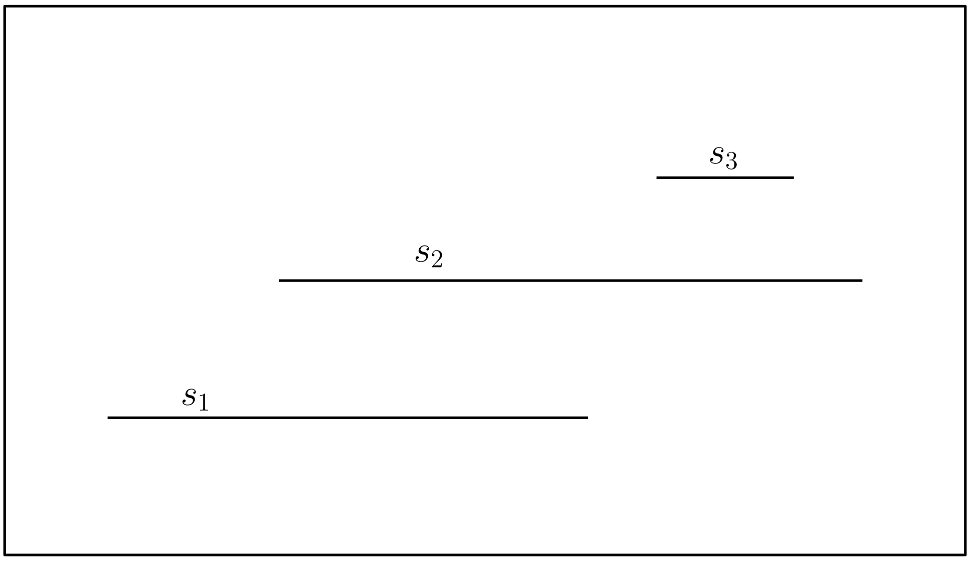
Algorithms Group **Summer '15**
Departement of Computer Science - IBR
TU Braunschweig

Michael Hemmer

Geometric Algorithms
Exercise 4
June 19, 2014

This sheet is comprised of several exercise. The solutions should be handed in by **June 25, 2015** until 19:00pm. This can be done by placing them in the appropriate box of the usual exercise locker.
In order to achieve the *"Studienleistung"*, you must achieve 50% of the points over all exercise sheets and you must have presented at least two exercises until the end of the term. Please mark those exercises that you would like to present during the tutorial.

Exercise T1 (Trapezoidal Map – Search Path):



Insert the segments according to sequence $[s_1, s_2, s_3]$.

a) Draw $T(S_i)$ and $D(S_i)$ for $i \in \{1, 2, 3\}$ separately.

Now observe that not all paths in $D(S_3)$ are actually valid search paths. Let $\mathcal{D}$ be the maximal length of all paths in $D(S)$ and $\mathcal{L}$ be the maximal length of all valid search paths.

b) Indicate a path of length $\mathcal{D}$ in $D(S_3)$.

c) Indicate a valid search path of length $\mathcal{L}$ in $D(S_3)$.

(10+2+2 P.)

Exercise T2 (Planar Point Location - Trapezoidal Map):

Let n be a square, that is, $n = k^2$ for some $k \in \mathbb{N}$. Sketch a set of segments S of cardinality n and give an insertion order such that the length of the longest path in $D(S)$ is $O(n)$ whereas the length of the longest search path is in $O(\sqrt{n})$.

(5 P.)

Exercise T3 (Planar Point Location - Trapezoidal Map):

Let n be a power of 2, that is, $n = 2^k$ for some $k \in \mathbb{N}$. Sketch a set of segments S of cardinality n and give an insertion order such that the length of the longest path in $D(S)$ is $O(n)$ whereas the length of the longest search path is in $O(\log n)$.

(5 P.)

