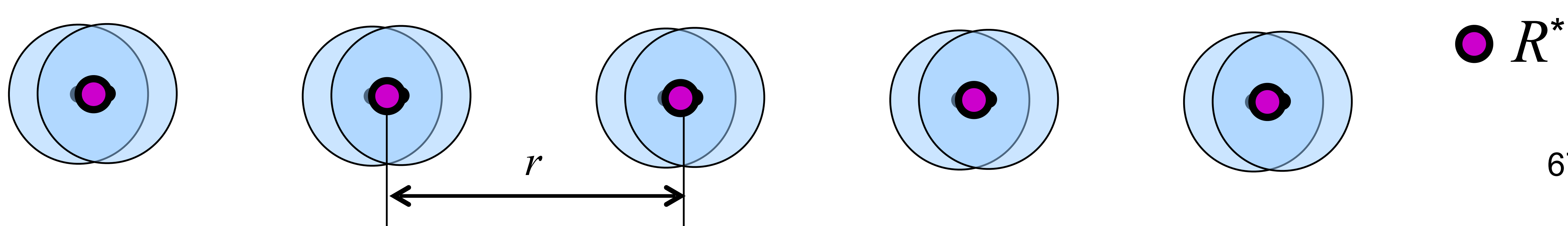

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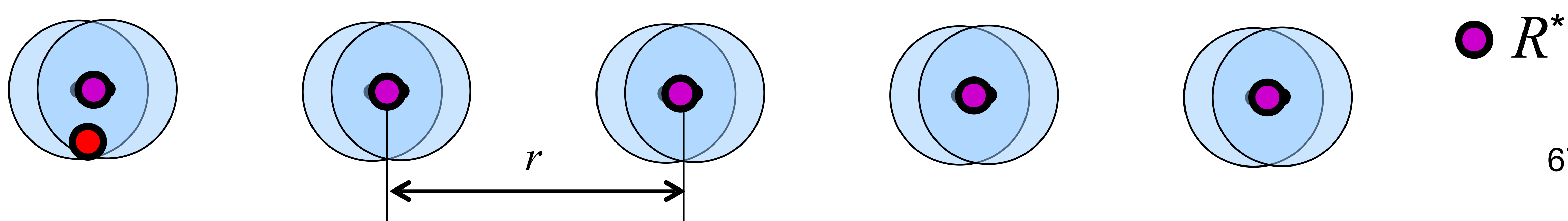

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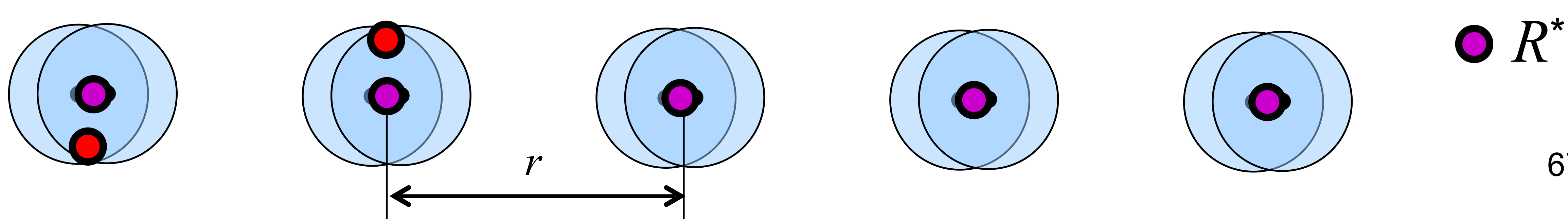

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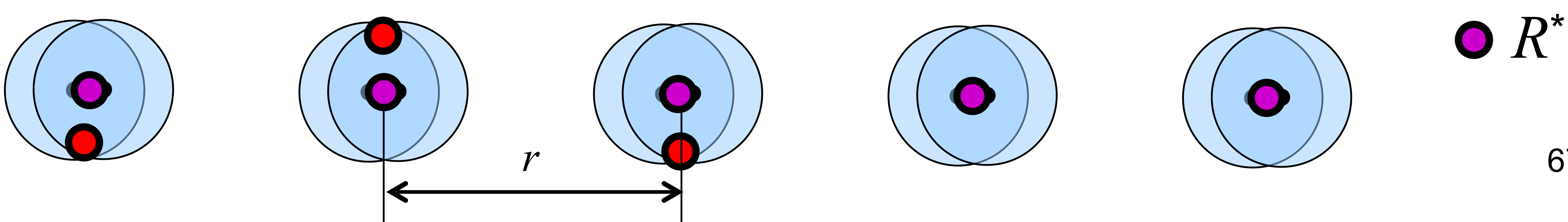

