Exercises

Exercise 1 (Marking Algorithms):
Given: \( k = 4 \) pages, numbered 1,2,3,4, and a cache of size 3. In the beginning, the pages 1,2,3 are residing in the cache. Consider the sequence \( \sigma = (41234) \) of requests.

Wanted: For the marking algorithm, compare the strategies FIFO (first in - first out) and LFU (least frequently used).

- Which strategy generates more page faults for the sequence \( \sigma \) given above?
- How many page faults occur for FIFO and LFU, respectively?
- For each step, indicate which pages are residing in the cache.

(20 points)

Exercise 2 (Bahncard Problem):
During the tutorials we presented the online algorithm SUM for the Bahncard Problem. This algorithm has a competitive ratio of \( 2 - \beta \). Construct a sequence \( \sigma \) that reaches this competitive ratio (i.e., a worst-case example).

(20 points)

Exercise 3 (Paging):
Prove the following statement:

Let \( ALG \) be any marking algorithm as presented in the lecture with a cache of size \( k \), and let \( OPT \) be an optimal offline-algorithm with a cache of size \( h \leq k \). Then \( ALG \) is \( \frac{k}{k-h+1} \)-competitive.

Hint: Analogous to the problem in the lecture: consider a decomposition of a sequence \( \sigma \) into phases of length \( k \).

(20 points)