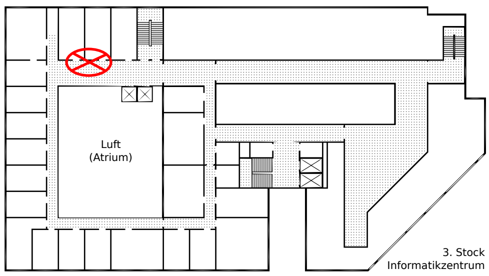
**Algorithms Group** **Summer '14**
**Departement of Computer Science - IBR**
**TU Braunschweig**

Michael Hemmer

# Geometric Algorithms
# Exercise 1
# April 15, 2014

This sheet is comprised of several exercise. Most of them are theoretical but some are also applied, i.e., coding tasks. The solutions should be handed in by **April 22.**. This can be done either at the beginning of the lecture or by placing them in the appropriate box of the exercise locker, see floor plan on the right. Applied exercises should be handed in via email to `hemmer@ibr.cs.tu-bs.de`. Please mark those exercises that you would like to present during the tutorial.




In order to achieve the *"Studienleistung"*, you must have presented at least three exercises until the end of the term. Moreover, you must pass the midterm exam at the beginning of June.

### Exercise P0 (Registration):

Write an email to `hemmer@ibr.cs.tu-bs.de` containing the following information.

- name
- matriculation number
- degree program

### Exercise T1 (Convex Combination of Disks):

Let $D_1 = D(q_1, r_1)$ and $D_2 = D(q_2, r_2)$ be two disks such that $D_1 \cap D_2 \neq \emptyset$. Define disk $D_\lambda$ for $\lambda \in [0, 1]$ via function:

$$f_\lambda(p) = \lambda f_1(p) + (1 - \lambda) f_2(p) \leq 1,$$

where

$$f_i(p) = \frac{1}{r_i^2} \cdot ||p - q_i||^2.$$

Show $\forall \lambda \in [0, 1]$:

- $D_0 \cap D_1 \subset D_\lambda$
- $\partial D_0 \cap \partial D_1 \subset \partial D_\lambda$
- $D_\lambda$ is a disk
- the radius $r_\lambda$ of $D_\lambda$ is smaller than $\max(r_1, r_2)$

**Exercise T2 (Degree of Freedom):**

Show that a disk in $\mathbb{R}^2$ is uniquely defined by three non collinear points on its boundary.
Hint: Obtain linear equations by symbolic evaluation of $f(p)$.

**Exercise T3 (Complexity of RIC Algorithm for Smallest Enclosing Ball in $\mathbb{R}^d$):**

Show by induction that

$$t_j(n) = nj! \sum_{k=1}^{j} \frac{1}{k!}$$

and

$$s_j(n) \leq (1 + H_n)^j,$$

where $H_n = \sum_{k=1}^{n} \frac{1}{k}$ and $s_j$, $t_y$ as defined in the lecture.

**Exercise T4 (Smallest Enclosing Axis Aligned Bounding Box (AABB) in $\mathbb{R}^2$):**

a) Formulate the randomized incremental algorithm for smallest AABB in $\mathbb{R}^2$.
b) Give a complexity analysis of this algorithm.
c) This is also an $O(n)$ algorithm, but why would one consider it as inefficient ?

**Exercise P1 (Implementation):**
Implement algorithm for smallest enclosing disk in $\mathbb{R}^2$.