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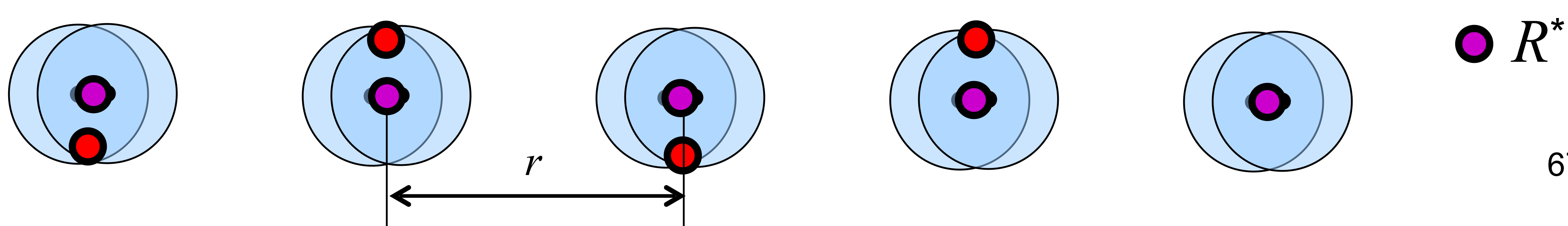

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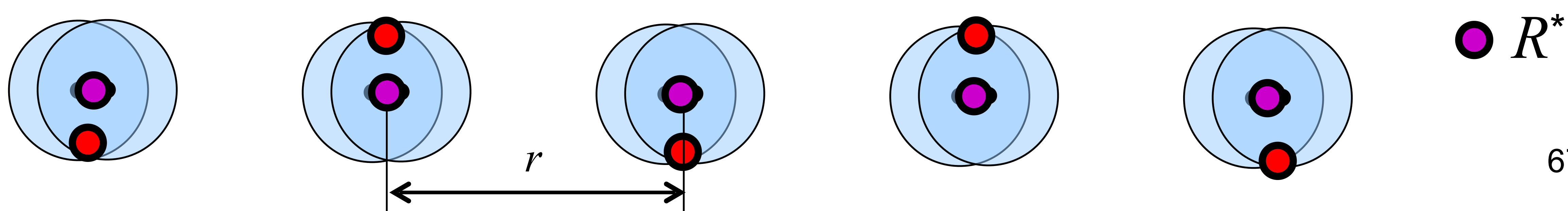

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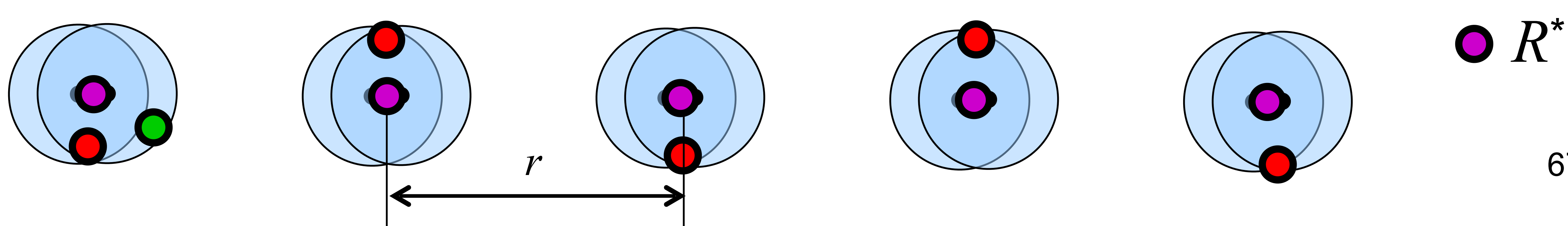

