Abteilung Algorithmik
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Online-Algorithms
3rd Homework Assignment, 14. May 2013

Due on 05. June 2013 until 13:00 in the box in front of IZ 338
Don't forget to label each sheet with your name!

Exercise 1 (Online Single Machine Scheduling): In the tutorial on May 14 we considered the FCFS algorithm for the Online Single Machine Scheduling Problem. Consider another algorithm for this problem, the Shortest-Available-Job-First (SAJF) algorithm. As FCFS, it maintains a queue to contain all the jobs that have arrived but have not been executed. For SAJF, the jobs in the queue are listed according to nondecreasing p_j. Similar to FCFS, SAJF has the following property: when the machine becomes idle after completing the execution of a job, the first job in the queue is assigned to the machine for execution. When a new job arrives, it is inserted into the correct position in the queue.

a) SAJF is n-competitive for any instance I of n jobs (as is FCFS). Show that this competitive ratio is tight, that is, give an instance for which SAJF actually achieves this competitive ratio.

b) Let $\mu_{SAJF}(I)$ and $\mu_{FCFS}(I)$ be the maximum completion times (also called *makespans*) of any instance I in SAJF and FCFS, respectively. Show $\mu_{SAJF}(I) = \mu_{FCFS}(I)$

(5+10 points)

Exercise 2 (Heuristic for Robot Navigation):
Consider the robot navigation problem "looking around a corner", in which we want to find the length of each step on the trajectory for the robot. The robot follows a polygonal path that is inscribed in a semicircle of diameter d (spanned by the starting point of the trajectory and the corner). We want to determine the points where the robot will stop to perform the scan. For this, we can use the following equation presented in the lecture:

$$x_{i+1} = c \cdot (1 + d_i) - (1 + i) - \sum_{j=1}^{i} x_j. \tag{1}$$

In the jth step the robot moves along a chord of length x_j. This chord is visible from the corner at an angle of $\varphi_j = \arcsin \frac{x_j}{d}$. The chord connecting the starting point with the ith position of the robot has a length of

$$d_i = d \cdot \sin \sum_{j=1}^{i} \varphi_j. \tag{2}$$

Using the equation 2 for d_i in equation 1, the result is a recursion for the step length. For a given $c > 1$, the first value is chosen as $x_1 = c - 1$. As soon as the computed length of the last step is smaller than zero, i.e., $x_j < 0$, the computation can be aborted.

a) Why is it necessary that the robot moves all the way to the corner?

b) Why is the strategy *"move directly to the corner"* not competitive?

c) How is it possible to decide whether the strategy with the computed step length will arrive at the other side of the semicircle?

d) Consider a diameter of $d = 40$ and for the constant c the two values 2.0015 and 2.0016.
Is it possible to achieve a competive factor of c in either case?

For your convenience, we provide the following intermediate results:
For $c = 2.0015$ we have $\sum_{j=1}^{19} x_j = 61.7568$ and $\sum_{j=1}^{19} \varphi_j = 1.5541$.
For $c = 2.0016$ we have $\sum_{j=1}^{18} x_j = 61.2617$ and $\sum_{j=1}^{18} \varphi_j = 1.5432$.

(5+10+10+20 points)