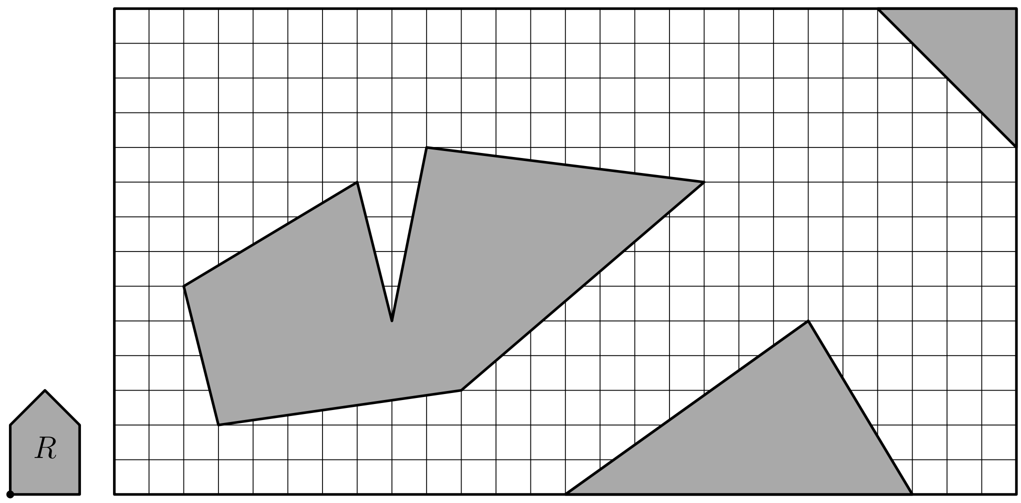
Exercise T4 (Randomized Incremental Construction - Backward Analysis):

Consider an algorithm that constructs a binary search tree T via randomized incremental construction. That is, the algorithm selects a random insertion order in the beginning and then constructs the tree by inserting the elements without any balancing operations. Given a set of elements S of cardinality n and another query point p. Show that the expected query time to located p in T is $O(\log n)$.
Hint: Consider the path in T to p already during construction. What is the probability that the length of the path increases due to an insertion of an element ?

(10 P.)

Exercise T5 (Exact Robot Motion Planning):

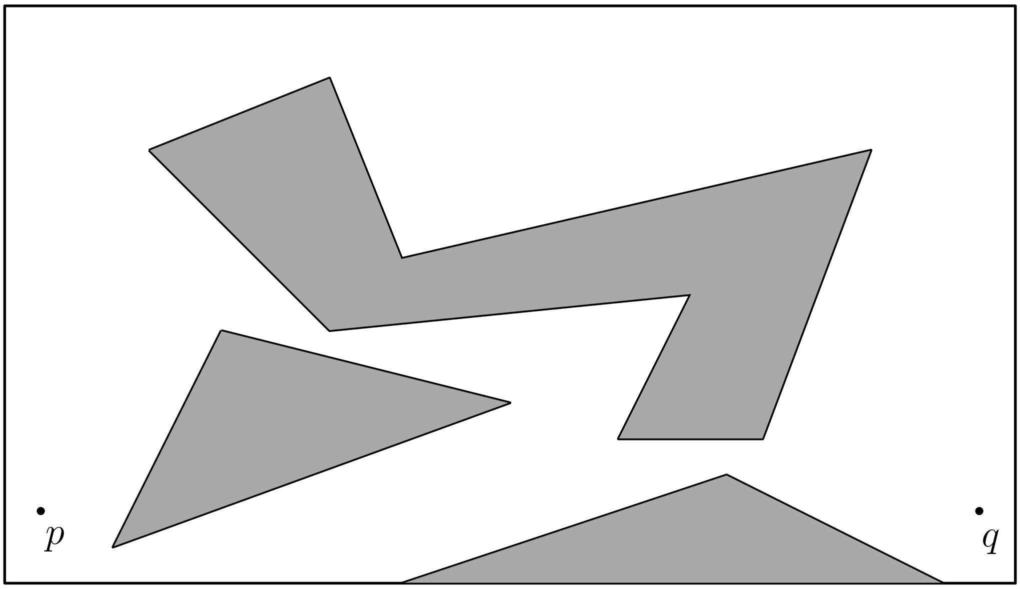
a) Given the workspace and translating robot R with reference point as indicated below, draw the subdivision of the configuration space of R.



(10 P.)

Exercise T6 (Road Maps):

a) Give the road map as defined in the lecture of the configuration space given below.



b) Indicate the path from p to q that would be found by the PRM algorithm.

(4+2 P.)