Exercise 1 (Independent Set): Let $G = (V, E)$ be a graph. A set of vertices $I \subseteq V$ is called independent if for all $u, v \in I$: $\{u, v\} \notin E$. The Independent Set Problem (IS) asks for an independent set of maximum cardinality. (1) Show that $C$ is a Vertex Cover of $G$ iff $I = V \setminus C$ is an independent set. (2) Prove that IS is NP-Complete. 

Exercise 2 (Vertex Cover): We have seen in the lecture that the (minimum) Vertex Cover Problem (VC) is in general NP-complete. Show however that, when the input graph is a tree, VC can be solved in polynomial time.

Exercise 3 (Vertex Cover): We consider two greedy algorithms for the Vertex Cover problem in a graph $G = (V, E)$:

**Greedy 1:**

$C := \emptyset$

while $E \neq \emptyset$
do

Choose an edge $e \in E$ and choose a vertex $v$ of $e$.

$C := C \cup \{v\}$

$E := E \setminus \{e \in E : v \in e\}$

end

return $C$

Show that for both algorithm a constant approximation factor cannot be guaranteed, not even in bipartite graphs.
Greedy 2:

\[ C := \emptyset \]

\[ \text{while } E \neq \emptyset \text{ do} \]

\[ \quad \text{Choose a vertex with maximal degree in the current graph.} \]

\[ \quad C := C \cup \{ v \} \]

\[ \quad E := E \setminus \{ e \in E : v \in e \} \]

\[ \text{end} \]

\[ \text{return } C \]

Exercise 4 (Diameter of Sets of Points): Let \( P \) be a set of \( n \) points in \( \mathbb{R}^d \) (assume \( d \) is constant). The diameter (\( \Lambda \)) of \( P \) is a pair of points \( p, q \in P \) that realizes the maximum distance between any two points of \( P \) (two points that are furthest apart). The diameter of \( P \) can trivially be computed in \( O(n^2) \) time (assuming that the distance between points can be computed in \( O(1) \)). However, show that in \( O(n) \) time a 2-approximation of the diameter can be computed. That is, a number \( \Lambda' \) such that:

\[ \Lambda' \leq \Lambda \leq 2 \cdot \Lambda'. \]

10pts.