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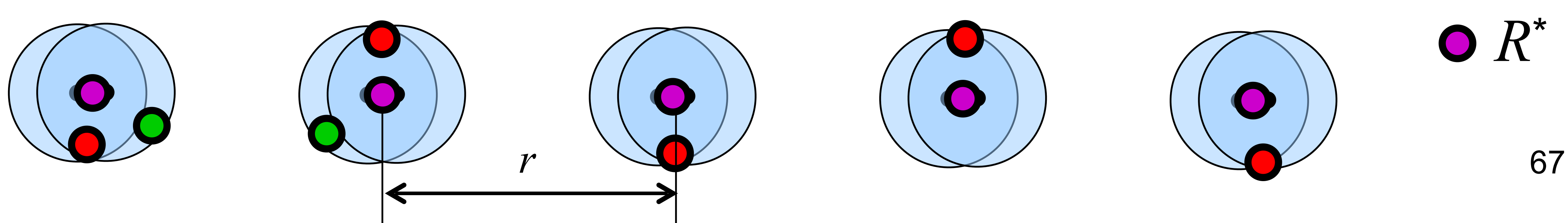

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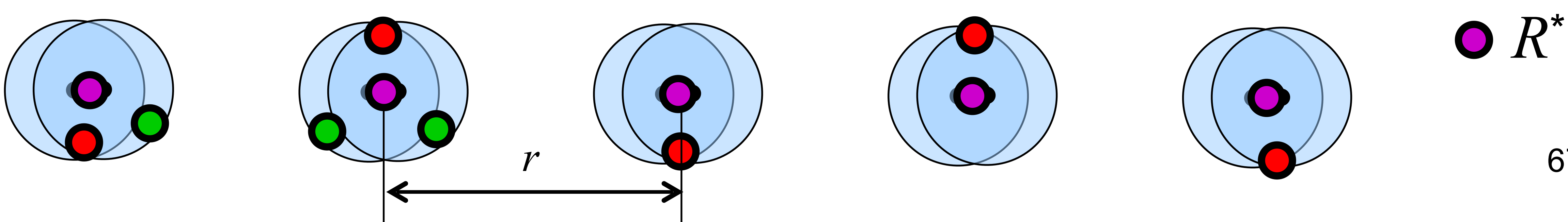

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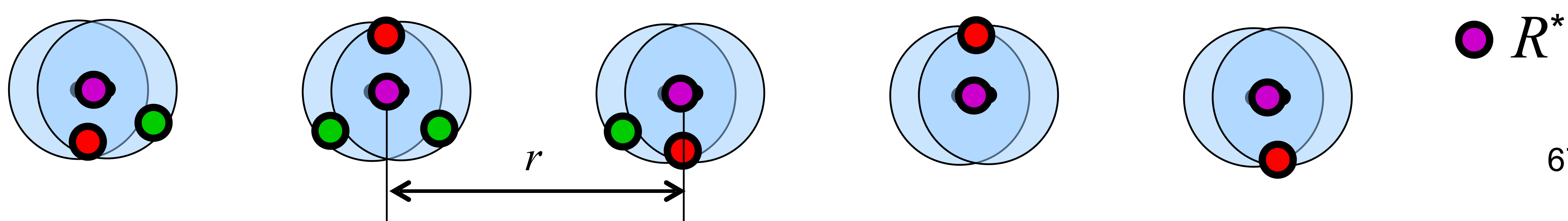

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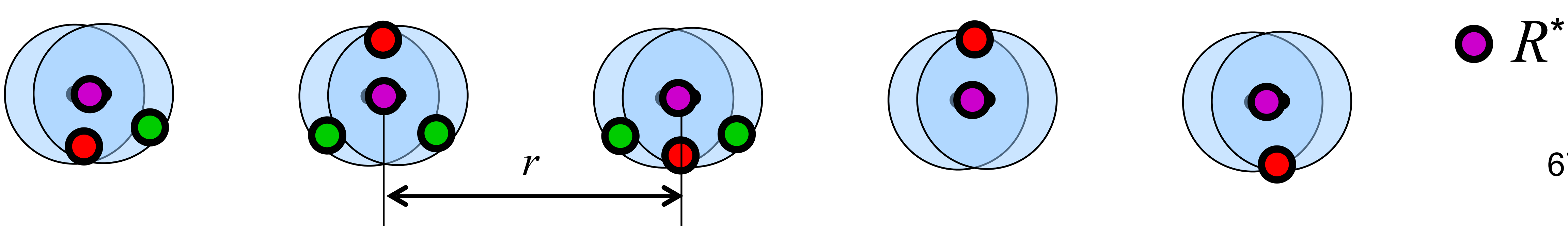

