Abteilung Algorithmik
Institut für Betriebssysteme und Rechnerverbund **SS 13**
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Online-Algorithms
1st Homework Assignment, 16. April 2013

Due on 2. May 2013 until 13:00 in the box in front of IZ 338
Don't forget to label each sheet with your name!

Exercise 1 (Marking Algorithms):
Given: $k = 4$ pages, numbered 1,2,3,4, and a cache of size 3. In the beginning, the pages 1,2,3 are residing in the cache. Consider the sequence $\sigma = (41234)$ of requests.

Wanted: For the marking algorithm, compare the strategies FIFO (first in - first out) and LFU (least frequently used).

- Which strategy generates more page faults for the sequence σ given above?
- How many page faults occur for FIFO and LFU, respectively?
- For each step, indicate which pages are residing in the cache.

(20 points)

Exercise 2 (Bahncard Problem):
During the tutorials we presented the online algorithm SUM for the *Bahncard Problem*. This algorithm has a competitive ratio of $2 - \beta$. Construct a sequence σ that reaches this competitive ratio (i.e., a worst-case example).

(20 points)

Exercise 3 (Paging):
Prove the following statement:

Let ALG be any marking algorithm as presented in the lecture with a cache of size k, and let OPT be an optimal offline-algorithm with a cache of size $h \leq k$. Then ALG is $\frac{k}{k-h+1}$-competitive.

Hint: Analogous to the problem in the lecture: consider a decomposition of a sequence σ into phases of length k.

(20 points)