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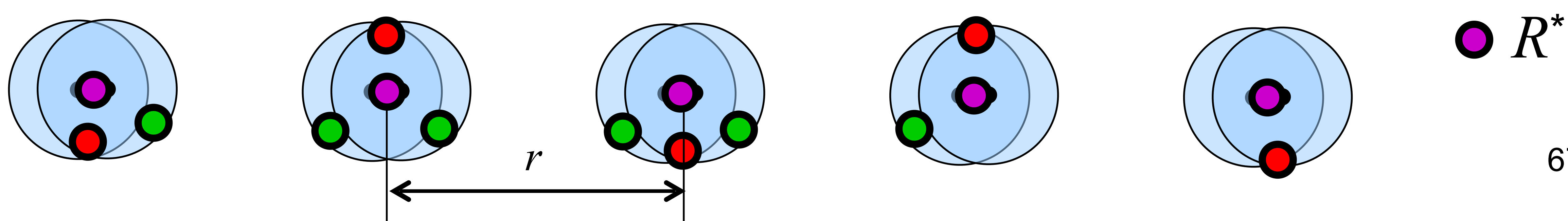

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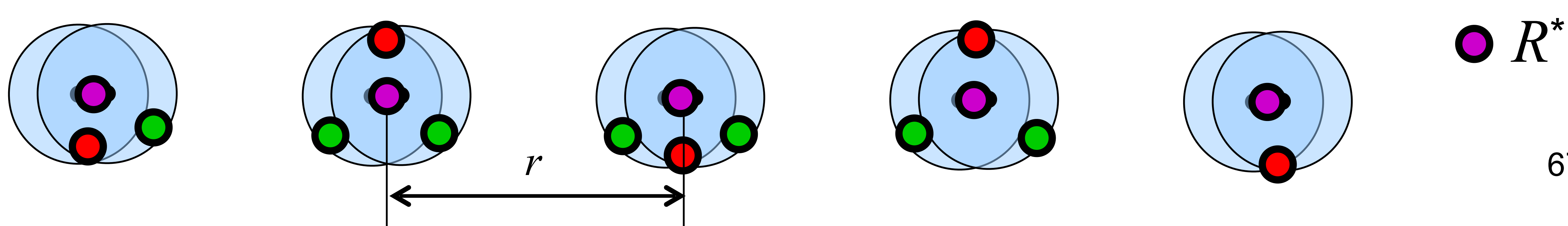

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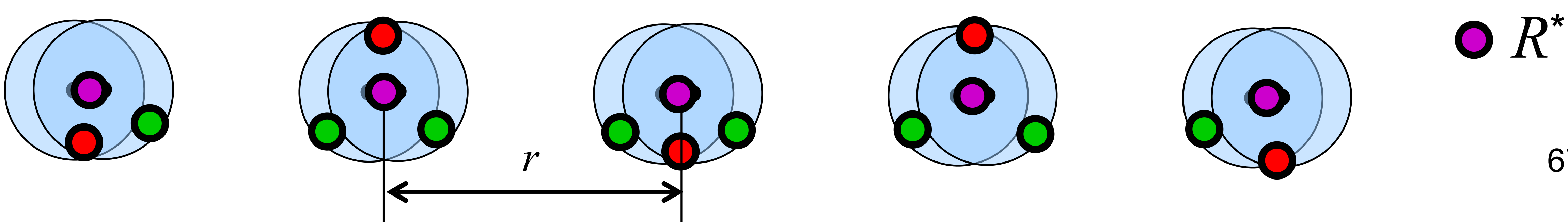

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  - [Bredin, Demaine, Hajiaghayi, Rus, MobiHoc'05, Zhang, Xue, Misra]

1. Introduction
2. Review
3. Extra Packing: Dispersion
4. Extra Tours: Lawn Mowing
5. Relay Placement
6. Coordinated Motion Planning

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Video!

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Thank you!